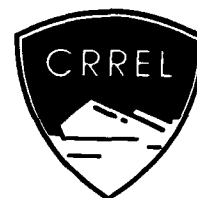


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Historical Perspectives in Frost Heave Research

The Early Works of S. Taber and G. Beskow

Patrick B. Black and Mark J. Hardenberg, Editors

December 1991

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**U.S. Army Corps
of Engineers**
Cold Regions Research &
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PREFACE

Sixty years have passed since Gunnar Beskow and Stephen Taber published their pioneering works on the mechanics of frost heave. Since then, there has been periodic interest in attempting to understand the principles underlying moisture movement in freezing soil. Much of that work appears to be an extension or duplication of Beskow's and Taber's original work. With that in mind, these papers and monograph are presented together for the benefit of those working or starting to work on the mechanics of frost heave.

This report contains a historical perspective of frost heave research conducted in North America and Europe since the early 1900s, Stephen Taber's two papers entitled *Frost Heaving* and *The Mechanics of Frost Heaving* published in the *Journal of Geology*, and J.O. Osterberg's translation of Gunnar Beskow's monograph, *Soil Freezing and Frost Heaving With Special Application To Roads And Railroads*.

In the interests of creating a uniform document, the text of Taber's and Beskow's papers was retyped and this report was produced on CRREL's electronic publishing system. We have tried to reproduce these papers exactly without introducing new errors, a perhaps impossible task. Some minor stylistic changes were made during the process of retyping and reformatting, but, on the whole, these works are as they were when they were originally published.

We thank the University of Chicago Press for granting us permission to reprint Taber's papers and R.J. Krizek of Northwestern University for allowing us to reprint the translation of the work of Beskow. B. Sivertsen of the *Journal of Geology* was particularly helpful, and J.O. Osterberg willingly gave additional insight into the history of his translation. Also appreciated are the careful review and suggestions offered by E. Chamberlain and R. Berg, both of CRREL. In addition, P. Bosworth and D. Valliere of CRREL are acknowledged for their efforts in typing and creating new pages for Taber's and Beskow's papers.

P.B. Black
Mark J. Hardenberg
Hanover, N.H.
Summer 1991

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Historical Perspective of Frost Heave Research

P. B. Black

**U.S. Army Cold Regions Research and Engineering Laboratory
Hanover, New Hampshire**

Historical Perspective of Frost Heave Research

PATRICK B. BLACK

INTRODUCTION

Stages in the evolution of current concepts of soil freezing and frost heaving overlap in time, partly because of both professional isolation and the divergent interests of the people involved. They include geologists, geotechnical engineers, soil physicists, physical chemists, chemical engineers, applied mathematicians and metallurgists. It is instructive to highlight some of the progress that has been made in our understanding since the time of Beskow (1935) and Taber (1929, 1930) and note how important their work remains today.

DISCUSSION

The earliest work meriting mention was that of soil physicist G. J. Bouyoucos (1920) of Michigan State University, who discovered that water in soils does not all freeze at one temperature. Next came the work of geologist S. Taber (University of South Carolina) and G. Beskow (Swedish Road Institute), the first to think clearly about the process of frost heaving and to conduct appropriate experiments. Taber completely discredited the general assumption that frost heave is caused by expansion of freezing water. He suggested a plausible mechanism for the growth of ice lenses in soil that, as far as it went, was correct. Beskow reinforced and greatly extended Taber's work and produced his classic monograph on the mechanics of frost heaving. It is because of their significance that the works of these two researchers are still used today and are presented together in this document.

At about the same time as Taber and Beskow, R.K. Schofield (1935), a soil physical chemist at the Rothamstead Agricultural Experiment Station in England, proposed the "pF scale" for the Gibb's free energy of water (specifically, H_2O) in moist soils, giving an equation for computing pF from the measured freezing point depression. Soil scientists N.E. Edlefsen and A.B.C. Anderson (University of California, Davis), clearly upset by

Schofield's superficial treatment, proposed a rigorous analysis of the thermodynamics of soil moisture, while other soil scientists were busy with laboratory evaluations of pF. After they corrected suspected typographical errors in Schofield's equations, Edlefsen and Anderson's (1943) exhaustive analysis revealed unresolved ambiguities in Schofield's vague interpretation of freezing point measurements, which could not be described as being a result of either the presence of impurities or a difference in pressure between the water and ice phases. Despite initially encouraging results, the laboratory studies found ambiguities as well, leading soil physicists to abandon soil freezing studies.

During the 1950s, geotechnical engineers at the U.S. Arctic Construction and Facilities Engineering Laboratories accumulated the results of systematic empirical heaving tests with many kinds of soils (Linell and Kaplar 1959), using apparatus based on Taber's classic demonstrations. Unfortunately, they were unable to correlate frost heave with other engineering properties that were significantly better than the commonly used simple design criterion of A. Casagrande ([1931] Harvard University) that was based on the percentage of soil particles finer than 0.02 mm. This resulted in spite of Casagrande never intending that his field observations of frost heave in a few silty soils be used as a general specification for frost susceptibility.

In the late 1950s, an interest in the mechanism of frost heaving was reborn. Metallurgists K. Jackson and B. Chalmers (1958) at Harvard proposed a mechanism of heaving based on the kinetics of solidification. They perceived an irreversible process dependent on the freezing of supercooled water. At about the same time, L. Gold ([1957] National Research Council of Canada) proposed a different mechanism based upon the surface tension of the ice/water interface. At the end of the 1950s, L. Cass of the U.S. Army Snow, Ice and Permafrost Research Establishment and soil physicist R.D. Miller (Cornell University) suggested a role for the osmotic properties of diffuse electrical double layers on the grain surfaces (Cass and Miller 1959). All of these

models were in the context of lens growth perceived by Taber.

The 1960s brought about a period of great interest, advancement and premature confidence in our understanding of frost heaving. Miller and his students at Cornell produced a series of experimental observations and theories on the mechanisms involved in soil freezing. Using certain geometric approximations for soil composed of uniform spheres, they showed that, when appropriately married, an osmotic model of the unfrozen film and a surface tension model of the interface at the base of the ice lens, supported by grains, allowed experimental validation of an appropriate form of the Clapeyron equation (Miller et al. 1960). This result seemed to substantiate a quantitative version of Taber's qualitative description of the mechanism of heaving if, but only if, heaving involved the formation of a single ice lens rather than the sequence of lenses normally observed in freezing soils. At the University of Bristol (England), physical chemist D. H. Everett (1961) offered a rigorous analysis of the surface tension model in relation to pore necks between soil particles. This derivation avoided geometric approximations except that it implied an upper limit on the pressure at which an ice lens could grow in a given soil in the manner Taber envisioned. This theoretical basis, coupled with the experimental work on heaving pressures by E. Penner ([1959] National Research Council of Canada), led some to prematurely feel that the mechanics of frost heaving were now explained.

Meanwhile, Miller and his students were exploring other problems. They were able to quantify relationships between the freezing-thawing characteristics of colloidal or colloid-free soils (confined to suppress heaving) and their drying-wetting characteristics. At the same time, they were able to obtain an experimentally determined value (2.20) for the ratio of the surface tension of a water/air interface at 20°C to the surface tension of an ice/water interface near 0°C (Koopmans and Miller 1966). The phenomenon, now known as thermally induced regelation, was anticipated and demonstrated for the case of an isolated grain embedded in ice. The experiment that showed that the particle migrates up a thermal gradient was performed both at Cornell (Romkens and Miller 1973) and at CRREL in conjunction with P. Hoekstra (Hoekstra and Miller 1967). Other work at Cornell (Dirksen and Miller 1966) discovered that freezing-induced redistribution of water in moist soils exceeded, by orders of magnitude, the amount that could be explained by the otherwise successful Phillip and DeVries (1957) theory of thermally induced redistribution. Hoekstra (1966) conducted similar work that advanced our understanding of diffusion processes in the frozen soil and began to incorporate

concepts of irreversible thermodynamics, but the picture was still far from complete.

Reflection on these results led Miller (1977) to suggest that Penner's (1959) "confirmation" of agreement between experimentally measured maximum heaving pressures and theoretical limits of the conditions for growth of a single ice lens was in fact disagreement. He suggested that intrusion of ice into pores below an ice lens would not "anchor" the lens to the soil below as assumed. Instead, ice in a "frozen fringe" beneath an ice lens could move (by regelation) at the same velocity as the lens in a process he identified as "secondary heaving" (fringe present) as opposed to "primary heaving" (fringe absent). What had previously been regarded as a theoretical maximum for heaving pressure became the transition from primary heaving to secondary heaving as the load to be heaved was increased. This hypothesis was tested and appeared to be confirmed in experiments done at Cornell by G. Loch and Miller (1975).

Research on frost heaving took a side road in the 1970s when R.L. Harlan (1973), a hydrologist from Canada, developed a model for freezing-induced redistribution based on the already discredited Schofield equation. This "hydrodynamic" model was appealing because its premises were so easily adapted to the numerical procedures that had been developed for simulating transient flow processes in unsaturated, unfrozen soils. Despite objections from the Cornell group (Bresler and Miller 1975), other hydrologists and soil physicists continued to develop this model. They transformed it into a model for simulating frost heave only to discover that, to achieve any degree of agreement between experimental results and simulations, a very potent and wholly arbitrary correction function ("impedance factor") had to be introduced and "tuned" for the occasion. Meanwhile, Miller (1973) had proposed his own "capillary sink" mechanism to account for freezing-induced redistribution. Experiments by CRREL snow physicist S. Colbeck (1982, 1985) subsequently confirmed Miller's analysis, but a complete set of defining equations for ice segregation was still needed.

The 1970s also brought about further studies of the general processes involved in heat and mass transfer in soil, both ice-free and frozen. Oil company researchers F.J. Radd and D.H. Oertle (1973) conducted a series of frost heave experiments that appeared to them to demonstrate a Clapeyron type of pressure and temperature relationship between the temperature in the unfrozen soil and frozen soil. Soil scientist P.A.C. Raats ([1975] United States Salinity Laboratory) developed a general set of transport equations for unsaturated, ice-free soil using the framework of irreversible thermodynamics. Canadian soil scientists P.H. Groenevelt and B.D. Kay, following a similar approach, formulated another set of

transport equations (Groenevelt 1974, Groenevelt and Bolt 1969, Groenevelt and Kay 1974, Kay and Groenevelt 1974). All of these relationships were in terms of yet to be determined phenomenological coefficients and only slightly addressed the influence of ice. Miller et al. (1975) approached the problem from a different perspective. Starting with expressions that described the transport of heat and mass through a frozen soil and water system that were in terms of experimentally measurable coefficients, they developed another set of generalized transport equations that were similar to those from irreversible thermodynamics. The phenomenological coefficients in their expressions though were now in terms of commonly accepted soil functions. No further research on frozen soil in terms of irreversible thermodynamics was conducted in the 1970s.

The last half of the 1970s brought about the final development of a model of frost heaving in the secondary mode. As envisioned by Miller (1978), the criterion for the initiation of a new ice lens within the frozen fringe below a growing lens was simply the same criterion for initiation of an air-filled crack in unsaturated ice-free soil used by geotechnical engineers, i.e., the condition for tensile failure of moist soils (Snyder and Miller 1985). This made it possible to compile a complete list of simultaneous equations for the conservation of mass and energy to be satisfied for a freezing front descending through an air-free, solute-free incompressible soil. This model, now known as the "rigid ice model," is hardly ready for use on practical problems. Computational difficulties are unusually formidable and, of course, it is necessary to have, as inputs, a series of functions that describe the relevant properties of a given soil.

The 1980s saw a flurry of activity in frost heaving attributable in part to the economic influence of the proposed natural gas pipeline in Alaska. Many "new" models were developed (Gilpin 1980, Konrad and Morgenstern 1980, Takagi 1980), some of which actually had sound scientific basis, but the problem of predicting the influence of frost heaving still eluded our best efforts. This was also a time of questioning the basic physical principles that were just starting to be accepted. The most fundamental objection to the growing thinking about soil freezing was the relationship between temperature and pressure. The prior work by the Cornell group (Miller et al. 1960) showed them to have a one-to-one correspondence that could be explained by the Clapeyron equation for the static case of no water flow. The work of Radd and Oertle (1973) appeared to show a lack of direct correspondence between temperature and pressure in the dynamic case that contained moving water. Physical chemists T. Førland and S. Ratkje (1980) of the Norwegian Institute of Technology

(Trondheim) renewed the belief that irreversible thermodynamics, and its rejection of the suitability of the Clapeyron equation, must be applied if any hope of solving the phenomena of frost heaving was to be made. Their work, like the prior work of Hoekstra in the 1960s and Raats in the 1970s on irreversible thermodynamics, suffered from even more formidable mathematical and experimental difficulties than the rigid ice model.

As the 1990s arrive, we find that we have no satisfactory explanation for the mechanics of frost heaving. There are many models that purport to explain it, but they all suffer from the common fault of little or no experimental verification. The mood appears to be one of emphasizing the experimental nature of research on frost heaving, while reviewing the various assumptions and approaches all the models make in order to more intelligently determine how they are alike and differ. This latter aspect is progressing with the work of Y. Nakano (1990) and K. O'Neill (1983), both of CRREL, on defining the general formulation of the balance laws and how each model fits into them. The experimental aspect of frost heaving is being investigated in two ways. First, detailed frost heave experiments are being conducted in laboratories using apparatus very similar to Taber's original setup, while detailed, large-scale tests are being conducted that are reminiscent to the original work of Beskow. Second, detailed laboratory experiments are being conducted and planned to accurately measure moisture movement through frozen soil in response to water content gradients, temperature gradients, chemical potential gradients and hydraulic gradients as is envisioned to take place in the frozen fringe. It is from these carefully measured experimental data that we hope to determine which existing model is correct or whether further model development is required. This approach is exactly as that used by Taber and Beskow, but one that we hope will yield more fruitful results.

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Frost Heaving

Stephen Taber

University of South Carolina
Columbia, South Carolina

Frost Heaving*

STEPHEN TABER

ABSTRACT

This paper describes a laboratory investigation of problems connected with frost heaving. The principles developed have applications in geology, plant physiology, and engineering.

Pressure effects accompanying the freezing of soils are due to the growth of ice crystals and not change in volume. Pressure is developed in the direction of crystal growth, which is determined chiefly by the direction of cooling. Heaving is often greater than can be explained by expansion. It is due to the segregation of water as it freezes, more water being drawn up by molecular cohesion. The chief factors controlling segregation and excessive heaving are: size of soil particle, amount of water available, size and percentage of voids, and rate of cooling. The pressure developed is limited by the tensile strength of the water.

More water may be concentrated in the surface soil through ice segregation followed by thawing than in any other way. Segregation causes shrinkage cracks below if the supply of water is limited or the soil very impermeable.

Differential heaving is due chiefly to differences in soil texture and in the amount of available water, but differences in the kind and amount of soil cover are also factors.

INTRODUCTION

Heaving and subsidence of the surface, due to freezing and thawing of soils, have caused much damage in cold climates to hard-surfaced roads, foundations, and also to certain agricultural crops. The uplift of the sur-

face soil has usually been attributed to the change in volume that takes place when water freezes, the expansion being upward because of less resistance in that direction.

Several observers have called attention to the columnar ice crystals that uplift pebbles and clods of earth as evidence of a force which could not be attributed to expansion in volume, one of the earliest and clearest statements having been made by Volger (1854). This evidence, however, has failed to convince most writers, and, so far as I can ascertain, almost no experimental work has been done on the problems connected with frost heaving.

Experiments carried out by me on cold nights during the winter of 1914-15 indicated that the pressure effects accompanying freezing are due to the growth of ice crystals and that excessive heaving is to be explained by the segregation of water as it freezes. This segregation was obtained in clay but not in sand. After publishing brief descriptions of some of these experiments together with certain conclusions (Taber 1916, 1918a,b), I laid the investigation aside temporarily, because of lack of facilities for obtaining and maintaining low temperatures.

In March 1927, suitable low-temperature apparatus was placed at my disposal by Mr. E.W. Allen, district manager of the Frigidaire Corporation, and I began an investigation to determine the factors involved in excessive and in differential frost heaving. I am indebted to my colleagues in the science departments of the University of South Carolina for suggestions and assistance during the progress of the investigation. Since November 1927, the work has been carried on through cooperation with United States Bureau of Public Roads.

The experiments have been conducted in a room having nearly constant temperature. The freezing cabinet, cooled by expansion coils that extend across the top, contains a box of sand, suitably insulated, in which the soil containers are packed so that freezing takes

* Presented in part at the New York meeting of the Geological Society of America, December 28, 1928; published in *Journal of Geology*, vol. 37, 1929.

place from the top down just as in the ground. A thermograph records the temperature of the air in the cabinet, and a thermocouple is used to determine soil temperatures below the surface. In most tests the soils are frozen in cylindrical pasteboard containers, made impervious with paraffin or shellac. At the end of a test, the container is quickly cut away from the frozen soil cylinder, which may then be examined, measured, and photographed.

In some tests gallon containers have been used, but for most purposes the standard quart cartons, 17 cm high and 8.5 cm in diameter, have proved more satisfactory, as a larger number of simultaneous tests can be run.

Since the ice in some soils is formed in part of water drawn from the water-table, this condition was reproduced in experiments. Holes were punched in the bottoms of cartons and covered with filter paper before introducing the soils; then the cartons were set in sand kept saturated with water. Collars fitting snugly around the lower part of the cartons prevented the wet sand from coming in contact with the dry sand in which the tests were packed.

EXCESSIVE FROST HEAVING

The maximum amount of frost heaving that occurs during cold winters in the northern part of the United States is not known, very few accurate observations being available. An uplift of 6 in. has been observed in New York State, but the maximum is probably two or three times that amount.

When water present in the voids of a soil freezes without the introduction of additional material, the amount of heaving is necessarily limited by the change in volume that accompanies freezing; and, since the water-content of the average soil is seldom as much as 50% while the depth of freezing in the United States seldom exceeds 2 or 3 ft, the amount of heaving under these conditions should not be more than 1 or 2 in., for the expansion in volume of water on freezing is only about 10%. Moreover, in most soils having a high water-content a considerable percentage of the water does not freeze.

Surface heaving appreciably in excess of that which would result from the expansion in volume of the water contained in the soil means that additional material must have been introduced as a result of the freezing. Laboratory experiments under favorable conditions give a surface uplift of over 60% of the depth of freezing. Excessive heaving, in the laboratory, is always accompanied by the introduction of additional water and the segregation of some water to form masses or layers of more or less pure ice. Such ice, while usually solid and transparent, has a satiny luster and fibrous structure



Figure 1. Frozen clay cylinder with sand layer in middle. A—frozen clay; B—sand; C—ice; D—un-frozen clay.

because of the orientation of the tiny ice prisms normal to the cooling surface (see Fig. 1, 6, 8, 12, and 18). Similar segregated ice has been found in the ground when excavations have been made in investigating frost heaving (Wyckoff 1918).

FACTORS INFLUENCING FROST HEAVING

In the investigation described in this paper an attempt has been made to determine the effect of different factors on the freezing of water in soils, especially of the factors involved in segregation and excessive heaving, by varying one factor at a time while the others are kept as nearly constant as possible.

SIZE AND PERCENTAGE OF VOIDS IN SOIL

The size of the capillary spaces in soils is an important factor, for it determines the height to which water may be lifted above the water table by surface tension. Other things being equal, the height to which water rises varies inversely as the diameter of the capillaries; in extremely fine material it may be 3 m (10 ft) or more, but most very fine soils have a high percentage of colloids, the effect of which is to decrease permeability and prevent water from rising as high as it otherwise would.

A relatively thin layer of coarse sand will stop the upward movement of water by capillarity and thus prevent the formation of ice layers. This was illustrated by an experiment. A carton with perforated bottom was filled with tightly packed dry clay except for a layer of standard Ottawa sand, 2.5 cm thick, in the center. It was placed in sand kept saturated with water and compacted several times a day until, at the end of four days, it became slightly moist on top; then it was subjected to freezing from the top down for four days. The result is shown in Figure 1. Because of lack of water very little segregated ice formed above the sand, and in the sand there was insufficient ice to bind the grains together. The cylinder was photographed on its side with only part of the carton removed in order to avoid complete

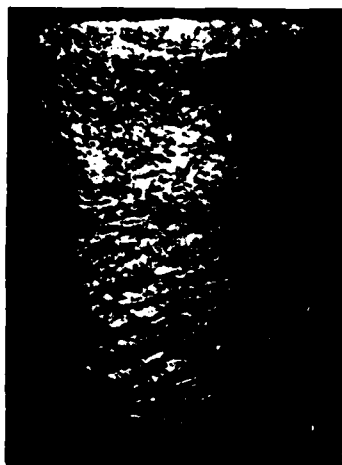


Figure 2. Spiral ice lenses in frozen clay cylinder. Unfrozen clay below.

slumping of the sand. Below the sand and resting on unfrozen clay a layer of practically pure ice, 2.5 cm thick, was formed.

In previous papers attention has been called to the fact that water does not freeze as readily in very small capillary spaces. In Figure 2 the ice lenses extend for about 1 cm obliquely downward into unfrozen clay. Since cooling took place from the top down the isothermal surfaces should be nearly horizontal.

In material having very small pore spaces a portion of the contained water remains unfrozen after long exposure to a temperature several degrees below freezing. The accurate determination of the percentage of water freezing in certain types of soil is a difficult problem. Previous investigators (Bouyoucos 1917, Wintemeyer 1925) have used small samples of soil (usually about 25 gm); freezing has been rapid, heat being abstracted in all directions; and, usually, the sample has been agitated. These conditions are not conducive to ice segregation; therefore it was necessary to modify the methods hitherto used.

The soils were frozen in a modified dilatometer consisting of a glass cylinder with tightly fitting top having a long-drawn-out stem (see Fig. 3). Meta-xylene was placed on top of the soil, and the rise of this liquid in the glass stem gave a measure of the change in volume. Xylene was substituted for the ligroin, previously used, because the loss through vaporization was less during long experiments and it did not congeal at the low tem-



Figure 3. Dilatometers containing frozen clay and frozen lump of clay.

perature required. The apparatus has a capacity of about 300 cc. Because of the size of the ground-glass joint between the cylinder and its top, much trouble was caused by leakage; but, after considerable experimenting, it was found that a perfect seal could be obtained with a little paste made of powdered Castile soap and water.

The apparatus was buried in sand with only the top exposed and the air temperature reduced to between -12° and -15°C . Freezing was continued until the volume became constant. The temperature reached at the bottom of the cylinders was between -5° and -7°C , depending on the length of the tests, which varied from two to five days. The apparatus was tested by freezing 100 cc of boiled distilled water, and during every test a blank was run in order to correct for the decrease in volume of the xylene.

When clean sand, or other material having relatively large voids, is saturated with water the change in volume on freezing indicates that all of the water is converted into ice; but in every test with clay, quartz flour, and other material having a large number of extremely small voids the increase was less than that which would have occurred if all the water froze.

In obtaining accurate results with clay and similar material all air must be carefully eliminated. It is impossible to pack a uniform mixture of clay and water in a cylinder without including some air. When holes were bored in the bottom of a cylinder so that water could enter dry clay from below, the free air was driven up-

ward out of the clay, but a few hours later adsorbed air separated out, forming cracks and bubbles.

Powdered clay was stirred into boiling water to form a thin mixture which was poured into a cylinder. Excess water was removed by siphoning and evaporation, the material being jarred and tamped occasionally to prevent development of shrinkage cracks. On freezing, segregated ice layers were formed in the upper part of the clay, but in the lower part shrinkage cracks ranging in width up to 3 mm developed. Since air might have been absorbed during evaporation, a similar test was run in which the excess water was removed by evaporation under a vacuum with sulfuric acid as absorbent. Freezing resulted in no cracks, but the lower part of the clay shrank away from the glass (see A in Fig. 3). This indicates that a vacuum may be formed by the shrinkage of clay due to removal of water to form ice masses during freezing. The percent of water present was determined by drying the clay after the close of the experiment.

A cylinder weighing 255 gm, cut from a lump of somewhat indurated Cretaceous white clay containing 20.4% water as it came from the mine, was placed in xylene in a dilatometer and cooled as before. A layer of fibrous ice, 0.5–1.0 cm in thickness, formed on top of the clay, but very little ice was visible elsewhere; shrinkage cracks developed in the lower part (see B in Fig. 3).

Another clay cylinder from which half the water had been removed by evaporation gave similar results except that the ice layer was thinner. It was necessary to delay freezing until xylene had displaced the air which partly filled the voids in the clay.

In all of these tests the change in volume indicated that for each 100 gm of dry clay about 6 gm of water failed to freeze. The amount of water present seems to make little or no difference in the results, but definite conclusions should not be drawn without more data.

In tests made on large fragments of porous porcelain saturated with water about seven parts of water failed to freeze for each one hundred parts of the solid present. The failure of part of the water to freeze is determined by pore space rather than size of particle, for the pieces of porous porcelain behave in other freezing tests like large particles. The water that does not freeze is probably adsorbed water, and adsorption is a surface phenomenon.

SIZE OF SOIL PARTICLE

Size of the soil particle is one of the most important factors controlling segregation of water during freezing, and the effects are clearly distinguishable from those due to size and amount of pore space. Early in the investigation it was observed that excessive frost heav-

Figure 4. Quartz flour passing 200-mesh sieve, showing slight segregation of ice near top; lower part of cylinder unfrozen.



ing occurred on clays but not on clean sands. Even when the sand is so fine that water will rise in it to a height of 20 cm or more, segregation and excessive heaving do not occur.

In an attempt to ascertain the maximum size of particle that would permit segregation, clean quartz was ground in a ball mill and sized by screening. The quartz dust passing a sieve with 200 wires to the linear inch (maximum particle size about 0.07 mm in diameter) when tested under the most favorable conditions gave only the faintest evidence of segregation (see Fig. 4). After regrinding in an agate mortar and bolting through silk, segregation was slightly increased. Microscopic examination showed that the average diameter of the quartz particles was probably between 6 and 10 microns.

Because of the difficulty of grinding quartz fine enough and sizing it accurately, chemically precipitated crystals of substances with behavior similar to quartz were substituted in some of the tests. Precipitated barium sulphate,* very uniform in size, having an average particle diameter of about 2 microns and a maximum diameter of 3 microns, gave well-defined segregation under favorable conditions of cooling (see Fig. 5), but no segregation under unfavorable conditions. In materials having a particle size of about 1 micron or less segregation took place without difficulty. The results obtained with lithopone, which has a particle size of

* The barium sulphate used in this investigation and also the lithopone (71% BaSO₄ and 29% ZnS) and kadox (ZnO) were obtained from the research laboratories of the Du Pont de Nemours Co. through the courtesy of Mr. Marion L. Hanahan.



Figure 5. Barium sulphate with ice layer at lower limit of freezing; faint bands of segregated ice above. Iron weight on top.

approximately $\frac{1}{2}$ micron, and with kadox, having a probable particle size of $\frac{1}{4}$ micron or less, are shown in A and B of Figure 6.

All of the tests were carried out under exactly the same conditions. The dry material was tightly packed in cartons, having perforated bottoms covered with filter paper, which were then stood in sand kept saturated with water. As the water rises by capillarity through very fine



Figure 6. Frozen cylinders of (A) lithopone, with particle size of $\frac{1}{2}$ micron; and (B) kadox, with particle size of $\frac{1}{4}$ micron. Note layer of pure ice 5 cm in thickness.

material, driving the air upward, the small particles are rearranged and more closely compacted so that the wet material below becomes separated from the dry material above by an air space large enough to stop the ascent of the water. Therefore, it is necessary to tamp the material occasionally with a heavy iron cylinder, and it takes several days for the water to reach the top. During the process of freezing the water ascends much more rapidly or it would not be possible to build up such thick layers of ice.

The sedimentary clay used in these tests consists largely of particles having apparent diameters of between 1 and $\frac{1}{2}$ micron, but it is very difficult to distinguish the ultimate particles from aggregates. Part of the clay is much finer. A few flakes of mica ranging up to 0.1 mm are present, and more rarely quartz grains having diameters up to 0.01 mm.

Tests were run on different mixtures of pure clay and standard Ottawa sand. This sand is very uniform, the grains having an average diameter of about 0.7 mm, and when well packed has 32.9% voids. Mixtures containing slightly less than 20% clay by weight would have just enough clay to fill the voids between the sand grains so that the size of the pore spaces would be the same as in pure clay. On freezing no segregated ice could be seen in mixtures containing less than 30% clay, and even with 50% clay the amount of segregation is small as compared with tests made on pure clay. The results obtained by freezing sand-clay mixtures containing 50, 40, and 30% clay, respectively, are shown in Figure 7.

The tests were prepared by weighing and thoroughly mixing the dry materials which were then tightly packed in cartons with perforated bottoms. The cartons were stood in sand kept saturated with water until the mixtures became wet on top; then they were removed and



Figure 7. Frozen cylinders containing (A) 50%, (B) 40%, and (C) 30% clay mixed with sand.

the material further compacted by tamping and evaporation. When shrinkage was nearly complete the cartons were again placed in the wet sand, and, after the mixtures were wet on top, they were frozen from the top down.

The influence of large particles in preventing segregation during freezing is probably to be explained as follows:

During the growth of a thick ice layer in soil the freezing isotherm does not move downward; as water, drawn up through the capillaries, crosses this isothermal surface it is converted into ice, and the overlying material is uplifted through a distance equal to the thickness of the ice. In order to maintain the position of the freezing isotherm relative to the unfrozen soil, the heat conducted away through the frozen material must balance the heat conducted up through the unfrozen material together with the heat brought up by the ascending water and the heat set free in converting water into ice.

The freezing isotherm cannot advance as rapidly in water as in the minerals found in soils, for it is a poorer conductor of heat, has a higher specific heat, and also much heat has to be removed in converting water into ice. Hence, if ice crystals in growing downward come in contact with the top of a large soil particle and begin to surround it so that the temperature at the top of the particle drops below freezing, then the temperature of the bottom of that particle will reach the freezing point before the water with which it is in contact. Therefore, freezing takes place in part outward from the surfaces of large mineral particles that are in contact with water, and not merely through the downward growth of ice crystals as in pure water.

When a growing ice crystal closely approaches a soil particle, the water separating them is gradually reduced to a very thin film and further growth of the crystal in this direction can take place only as molecules of water enter this film. If the soil particle is relatively small so that the molecules do not have far to travel through the film, and if growth is relatively slow, so that they have time to enter between the ice and the particle, then the growing crystal will exclude the particle; but if the particle is relatively large and if freezing is relatively rapid, the particle is gradually surrounded by the ice. To build up a layer of ice, which consists of many prismatic crystals, the capillaries supplying the water must be closely spaced.

The space occupied by a horizontal glass rod buried in clay was elongated in a vertical direction to form a large open space by the growth of an ice layer. Even the hole left by the removal of a rod was similarly elongated (see Fig. 8).

Another experiment affords confirmatory evidence in support of the theory outlined above. A porous bat-



Figure 8. Frozen clay cylinder, showing elongation of hole by growth of ice layer.

tery cell was placed bottom side up in a shallow vessel of water and surrounded with insulating material so that it could be cooled from the top down. When the temperature of the cell was above freezing and the air in contact with the exposed upper surface of the cell only a little below freezing, a layer of fibrous ice was formed on top of the cell; but when the air temperature was low, so that the freezing isotherm passed rapidly down into the cell, very little ice formed on its surfaces, and the cell did not break.

The battery cell is similar to clay in having closely spaced capillaries through which water may be fed up to growing ice crystals and in which only part of the water freezes; but it is similar to coarse material in preventing the formation of ice layers within. The failure to break can hardly be attributed to the greater tensile strength of the cell, for, as will be shown later, the pressure developed by growing ice crystals may be very great.

Porous brick, cement, and stone even when saturated with water usually offer considerable resistance to fracture at low temperatures, especially if heat is conducted away in only one direction. This is probably because they are so thoroughly fused or cemented together that they behave as a single solid and not as an aggregate of small discrete particles. If flaws, cracks, or large cavities are present they may, under favorable

conditions, be enlarged. Repeated freezing and thawing especially when cooling takes place from all directions, tends gradually to cause disintegration.

WATER-CONTENT OF SOIL

The amount of heaving that takes place when soils freeze is limited by the supply of available water; not merely the water present in soils before freezing, but also the water that can be drawn from points below the depth of freezing. The effect of moisture content on segregation was determined through a series of tests on samples prepared with different percentages of water.

Dry powdered clay was packed in cartons with perforated bottoms and weighed. The cartons were then stood in wet sand until the clay was saturated, after which the excess water was removed by slow evaporation together with compression and tamping. This method prevented the development of shrinkage cracks. The cartons were weighed daily until their weights indicated that the desired moisture content had been reached, when they were sealed, top and bottom, with paraffin to prevent further loss. The clay cylinders were about 15 cm high and 8.5 cm in diameter. The tests were buried in sand and frozen from the top down, after which samples were taken from the bottom of each cylinder and the moisture content determined. The results are shown in Table 1.

Table 1. Percentage of water at bottom of clay cylinders.

Test no.	Before freezing	After freezing
1	29.0	20.8
2	25.8	19.9
3	24.2	18.3
4	23.2	18.3
5	19.9	16.8
6	16.8	12.3
7	9.5	7.7

The small irregularities shown in the table may be attributed to slight differences in the rate of freezing and in the height of the cylinders as well as the difficulty of obtaining uniform samples. Samples from near the tops of the cylinders were useless for comparison because of the presence of segregated ice layers in tests 1, 2, 3, and 4. These ice layers had a maximum thickness of 3 mm in test 1, and were somewhat smaller in the other three. Tests 5, 6, and 7 showed no visible evidence of the segregation revealed by the moisture determinations. Proof that the segregation was due entirely to freezing,

and not to previous treatment, was obtained by freezing check samples with the ends of the clay cylinders reversed.

A cylinder of clay, cut from a lump at the mine and imbedded in paraffin without loss of moisture, was tested in the same way. Samples taken after freezing showed a moisture content at top and bottom of 34.76 and 19.06%, respectively.

When freezing tests were made on closed systems from which air and additional water were excluded, the clay reabsorbed the water from the ice layers soon after melting, but in tests where air and water were not excluded water often stood on top of the clay after thawing.

Another experiment sheds some light on the movement of ground-water during freezing. A cylinder 7 cm in diameter and 15 cm high, cut from a lump of somewhat indurated clay, was placed in a carton with perforated bottom and surrounded with paraffin. The bottom of the carton was kept in saturated sand while the clay was being frozen from the top down. As obtained from the mine the clay contained 20.46% water. When the cylinder had frozen to a depth of 11 cm, with heaving of 4 cm (see C in Fig. 11), a sample of frozen clay was obtained 8 cm from the top between two ice layers, and another was taken in the unfrozen clay about 7 cm above the bottom. The moisture content of these samples was 20.68 and 20.64%, respectively.

The thoroughly consolidated clay used in these experiments has extremely small capillaries and the colloidal content is relatively low; therefore, water would rise higher above the water-table than in most soils. While the upward movement of water during freezing is very slow, there must be considerable frictional resistance to its passage, so that, other conditions being equal, frost heaving decreases as the depth of the water-table increases. However, the effect of a low water-table could be in part compensated by slower cooling.

RATE OF COOLING

The formation of segregated ice layers during the freezing of soils is favored by a very slow lowering of the freezing isotherm; therefore, the most favorable places for the development of ice layers are close to the surface and near the lower limit of frost penetration. Needle ice, or frost columns, develop at the surface of moist, clayey soils when the temperature of the ground immediately below the surface remains above freezing, while the air temperature is below freezing. They do not form when previously chilled soils are rapidly frozen during a sudden drop in temperature.

In material barely fine enough to permit segregation, such as the barium sulphate previously described, well-defined layers of ice were obtained only near the top and

at the bottom of the frozen material. The absence of well-defined ice layers near the top of the barium sulphate shown in Figure 5 was due to rapid conduction of heat by a 4-lb iron weight that rested on it during the experiment. The development of ice layers was entirely prevented by packing the carton of barium sulphate, together with the wet sand in which it stood, in ice for eighteen hours to bring the system to 0°C , and then transferring it quickly to the sandbox, which had been brought to approximately the same temperature. The freezing isotherm moved rapidly downward as the air temperature was about -15°C . Pure clay, when treated in this way, was not appreciably affected, and ice layers developed as usual.

The rapidity with which soil temperature changes depends on many factors, the most important being the temperature gradient. Dark-colored soils absorb and radiate heat more rapidly than light-colored soils. Other things being equal, the rate of cooling is determined by the thermal conductivities and specific heats of the different materials present, the former being most important. Soils containing much organic matter change temperature more slowly than soils made up chiefly of minerals. The soil minerals differ from one another little in their thermal properties, but they differ greatly from air and water, the other constituents of soils. Therefore, at temperatures above freezing, the rate of cooling of mineral soils is determined chiefly by the porosity and the water content.

When the soil temperature drops below freezing, another factor helps to determine the rate of cooling, i.e., the amount of water converted into ice. In sand this is limited to the interstitial water, but in clay much of the ice may be formed of water drawn from below during the freezing process. The difference in rate of cooling caused by difference in soil texture is well illustrated by the graphs shown in Figure 9, which were plotted from data obtained as follows:

Barium sulphate and clay were packed dry in separate containers, which were then buried in insulating material in a constant-temperature room. The temperature at the surface of the material was maintained at 0°C , while at half-hour intervals readings were made to the nearest 0.05°C on the partial immersion thermometers inserted 8 cm below the surface. The clay, containing 58.7% voids, cooled down more rapidly than the barium sulphate with 69% voids and reached a minimum temperature 1° lower.

The voids were next filled with water, which made possible somewhat closer packing, especially of the clay, as it is in large part composed of tabular-shaped particles, and then the experiment was repeated with the thermometers 7 cm below the surface. Both cooled down more rapidly than when the voids were filled with

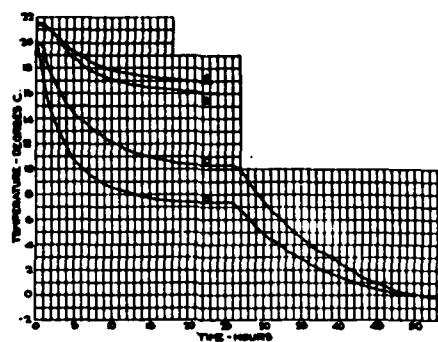


Figure 9. Cooling curves for barium sulphate and clay. A—barium sulphate; voids filled with air. B—clay; voids filled with air. C—barium sulphate; voids filled with water. D—clay; voids filled with water. Surface temperature 0°C for first 26 hours, and then -21.3°C until close of experiment.

air; but the clay, containing 50.5% water by volume, cooled down much more rapidly than the barium sulphate with 67% water and reached a minimum temperature 3° lower.

After the temperature had become constant the surface temperature was lowered to -21.3°C . Under these conditions the clay cooled down more slowly than the barium sulphate, so that the graphs converge instead of diverging as before. This change in the relative rates of cooling was due to the formation of segregated ice near the surface of the clay, while little or no segregation took place in the barium sulphate. In this experiment the cartons were sealed at the bottom with paraffin. If water had been permitted to enter from below, the difference in cooling would have been even more noticeable.

DEPTH OF FREEZING

The amount of surface heaving is not proportional to the depth of freezing, but the latter is a limiting factor. The uplift may vary from zero in certain soils having a low water-content up to 100% of the depth of freezing in instances where a layer of ice is formed at the surface with no freezing at points within the soil. The depth to which freezing extends depends on the minimum temperature reached, the length of the cold spell, the amount of heat present in soil and rock near the surface, the thermal properties of the soil and soil cover, and the changes produced in the composition and thermal properties of the soil because of segregation during freezing.

In laboratory experiments conducted at the same room temperature, the thick layers of segregated ice that mark the lower limit of freezing in clay developed at greater depth when the air in the refrigerator was kept at lower temperature. In similar tests in which the temper-

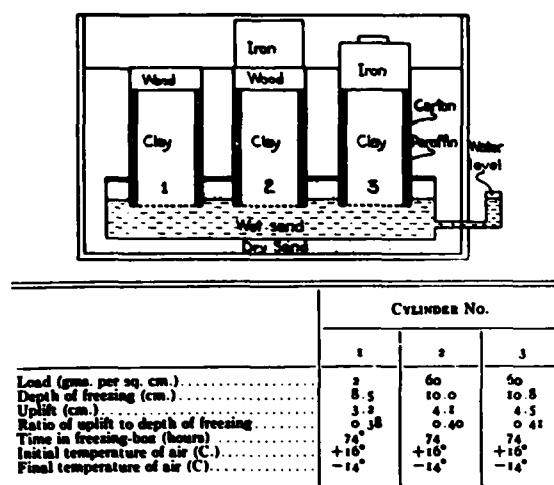


Figure 10. Apparatus for freezing clay cylinders when weighted with different materials.

ature of air in the refrigerator was kept the same, the depth of freezing was greatest when the room temperature was lowest.

The effect of soil cover as a factor in frost heaving was investigated experimentally. Three tests could be conducted simultaneously, and care was taken to secure comparable results. Cylinders 14.25 cm high and 7.8 cm in diameter were cut from moist clay of uniform texture and placed on filter paper in cartons with perforated bottoms. The top and bottom of the cylinders were made smooth and parallel. Molten paraffin was poured between the cylinders and the sides of the cartons after a little clay had been tamped around the base of the cylinders to prevent intrusion of paraffin under them, as this would interfere with the entrance of water.

The cartons were placed in sand kept saturated with water during the experiment. A disk of wood, soaked in paraffin and weighing 0.1 kg, rested on one of the clay cylinders; a similar disk of wood and an iron weight totaling 3.1 kg rested on the second cylinder; while the third was loaded with 3.1 kg of iron (see Fig. 10). The depth of freezing and the ratio of uplift to a depth of freezing were both greatest where the iron, a good conductor of heat, was in contact with the clay, and were least where wood was the only covering.

Similar tests were run on three clay cylinders, one of which supported 7.4 kg of iron, while another with the same load had a wooden disk between the clay and iron, and the third had nothing on it. The depth of freezing and ratio of uplift to depth of freezing were greatest where the iron rested on clay and least where the wood insulated the clay from the cold air (see Fig. 11).

The result of a test conducted in the same way, except that tightly packed clay was substituted for the cylinders cut from undisturbed clay, is shown in Figure 12. The



Figure 11. Clay cylinders frozen (A) under no surface load; (B) under iron weight insulated from clay by wooden disk; and (C) under iron weight in contact with clay.

clay supported a maximum load of 14 kg (7 lb/in.²), but it was necessary to remove some of the iron weights, because of lack of space, as heaving progressed. The uplift was 6.4 cm and the depth of freezing 15 cm, the test being discontinued after 114 hours.

Tests show that at temperatures above freezing concrete pavements are better conductors of heat than clay, even when the latter is saturated with water.



Figure 12. Clay cylinder frozen under 30 pounds of iron; shows excessive segregation of ice.

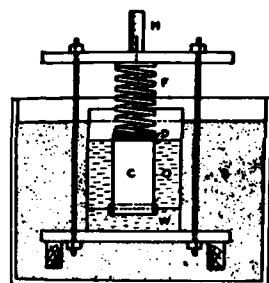
DIRECTION OF COOLING

It is commonly stated that frost heaving is upward because of less resistance to expansion in that direction, but this is not the true explanation. The pressure effects that accompany freezing are not due to change in volume, but to the growth of ice crystals. Crystals perform work in any particular direction by growing in that direction and overcoming the resistance to growth. The upward heaving that accompanies the freezing of soils is due to the growth of ice crystals in a vertical direction, and this is usually determined by the direction in which heat is conducted away most rapidly, and by the availability of the water necessary for growth. The importance of the direction of heat conduction is illustrated in the experiments described below.

Mixtures of white clay and water in different proportions were frozen in thin glass test tubes, half of them being buried in sand so that freezing was from the top down, while the others were exposed so that freezing took place from the sides inward. All of the latter were broken, longitudinal cracks extending the full length of the test tubes. But where frozen from the top down none was broken, for the ice crystals grew only in a vertical direction. Alternating layers of clear ice and frozen clay were formed in the upper parts of the mixtures containing sufficient water, while shrinkage cracks developed below as water was withdrawn to build the ice layers above.

When test tubes filled with clear water are exposed to low temperatures they do not usually break unless there is a constriction in the tube, for the density of water decreases on cooling from 4° to 0°C, causing the coldest water to rise and come in contact with the downward-growing ice crystals. The clay not only prevents convection currents, but it also supplies capillaries through which water reaches the growing ice crystals.

Apparatus was devised in which it was possible to freeze clay cylinders from the top down under heavy vertical pressure and with practically no lateral support (see Fig. 13). A cylinder 6 cm in diameter, cut from hardened clay, was placed in a pasteboard tray with



C—Clay cylinder
O—Viscous oil
W—Sand saturated with water
D—Steel disk
F—Steel spring
M—Graduated scale
S—Dry sand

Figure 13. Apparatus for freezing clay cylinders under heavy vertical pressure and no lateral support.

perforated bottom, and moist clay packed around it to hold it in place, a strip of adhesive tape, 2 cm wide, having been wrapped around the bottom of the cylinder to prevent softening by the rapid absorption of water. The tray was placed in a large carton containing a layer of sand saturated with water, and oil was then poured in until level with the top of the clay. Carton and tray had been made impervious to water and oil by coating with shellac. The oil used was very viscous when cold, thus eliminating convection currents, yet it did not solidify at the lowest temperature reached.* A metal disk resting on the clay cylinder supported a stiff steel spring which was compressed against a board held in place by steel bolts. The amount of heaving and the pressure developed were indicated on a scale by a wire that extended from the top of the clay up through the spring.

When ready for freezing, the apparatus was buried in dry sand, the bolts having been tightened to give an initial pressure of 101 lb. Five hours after cooling was started, the pressure had dropped to 94 lb because of

slow failure of the cylinder, but 15 hours later heaving had restored the pressure to 101 lb. The pressure increased gradually and reached a maximum of 140 lb (2.3 ton/ft²) 76 hours after the experiment was started. The pressure then remained constant as freezing had reached the bottom of the cylinder.

Examination of the clay cylinder (see Fig. 14) showed that heaving was largely due to the formation of horizontal veins of fibrous or columnar ice, ranging up to 0.5 cm in thickness. The large vertical cracks, due to failure under pressure, contained little ice, being occupied chiefly by oil, but a few cracks, less than 2 mm in width, were filled with ice.

In similar tests, conducted under less pressure, the clay cylinders showed no evidence of failure, and their appearance after freezing was essentially the same as the cylinders shown in Figure 11.



Figure 14. Clay cylinder frozen under heavy vertical pressure with no lateral support.

* "The Humble 'D' Pale Oil" used in these experiments was furnished through the courtesy of Mr. J.W. Saybolt of the Standard Oil Company of New Jersey.

In another experiment clay was packed around heavy copper bars standing in a carton with perforated bottom. The carton was placed on wet sand and then the entire apparatus was buried in the sand box for freezing as in previous experiments. When removed a layer of fibrous ice, about a centimeter thick, was found surrounding the copper, the crystals being normal to the copper bars. Ice veins, ranging up to 2 mm in thickness, branched off from the vertical ice layer and sloped steeply upward. Near the bottom of the carton a large mass of ice was formed which ruptured the carton and pushed part of the clay out into the inclosing sand.

The rapid conduction of heat by the copper caused the growth of ice crystals radially outward from its surface, and the pressure of these growing crystals ruptured the carton. In other experiments the cartons were never ruptured by the direct pressure of growing ice crystals because heat conduction, and therefore crystal growth, were limited to a vertical direction.

EXTERNAL PRESSURE

The determination of the maximum force that may be exerted by growing ice crystals is an interesting but difficult problem. Apparatus used in measuring pressures developed during frost heaving is shown in Figure 15. A carton containing a clay cylinder set in paraffin stands in sand kept saturated with water. The spring, supported by the clay, is compressed as heaving takes place, the amount of compression being measured as previously described. In most of the work a much heavier spring than the one shown in the photograph was used, and it was necessary to substitute steel plates

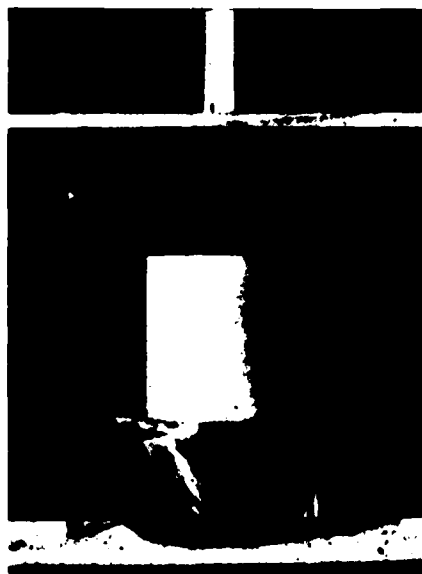


Figure 15. Apparatus for freezing clay cylinders under pressure.

for the boards. While the clay was being frozen the apparatus was buried in dry sand up to the top of the carton.

In tests with this apparatus an indicated pressure of over 155 lb/in.² (11 kg/cm²) was obtained before the sidewall of the carton was broken and pushed out near the bottom by softened clay. But the force exerted by the ice crystals must have been much greater, for in forming each layer of ice it was necessary to pry apart the clay cylinder and push it up against the friction of the paraffin in which it was embedded. The clay, containing 20% water as it came from the mine, had a crushing strength of 200 lb/in.² and a tensile strength of about 75 lb/in.², as determined in standard testing machines. Therefore, the maximum pressure developed by growing ice crystals in these experiments must have been well over 14 ton/ft² (14 kg/cm²).

In several tests with clay cylinders, vertical or steeply inclined planes of weakness developed against which the ice veins terminated abruptly, the growth of the veins causing local contemporaneous faulting along the flaws (see Fig. 16). While these faults were more common in tests made under high pressure, they also occur occasionally in tests made under no surface load. One of the clay cylinders showed an open vertical fissure when it was placed in the refrigerator. On freezing under pressure the fissure was filled with ice and contempora-



Figure 16. Clay cylinder frozen under heavy pressure, showing ice veins and contemporaneous faults.



Figure 17. Clay cylinder frozen under heavy pressure, showing breccia.

neous faulting occurred along it because of the growth of horizontal veins. In some tests the lower parts of the clay cylinders were more or less shattered by the high pressures together with insufficient lateral support, and typical breccias were formed (see Fig. 17).

From these tests it is clear that excessive heaving may occur under surface loads of 150 or 200 lb/in.². The ratio of the uplift to the depth of freezing was much less, however, than that obtained under lower pressures. What would be the effect of much higher pressures?

If the frozen clay cylinders had no lateral support the pressures developed would be limited by the crushing strength of the material. Little is known concerning the crushing strength of ice, but apparently it increases somewhat as the temperature is lowered, and is greater in a direction parallel to the principal axis of the crystals than in other directions. Barnes and Mackay found the crushing strength of St. Lawrence River ice, at or near the freezing point, to be about 363 lb/in.² (Barnes 1928). Tarr and von Engeln (1915), working with small blocks of pond ice at temperatures between 18° and 20°F, obtained a crushing strength of 1,000 lb/in.² in the direction of the principal axis of the ice crystals and 350 lb/in.² in a direction normal to them.

Tests on frozen clay cylinders, made by the writer at temperatures near freezing, show that the crushing

strength increases with the ice content, the maximum obtained being over 700 lb/in.². In all tests compression was applied in a direction parallel to the principal axis of the ice crystals, although in much of the material the ice crystals were too small to be visible to the naked eye.

During the freezing of soils the growing ice crystals are, as a rule, supported laterally by other crystals, so that the crushing strength of the frozen material does not limit the load which may be lifted.

If the crushing strength of the unfrozen material is small, flowage will prevent the development of high pressures unless the material be confined. When water freezes in a confined space the resulting pressure may still be attributed to growing ice crystals, for the crystals develop hydraulic pressure by displacing the water with which they are in contact.

The effect of pressure on ice is slightly to lower the melting point, thus making necessary a lower temperature if crystal growth is to continue. Bridgman (1912) found that a pressure of 1,000 kg/cm² lowered the freezing point to -8.8°C, while at 2,115 kg, where the freezing point is -22.0°, ice I and III are in equilibrium with each other as well as with water.

Extremely high pressure would compact the clay, reduce porosity, and make it less permeable to water, but the entrance of water would be retarded rather than prevented, and, if the water entered slowly, the removal of heat would likewise have to be slow in order that ice segregation take place. High pressure may bring part of the clay particles into such close contact with one another that they conduct heat essentially as though the clay were partly composed of larger particles.

With the apparatus available it was impractical to measure accurately pressures exceeding 150–200 lb/in.², but wet clays were frozen under much higher pressures by sealing them in wrought-iron cylinders, having a bursting strength of over 11,000 lb/in.². At first ice crystals grew radially inward from the exposed sides of the cylinder, but as the pressure increased and the wooden end plugs were slowly displaced, the ice crystals grew in the direction of least resistance, forming ice layers transverse to the axis of the cylinder. In the small iron containers cooling could not be limited to a single direction as in the other tests.

DIFFERENTIAL HEAVING

Simple heaving of the surface, if uniform, causes little cracking of pavements or disturbance of structures; but differential heaving is very destructive. Local differences in soil texture and in the amount of water available are the two major causes of differential heaving. Differences in the kind and amount of soil cover,

such as snow, paving and foundation material, vegetation, etc., are probably of importance. Local differences in insolation, air drainage in hilly regions, and differences in pressure due to static load are minor factors.

The effect of difference in soil texture is easily illustrated experimentally. A carton with perforated bottom was packed with fine sand on one side and clay on the other, by inserting a partition down the middle and gradually withdrawing it as the materials were rammed into place. On placing the carton in saturated sand, water rose slowly to the top displacing the air. Penetration was more rapid in the fine permeable sand, and therefore some water spread laterally into the clay. The carton was buried to the top in dry sand, so that freezing took place from the top down, but the bottom of the carton was kept in saturated sand throughout the experiment. At first only the clay was appreciably heaved, but later the sand also began to rise because unfrozen clay was being forced under it, and at the close of the experiment the bottom of the carton had been pushed out and to one side by the movement of the clay (see Fig. 18).

Another test was made in the same way except that, after saturating the sand and clay with water, the carton was sealed at the bottom by standing it in molten paraffin. On freezing, ice layers, 3-4 mm thick, were formed in the clay, the required water being drawn from the clay in the lower part of the carton and also from the sand. At the close of the experiment, when the carton had been cut away, the lower half of the sand was so nearly dry that it flowed readily, and careful handling was neces-

sary to prevent collapse until the photograph (see Fig. 19) could be taken. The upper part of the sand was thoroughly cemented by ice which filled all voids. Because of differential heaving, the soil cylinder was gradually tilted away from the clay side causing the wall of the carton to be pulled apart by tension.

Even though the soil texture is uniform, differential heaving will occur under favorable conditions where there are differences in the moisture content and especially differences in the position of the water-table relative to the surface. In experiments, the ratio of up-lift to depth of freezing showed little decrease as the height of the soil column above the artificial water-table was increased.

EFFECTS OF THAWING

It has been shown that freezing may result in the concentration of excessive amounts of water in the form of ice at, or near, the surface. Melting of this ice usually takes place from the surface downward, so that removal by downward percolation is largely prevented until after most of the ice has melted. In this way much water may be concentrated under pavements where it could not accumulate as a result of the rapid melting of snow. These conditions are conducive to unequal settling, especially on slopes, and the resulting damage to pavements is, in my opinion, greater than that which may be attributed directly to frost heaving. As the cracks may not develop except under heavy traffic and perhaps long after the freezing occurred, this cause of failure has not been appreciated.



Figure 18. Frozen cylinder, half sand and half clay. Much segregated ice in clay but not in sand.



Figure 19. Differential displacement of cylinder, due to segregation of ice in clay but not in sand. Cavity caused by dislodgement of dry sand.

EFFECTS OF THAWING AND REFREEZING

Experiments have shown that freezing tends to concentrate water near the surface, and that segregation is accentuated by a high water-content. Therefore, prompt refreezing after thawing should result in greater segregation than on the first freezing. This conclusion was confirmed by thawing and refreezing clays which contained too little water to form ice layers on the first freezing.

A sudden drop in temperature after a spring thaw, which has left an excessive amount of water in the surface soil, is a condition favorable for segregation and excessive heaving. Bituminous macadam pavements seem to be especially susceptible at such times, as are also certain agricultural crops, such as wheat.

SHRINKAGE OF CLAYS CAUSED BY ICE SEGREGATION

In every experiment in which no additional water entered the system, the withdrawal of water from the lower part of the container to build up ice layers above caused shrinkage in volume, and, usually, the development of tensional cracks. This shrinkage in volume furnishes an indication of the force with which water is drawn toward the growing ice crystals. Terzaghi (1925) has shown that the forces involved in the shrinkage of clay on evaporation are equal to the external pressure required to produce the same change in volume, and that these forces may be very great.

In soils that are very impermeable because of high colloid content, the formation of segregated ice sets up tension in the material immediately below, and, since the stress is uniformly distributed, vertical cracks tend to develop and form a polygonal pattern (see Fig. 20). The cracks are gradually filled with clear ice, and, as freezing progresses, the ice-filled cracks advance down-

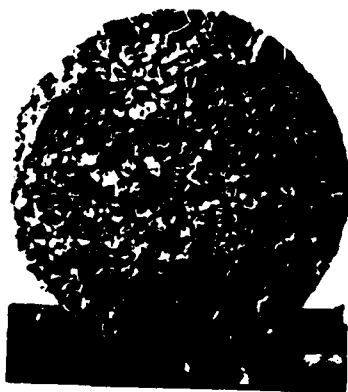


Figure 20. Polygonal structure formed in bentonite on freezing. For description, see text.

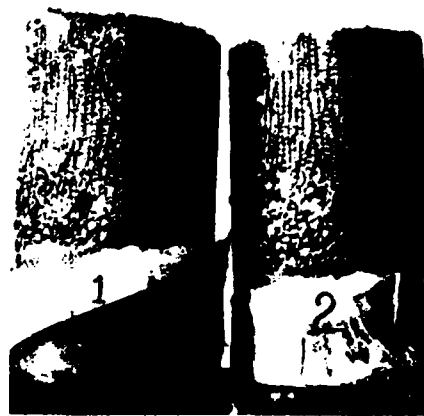


Figure 21. Cellular structure formed in bentonite on freezing. For description see text.

ward to form a columnar structure, and this combined with the usual horizontal ice layers results in the cellular structure shown in Figure 21. The clay forming the interior of the cells does not freeze easily, and therefore the divisional walls of clear ice are gradually thickened at the expense of the clay in the interior. With decrease in the percent of colloids the polygonal cracks become more widely spaced.

In tests requiring soils with a high percentage of colloids, bentonite and mixtures of bentonite with South Carolina Cretaceous white clay were used. The ice layers were thicker with depth below the surface (see Fig. 21) because cooling was slower and there was more time for the slow movement of water to the growing ice crystals. Carton no. 1 stood in sand saturated with water, while carton no. 2 was sealed at the bottom. The close similarity in the two tests proves that little water was drawn into carton no. 1, and this was confirmed by weighing the cartons before and after freezing.

EXPLANATION OF ICE SEGREGATION

The mechanics of the process by which an ice crystal grows under external pressure and displaces solids with which it is in contact is closely tied up with problems concerning the constitution of water and the nature of the changes occurring as the temperature is lowered and freezing takes place. Therefore, it is not possible to give a complete explanation at the present time, and only a tentative hypothesis is offered here.

During the growth of an ice layer in clay, water is supplied to the crystals through small capillary passages, but the upward flow of water should not be attributed to capillarity, as there is no free surface or meniscus. The uplift is due to the cohesive forces in the water. Askenasy's experiment shows that under suitable conditions a column of water may be placed under

tension (Palladin 1918). Conditions existing during the growth of an ice layer in clay are particularly favorable for the uplift of water by molecular cohesion.

A growing ice crystal is in contact with a thin film of water similar to the adsorbed layer that forms on other solids. As molecules are removed from the film and attached to the crystal, they are replaced by others from the surrounding water. When an ice crystal grows in a direction in which growth is opposed by a solid body, such as a clay particle, the pressure is exerted through this thin film which separates them. The film probably consists of little more than a single layer of molecules, for it is not expelled by pressure. After the available water has been exhausted, the film may be frozen, but it does not freeze easily. Cohesion is greater between the molecules in the film and between these molecules and the ice than it is between water molecules that are not close to ice crystals.

Orientation and attachment of a water molecule to an ice crystal is accompanied by a slight repulsion, proportional to the change in volume, and this results in a slight displacement of the whole crystal relative to the neighboring solid. Crystallization may be accompanied by the conversion of dihydrol into trihydrol molecules, as advocated by some physicists, but this would not materially alter the process here outlined. As the new molecule is attached to the ice crystal, another molecule is drawn into the film by cohesion, and this is the direct cause of most of the displacement.

The growth of ice layers in soils and the accompanying pressure effects are therefore attributed to molecular cohesion. The energy for the process is of course supplied by the removal of heat. Water is not only pulled into the film under an ice crystal with force sufficient to lift the overlying load, but considerable force must be exerted in pulling it through dense clay to the growing crystal; hence in these experiments, water has been placed under tension sufficient to lift a column of water over 150 m in height.

The maximum pressure that may be developed as a direct result of ice segregation in the materials used in these experiments is probably not much above 15 kg/cm². This limit is not due to the inability of ice crystals to grow under higher pressure, but to the failure of the water supply when the tensile strength of the water is exceeded. Water under such high tensile stress is superheated, even below 0°C, and the formation of vapor would immediately interrupt the supply. In one experiment water was forced into the clay under pressure, thus reducing the tensile stress at the points of crystal growth. This permitted segregation to take place under higher external pressure. When nitro-benzene, which boils at 210.85°C, was substituted for water, greater heaving was obtained under high pressure; but benzene,

with a boiling point of 79.6°C, heaved less than water. Both of these liquids freeze with decrease in volume at temperatures between 5° and 6°.

SUMMARY AND CONCLUSIONS

Freezing of soils often results in heaving much greater than that which can be attributed to change in volume of the water present. This excessive heaving is due to segregation of water as it freezes, additional water being drawn in by molecular cohesion. The pressure effects accompanying freezing result from the growth of ice crystals. Ice crystals develop pressure only in the direction of growth, which, in soils, is determined chiefly by the direction of heat conduction and the availability of water, differential pressure being of minor importance. In experiments a pressure of over 14 kg/cm² has been obtained through growth of crystals to form ice segregations, and water has been placed under tension sufficient to lift a column of water over 150 m in length.

The chief factors controlling segregation and excessive heaving are: size of soil particle, amount of water available, size and percentage of voids, and rate of cooling. Segregation occurs readily if the particle diameter is less than a micron, and under favorable conditions where particles are somewhat larger. High water-content favors segregation, and additional water may be drawn from the water-table to form very thick ice layers. Water occupying very small soil voids does not freeze readily and may be undercooled in the immediate vicinity of ice crystals. Rapid cooling when the temperature is near the freezing point prevents segregation in some soils, but has little effect if the soil particles are sufficiently small.

Segregation of water on freezing causes shrinkage cracks if the supply of water is limited, or if the soil is very impermeable because of high colloid content. In the latter case, the cracks are gradually filled and enlarged by ice.

Frost heaving, when uniform, can cause little damage to pavements or structures, but unequal heaving may be very destructive. Local differences in soil texture, and in amount of available water, are the major causes of differential heaving. Differences in the kind and amount of soil cover are important, and difference in load is a minor factor.

Freezing, followed by thawing, may result in the concentration of so much water near the surface as to cause flowage and unequal settling. A sudden drop in temperature after a thaw which has left an excessive amount of water in the surface soil is a condition favoring segregation and excessive heaving.

The principles developed in this investigation help to

explain certain geologic phenomena found in very cold regions, such as buried sheets, lenses and wedges of ice, and the soil polygons with convex surface. They also have many applications to engineering, especially in the construction of hard-surfaced roads. The resistance to freezing offered by water in very small pore spaces, and especially where colloids are present, may help to explain hardness in plants.

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The Mechanics of Frost Heaving

Stephen Taber

**University of South Carolina
Columbia, South Carolina**

The Mechanics of Frost Heaving*

STEPHEN TABER

ABSTRACT

The old theory that frost heaving is due to change in volume of frozen water was based on experiments with closed systems. Field observations and recent experiments indicate that soils, when subjected to freezing under normal conditions, usually behave as open systems. When the freezing of saturated soils results in little or no heaving, part of the water is forced through the soil voids below the zone of freezing, compressing or expelling air. Excessive heaving results when water is pulled through the soil to build up layers of segregated ice. These ice layers grow in thickness because water molecules are pulled into the thin film that separates the growing ice crystals from underlying soil particles. Since heavy surface loads may be heaved and much less force is required to pull water through impervious clay, the water is put under high tension.

Heaving is limited by the tensile stress that may be developed in the water and by downward growth of ice crystals in soil voids. These two factors also probably explain the rhythmic banding due to alternating layers of ice and clay.

In well-consolidated clays the surface uplift equals the total thickness of the ice layers, the water content of the clay between the ice layers remaining approximately constant; but heaving is continuous and regular instead of intermittent. Clay is soft near the lowest ice layer because much of the water is unfrozen, the hardness increasing higher up where the temperature is lower and freezing has gone on for a longer time. Additional evidence has been obtained by freezing in open systems other liquids than water.

MECHANICS OF FROST HEAVING

It has been generally assumed that frost heaving, as well as other pressure effects accompanying the freezing of water, is due to increase in volume of the frozen

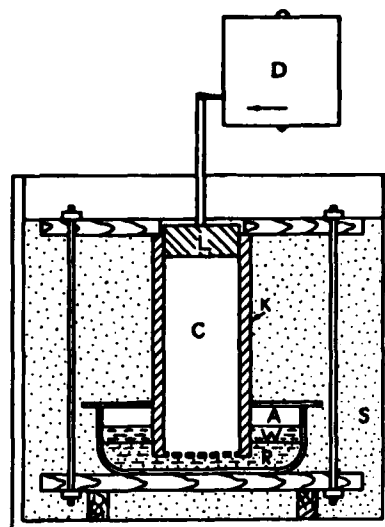


Figure 1. Apparatus for freezing soils in a system that is open with respect to water. C—soil; K—container; A—air; W—water; R—sand saturated with water; S—dry sand; L—lead disk with recording pen; and D—drum turned by a clock.

water. This assumption was based on experiments in which water was frozen in closed systems, i.e., systems from which the water present could not escape and into which additional water could not enter. The experiments of early investigators were limited to such systems.

In an investigation of frost heaving in 1914, the writer experimented with open systems on nights when the temperature was sufficiently low. The work was resumed in March, 1927, with an electric refrigerator, which made close control of temperature possible.*

Simple apparatus for freezing water in an open system is shown in Figure 1. A cylindrical container having a perforated bottom is tightly packed with the

* Published in *Journal of Geology*, vol. 38, 1930.

* The refrigerator was made available through the courtesy of the Frigidaire Corporation. Since November 1927, the U.S. Bureau of Public Roads has given financial cooperation.

soil to be tested, and is stood in a vessel containing sand, which can be kept saturated with water. This apparatus is placed in the bottom of the refrigerator and buried to the top in dry sand so that the soil is cooled from the top downward. A collar, fitting snugly around the container and resting on top of the vessel, prevents the entrance of dry sand. Container and vessel are held rigidly in place by a strong frame. A heavy lead disk resting on the soil in the container carries a recording pen which is in contact with a drum turned by a clock. This gives a graphic record of the amount and rate of movement of the soil surface during an experiment. Temperature is recorded on the same drum by another pen. Freezing extends gradually downward from the surface, and the interstitial spaces below the zone of freezing permit the movement of water either upward or downward according to the forces to which it may be subjected. This movement is without interference from outside forces, since air may readily enter the vessel, or be expelled, through the dry sand.

If a definite amount of water is cooled under atmospheric pressure, it freezes at 0°C , with an expansion in volume of about 10%. Greater pressures tend to prevent solidification, because the distance between the water molecules increases when ice crystals form; therefore, with increasing pressure the freezing point is lowered. If the water is confined so as to prevent expansion, freezing results in high pressure; but the maximum pressure obtainable is 2115 kg/cm^2 , for at this pressure and -22°C ice III, which is denser than water, begins to form (Bridgman 1912). In such a system pressure develops because openings through which water can escape are absent.

In experiments with open systems, it was found that some soils, when kept saturated with water, freeze with no uplift of the surface, while others give uplifts ranging up to more than 60% of the depth of freezing. An uplift of even 100% was obtained in some tests through the formation of a layer of ice at the surface. When no heaving occurs, some of the water must be pushed downward by the growing ice crystals and expelled from the soil container; when the surface uplift is in excess of that which could result from the expansion in volume of the water frozen, it is obvious that additional water must have entered the container as a result of the freezing.

When liquids that solidify with decrease in volume are substituted for water in freezing experiments, similar pressure effects are obtained in open systems, although these liquids could develop no pressure as a result of freezing in closed systems.

Numerous observations made out of doors indicate that soils, when subjected to freezing under normal conditions, usually behave as open systems rather than closed systems. Frost heaving is often too great to be

explained by change in volume of water present in the voids, and in other instances it is too little. Assuming the water content of soil to be 50%—and it is usually much less—the change in volume, if all of it froze in situ, could account for an uplift of 5% of the depth of freezing. The depth of freezing in the northern part of the United States seldom exceeds 2 or 3 feet; but a surface heaving of 6 inches is not uncommon, and an uplift of “a couple of feet” has been reported from Minnesota. On the other hand, the freezing of saturated soils is accompanied in many places by no appreciable uplift.

The entrance of additional water where soil heaving is excessive, and the partial expulsion of water from the soil voids where freezing results in little or no heaving, may be compensated for, in the one case, by the expansion and, in the other, by the compression of imprisoned air, or by the entrance and expulsion of air through pores and cracks. Most soils probably contain sufficient air to permit the free movement of water during freezing.

Soil may behave as a closed system with respect to water when the water-table is flat over large areas and practically coincides with the surface so as to exclude air; but even under these conditions, lenses and layers of segregated ice can form if the soil texture is favorable, and differential heaving may be produced if there is sufficient variation in the conductivity of soil cover so that freezing does not begin everywhere at the same time. The distinction between closed and open systems in the ground is not sharply defined. As the resistance to the movement of ground water increases, an open system tends to grade into a closed system.

In laboratory experiments excessive heaving is always accompanied by the introduction of additional water and its segregation to form layers or masses of more or less pure ice (see Fig. 2). Similar layers of segregated ice have been formed in the ground where excessive heaving has been observed. An excellent illustration of such ice layers (see Fig. 3) was sent to me by Mr. F.C. Lang, of the Minnesota Department of Highways. The lump of clay shown in the photograph was excavated in the spring of 1929 from a street where the heaving was “a couple of feet.” It is obvious that these ice layers could not have been formed by the freezing of water in situ.

An investigation of the factors involved in segregation and excessive heaving has been made (Taber 1929). The chief factors are: size and shape of soil particle, amount of water available, size and percentage of voids, rate of cooling, and the surface load or resistance to heaving. Segregation occurs readily if the particle diameter is less than a micron, and under favorable conditions if the particles are slightly larger. Tabular particles, such as are found in clay, probably give results

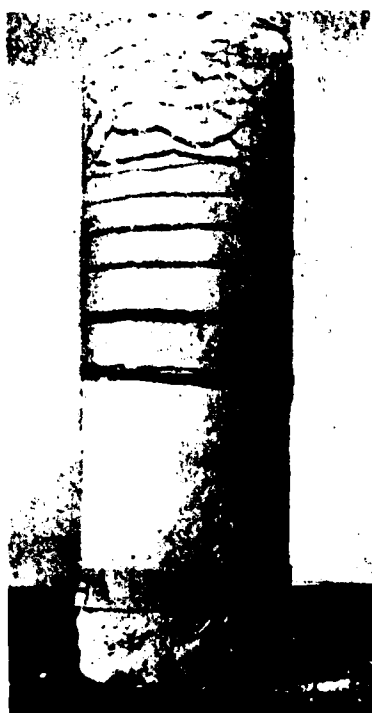


Figure 2. Cylinder of frozen clay. The surface uplift equals the total thickness of the ice layers. Scale in centimeters.



Figure 3. Clay containing ice layers from a badly heaved street in St. Peter, Minnesota.

similar to those obtained with somewhat smaller particles that are more nearly isodiametric. Water occupying very small voids in soil does not freeze readily and may be undercooled in the immediate vicinity of ice crystals. The water remaining unfrozen at low temperatures is probably adsorbed water. Rapid cooling, when the temperature is near the freezing-point, prevents segregation in some soils but has little effect if the soil particles are sufficiently small and the other factors favorable. Moderate loads have less influence on the heaving of clay than the heat conductivity of the material used in applying the pressure; heavier loads tend to reduce heaving, but in laboratory tests with pure clay, surface loads of nearly 15 kg/cm^2 were required to prevent heaving. Moderate loads prevent the heaving of relatively coarse material.

It is commonly stated that frost heaving is upward because of less resistance to expansion in that direction, but this hypothesis is not supported by the facts. Experiments prove that the pressure effects accompanying freezing are due to the growth of ice crystals. The pressure is developed only in the direction of crystal growth, which is determined chiefly by the direction of heat conduction and the availability of water. Differential pressure is a minor factor, of little importance when soils freeze under normal conditions.

A tentative hypothesis has been advanced to explain

the mechanics of the process by which ice crystals growing in open systems are able to exert pressure and overcome resistance (Taber 1929). A continuation of the investigation has led to the discovery of additional facts which shed more light on the problem.

A growing ice crystal is in contact with a thin film of water similar to the adsorbed layer which forms on many other solids that are in contact with water. As a molecule in the film is oriented and attached to the crystal, it is replaced by another from the adjacent liquid, thus maintaining the integrity of the film. If resistance to growth is offered by a solid body such as a soil particle, the pressure is exerted through the thin film of water that separates them. This film may consist of a single layer of molecules; but I am inclined to think that it is somewhat thicker, for the molecules in it possess considerable mobility. A unimolecular layer could not be expelled by pressure alone; thicker layers could be reduced through expulsion of some of the molecules under pressure, though this is probably resisted by the strong attractive forces. After the available water has been exhausted, the film may be frozen; but it does not freeze readily.

The orientation and attachment of a molecule to the crystal is accompanied by a slight repulsion, proportional to the change in volume; but this cannot be considered an essential factor in the process, for liquids that

freeze with decrease in volume give pressure effects in open systems similar to those obtained with water.

Cohesion is greater between the molecules in the film and between these molecules and the ice, than it is between water molecules that are not similarly located close to ice crystals. Since no outside force is competent to push molecules of water into the film between the growing ice crystal and the resisting solid, the crystal must be displaced relative to the adjacent solid because water is pulled in between them; and this is made possible by the high intermolecular attraction, which prevents the separation of the water molecules.

During the growth of an ice layer in soils, water is pulled up through the interstitial capillary passages; but the upward flow cannot be attributed to capillarity, for there is no free surface or meniscus. Since ice crystals growing in an open system are able to overcome a resistance many times as great as atmospheric pressure, the water, in such cases, is actually pulled into the nourishing film under high tension.

The ultimate tensile strength of water has been estimated, from the energy required to separate the molecules when it is converted into gas, as over 1300 atmospheres. A water column of large cross-section is placed under tension with difficulty, for the water is superheated when under negative pressure, even at temperatures below 0°C , and the formation of a gas phase immediately breaks the column. Extremely slender columns, or filaments, and thin films are more easily maintained under tension than larger ones; and this is the form in which water is present in clays.

Only rough estimates can be made of the maximum tensional stress developed in water during the freezing of soils. The surface load lifted can be measured in laboratory experiments; but to this it is necessary to add the frictional resistance of the frozen soil in contact with the container, the force necessary to separate the clay cylinder, and the force required to pull water through impervious clay to the growing ice crystals. Moreover, an ice layer must begin to form at some favorable point and then spread outward rather than form across the entire cross-section of the cylinder instantaneously.

Since the ratio of surface uplift to depth of freezing varies greatly with different soils under similar conditions of saturation, and with the same soils under different conditions of loading, the tensile stress that can be developed in the water is not the only factor limiting soil heaving. If tensile strength were the only factor, the amount of uplift would equal the depth of freezing or be zero. The principal additional factors are the rate of freezing and the downward growth of ice crystals due to the freezing of interstitial water in situ. The gradual formation of ice in the soil voids below a growing ice layer probably explains the rhythmic banding due to

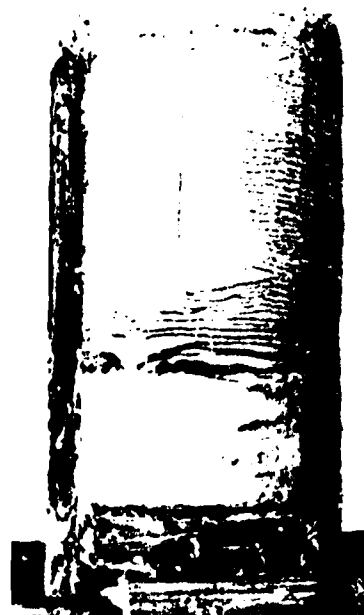


Figure 4. Clay frozen under pressure of 15 kg/cm^2 . Ice is clear and sharply separated from clay.

development of successive ice layers as freezing extends downward in clays (Fig. 2, 3, and 4).

In indurated clay, or in clay that has been thoroughly consolidated artificially, the layers of segregated ice are clear, for the most part, and very sharply separated from the frozen clay. The total thickness of these layers, as close as can be measured, is the same as the amount of surface uplift. The ice layers, because of slower cooling, tend to become thicker and more widely spaced with distance from the cooling surface. In unconsolidated clay the ice layers are not so sharply defined, and it is not usually possible to estimate accurately the thickness of the segregated ice. In tests with open systems where water can enter, moisture determinations show that the frozen clay between the ice layers contains the same percentage of water as clay below the depth of freezing. This observation confirms the conclusion that heaving in these tests is due to the formation of the ice layers and that the freezing of interstitial water causes practically no uplift. However, in spite of this, the rate of heaving, as shown by repeated experiments, is continuous under constant temperature, and never intermittent.

Using the apparatus shown in Figure 1, tests were made on indurated clay cut in the form of smooth, true cylinders and set in paraffin so as to equalize friction on all sides, and thus, as far as possible, prevent development of irregular ice layers. Temperature and heaving graphs for one of these tests, redrawn so that they start

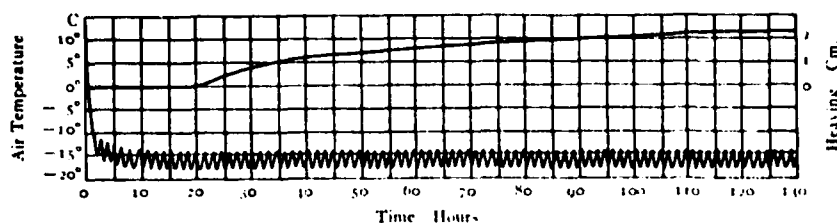


Figure 5. Graphs showing air temperature and rate of heaving during the freezing of the clay cylinder shown in Figure 2.

from the same vertical axis, are shown in Figure 5; and the distribution of the ice layers at the close of the experiment is shown in Figure 2. A sample of clay taken just below the frozen material and three samples taken between successive ice layers gave 22.0, 22.1, 21.8, and 22.0% water respectively. When tested with the point of a knife, the clay between the lowest ice layers was found to be soft, as compared with that higher up, the hardness tending to increase toward the surface where the temperature was lower and freezing had gone on for a longer time.

The hypothesis which best explains the facts here outlined is that during the growth of an ice layer the voids in the adjacent underlying clay, beginning with the larger ones, tend to fill gradually with ice and thus increase the work that must be performed in supplying water to the ice layer. When resistance becomes too great, the flow of water to this ice layer stops and a new layer of ice begins to form near the bottom of the zone of frost penetration. More of the water in the clay between the ice layers freezes as the temperature gradually becomes lower, some of it possibly migrating to the ice layers; but previous tests with this clay have shown that 6 parts of water per 100 parts of dry clay do not freezing on prolonged exposure to low temperatures.

An ice prism in one of the layers continues to grow at the base where it is exerting pressure against an adjacent soil particle as long as water molecules can enter the separating film and be attached to the crystal. If the soil particle is very small, the molecules travel only a short distance through the film to reach the points where they are attached. If the soil particle is larger, it takes longer for the molecules to reach their points of attachment; and meanwhile freezing may extend downward around the particle so as to gradually inclose it in ice. As soon as crystal growth is checked at any point, the temperature of the adjacent soil particle begins to fall, for heat is no longer liberated at this point by the conversion of water into ice; and, since water is a poorer conductor of heat and has a higher specific heat than the minerals present in soils, the temperature of the bottom of the soil particle will reach the freezing point sooner than the water with which it is in contact. This helps to bring about the inclusion of the soil particle in the ice.

The formation of layers of segregated ice is possible only when the soil particles are excluded by the growing crystals, a process which is aided by slow freezing, for then the molecules have more time in which to penetrate between crystal and soil particle.

In carefully sized material having a particle diameter of about 2 microns, segregated ice forms under conditions of slow freezing, but not when freezing is rapid. Even with slow freezing segregation may be prevented by applying a little pressure to the surface. As the diameter of the soil particles diminishes, higher pressures are required to prevent heaving. The effect of pressure in reducing and preventing heaving in open systems seems to be due in part to a decrease in the mobility of molecules in films that are under pressure.

If heat is conducted upward from the bottom of a growing ice layer just fast enough to remove both the heat set free when water is converted into ice and the heat brought up by the water, the ice layer will continue to grow in thickness; but if heat is conducted away more rapidly, the freezing isotherm will move gradually downward, causing ice to form in soil voids below. As freezing extends downward in clay, cooling is slower and there is less resistance to the upward movement of water; therefore, the ice layers tend to become thicker. Conditions most favorable for slow freezing are a relatively steep temperature gradient below the freezing-isotherm and a relatively gentle temperature gradient above.

Separation of the air dissolved in water, as freezing progresses, is a factor in interrupting the supply of water to the growing ice layers, but it is of relatively little importance. Examination of the ice with a strong magnifying glass shows that it contains small filiform cavities filled with air and oriented in the direction of crystal growth. These cavities, together with the parallel orientation of the ice prisms, give the ice a satiny luster. The cavities begin a short distance below the top of an ice layer and become more numerous lower down, but there is no difference in their abundance between the central and lower portions of thick ice layers. When boiled distilled water is used, with protective oil layers to prevent the absorption of air, the results obtained on freezing are not appreciably different. Also, when liquids which do not absorb air are substituted for water in

freezing experiments, the same rhythmic banding occurs.

In order to prove that the pressure developed through the growth of ice crystals in an open system is limited by the failure of the water supply, and not by inability of the crystals to grow under greater pressure, the following experiment was performed.

The upper part of a strong fiber container, water-proofed with paraffin, was filled with clay and the lower part with water, the two being separated by a perforated disk of hard rubber with filter paper resting on top. The disk was held in place by a stiff steel spring that rested on the bottom of the container. Before filling the lower part of the container with water, air was removed from the clay by displacement with water and clay thoroughly compacted for several days under a pressure of 14 kg/cm^2 . After introducing the water, the lower part of the container was stood in a can of impervious cement, and the upper part was strengthened by surrounding it with wood strips held in place by steel bands.

The container was placed in the refrigerator and buried to the top in dry sand. With the apparatus set up as described, and with the clay placed under a pressure of 14 kg/cm^2 by means of a piston inserted in the top of the container, the water in the lower part, because of the supporting action of the spring, was under a pressure of only about 10 kg/cm^2 . As freezing progressed, the pressure on the clay increased; and at the close of the experiment it was about 15 kg/cm^2 .

The walls of the container were sawed through and one-half was removed to expose the cylinder of frozen clay (see Fig. 4). The rubber disk in the photograph is slightly displaced. The total thickness of the ice layers was between 2 and 3 cm, whereas almost no heaving could be obtained in experiments with this clay under similar pressures but with the water under no outside pressure. The spring in the reservoir was completely compressed, for the water had been pulled into the clay to build up ice layers in spite of the high pressure on the clay.

There is some evidence that rupturing of the upward-moving filaments and films of water under tension is one way by which water supply is interrupted and growth of ice layers stopped. The inability of molecules to enter films that are under pressure, or to enter fast enough to prevent the downward freezing of interstitial water, is probably another way in which the growth of ice layers is stopped. The data now available are insufficient to determine which of these methods is the more important.

When water is under tension because of the growth of an ice layer in clay, the stress is ordinarily greatest right at the points where water molecules are being pulled into the film which separates the growing ice

crystals from the clay below, and it is therefore at this place that any break would occur. The breaking of a water filament, or film which is being drawn toward the ice layer, means that the molecules are separated beyond their range of attraction; in other words, a gas phase is formed between the liquid water and the ice. The gas phase, however, could not extend far below the ice layer, for the tensile stress in water a little below the ice is due to frictional resistance to the passage of the water through the small soil voids, and this stops as soon as the upward movement is checked.

On freezing clays that are extremely impermeable because of high colloid content and that contain a high percentage of water, tensional cracks of polygonal pattern develop and extend downward in advance of freezing. As freezing progresses, the cracks are gradually filled with ice (Taber 1929). These results were obtained with South Carolina Cretaceous white clay to which from 5 to 95% bentonite had been added.

The hypothesis that the growth of ice crystals stops because of interruption of the water supply by the formation of a gas phase receives some support from freezing experiments with open systems in which liquids having different boiling-points were substituted for water. The ratio of uplift to depth of freezing for nitrobenzene, which boils at 210.85°C , is greater than for water under the same surface loads and conditions of freezing; and this is true in spite of the fact that greater force is required in pulling nitrobenzene through the clay to the growing crystals, since its viscosity at low temperature is about 1.7 times that of water. Benzene, with a boiling-point of 79.6°C and a viscosity of 0.5 that of water, on freezing heaves much less than does water under similar conditions. Nitrobenzene and benzene both freeze with decrease in volume at temperatures between 5° and 6°C .

It would be interesting to know the maximum tensile stress that could be developed in water as a result of freezing in an open system. By using finer material and cooling with extreme slowness, it should be possible to obtain somewhat higher tensile stress in the water, and therefore lift heavier loads, than was possible in the writer's experiments. In an ideal system it is theoretically possible to obtain a tensile stress of about 1350 kg/cm^2 , but it is improbable that this stress could be approached in any attainable system.

The energy for frost heaving is supplied, of course, by the removal of heat. The greater the pressure on an ice layer the lower the temperature must be if growth is to continue. Therefore, if the films of water are not ruptured under tension, the load lifted is determined by the amount of undercooling possible in the clay immediately below an ice layer. The presence, between the base of an ice layer and the underlying soil, of a separat-

ing film connected with water below makes frost heaving a reversible action from the standpoint of thermodynamics.

When liquids that solidify with decrease in volume are substituted for water in freezing experiments with open systems, surface load, or pressure, reduces heaving, although removal of heat is the only source of energy; and the freezing point of such liquids is raised by hydrostatic pressure acting on both liquid and solid phases.

This anomaly might be explained if we assume that the base of a crystal where growth takes place is subjected to non-uniform pressure because of adjacent soil particles, for the effect of unequal pressure would be to lower, locally, the melting-point; but this assumption seems improbable, for crystal growth would tend to equalize the pressure, and as long as the pressure is transmitted across a separating film it should be practically hydrostatic.

An alternative hypothesis is that pressure tends to reduce the thickness of the nourishing film by expelling some of the molecules and, since the expulsive forces are increased, the attractive forces must likewise be increased, by lowering the temperature, else molecules cannot enter the film. Pressure decreases molecular mobility in the film and retards crystal growth.

Growth of ice crystals under resisting pressure in open systems is analogous to the growth of crystals from solutions under similar conditions (Taber 1916, 1917). One is possible because water occupying very small voids can be undercooled; the other, because solutions in such voids can be supersaturated. Water molecules are pulled toward a growing ice crystal to replace those that become attached to it, while molecules in solutions reach their points of attachment because of osmotic pressure. If water at low temperatures is considered to be a saturated solution of trihydrol, or ice molecules in dihydrol, the analogy would be even closer; but it would be unwise to draw conclusions until more data are available. Benzene and nitrobenzene have usually been classed as non-associated liquids; but, according to Schames, molecules of $(C_6H_6)_4$ begin to appear near the freezing point in liquid benzene, which corresponds to $(C_6H_6)_2$; and Dutoit and Moijou find nitrobenzene slightly associated (Turner 1915). Tension in water should favor the formation of trihydrol molecules, and therefore of ice, in soil voids below a growing ice layer; but tension should oppose the forma-

tion of associated molecules in benzene and nitrobenzene, and thus help to make undercooling possible.

For one whose conceptions of frost heaving have been derived from freezing experiments with closed systems, the results obtained in this investigation seem paradoxical. 1) Heaving on clay is greater than on sand, although part of the water does not freeze in clays, whereas practically all of it, within the zone of frost penetration, freezes in sand. 2) The pressure developed during heaving in an open system is limited by the tensile stress that can be developed in the water. 3) The boiling-point of a liquid seems to be more important than the freezing-point in determining the pressure developed by freezing in open systems. 4) While water expands on freezing, the freezing of saturated clays may be accompanied by the formation of shrinkage cracks, owing to withdrawal of water to build ice layers. 5) Heaving is upward because that is the direction of heat conduction rather than because it is the direction of least resistance.

It cannot be expected that the explanation of frost heaving here given is complete and without error in all of its details. It is a first attempt and will doubtless need modification as additional data become available. A better knowledge is needed of the constitution of water and the nature of the changes occurring as the temperature is lowered and freezing takes place. Also, more must be learned about the properties of thin films of water in soils. And, it is highly desirable that accurate field observations be made of the conditions under which soil freezing is accompanied by excessive heaving in some places and no heaving in others.

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Soil Freezing and Frost Heaving with Special Applications to Roads and Railroads

Fil. dr. Gunnar Beskow (Ph.D.)
Swedish Road Institute, Stockholm

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Special Applications to Roads and Railroads**

Gunnar Beskow

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TRANSLATOR'S PREFACE

Dr. Gunnar Beskow has devoted most of his life to the study of frost action of soils and is the author of numerous publications (see bibliography). This publication is a summary of his studies of the subject up to 1935 and is perhaps the only complete treatise on frost action.

The translator first became interested in the subject through his studies under Professor Arthur Casagrande, Graduate School of Engineering, Harvard University, and following his suggestion, translated this treatise, completing it in 1938. The manuscript was later re-edited, retyped, and made into a bound manuscript form with the suggestions and help of Professor W.L. Malcolm, Director, School of Civil Engineering, Cornell University.

Recently, due to the large airport program necessitated by our war effort, there has been a renewed interest in frost action and the translator has had numerous requests for this translation. The Technological Institute, Northwestern University, through Dean O.W. Eshbach and Professor P.C. Rutledge, in the interest of making worthy research of a valuable nature available to the American engineering profession, has undertaken the publication of this translation.

The translator has attempted to be as literal as possible in order to avoid any possible misinterpretation of the author's meaning. In cases where there is any obscurity or possible misunderstanding, the translator has inserted his comments indicated by brackets. Single parenthesis are the author's own remarks.

The Swedish soil classification of grain size, which is the same as the international classification, is somewhat different than the U.S. Bureau of Soils Classification. The Swedish word "mo" is used for grain sizes between 0.2 and 0.02 mm, and is translated as "silt." The word "majäla" is used for grain sizes between 0.02 and 0.002 mm, and is translated as "fine silt." Where adjectives are used, they are added to the terms; for example, "fin majäla" is translated as "very fine silt." Clays are those soils having grain sizes smaller than 0.002 mm and are designated as lean, medium or fat clay, fat clay being the real fine colloidal clay.

Soils indicated by letter and number, such as "GB20," refer to disturbed soils used by the author in his experiments. The mechanical analyses of all these soils are given in a summary table at the end of the text. References to publications as "Beskow 1932a," for example, are given in the text, and the full reference is given in the bibliography.

The translator is indebted to Mr. H.B. Edwards, Translator, U.S. Waterways Experiment Station, Vicksburg, Mississippi, for his careful proofreading of the equations and tables in the manuscript.

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J.O. Osterberg
Assistant Professor of Civil Engineering
Northwestern University
Evanston, Illinois
October, 1947

Soil Freezing and Frost Heaving with Special Applications to Roads and Railroads

GUNNAR BESKOW

I. INTRODUCTION—HISTORICAL

A part of the particular phenomena that appear in connection with the transition of groundwater from a liquid to a solid state, and vice versa, has long been a subject of observation, chiefly by the country people, although not until considerably later is it mentioned in literature. The early literature contains such descriptive terms as: frost boil, "pipkrake" [needle ice or hoar frost], and "freezing up of stones from the ground." The latter phenomenon is mentioned by Urban Hiärne (1694).^{*} One of the very unusual characteristics and startling nature of the phenomenon is the so-called "earth heaving" or "earth shooting" which is mentioned by Urban Hiärne (1694) and more completely described in 1706 (see Sjögren 1903). The usual frost heaving or the raising of the ground surface associated with freezing of the soil, seems to have been well known at least by the middle of 1700, when a full explanation was given by E.O. Runeberg (1765). Runeberg also mentions how he observed clean ice down in the frozen soil. He says "At the depth where the frozen soil stops, I found ice granules, lumps of clear blue ice in big pieces." He also understood this mechanical effect: "When such an ice layer lies underground, it is undoubtedly true that it forces out the frost crust at the place of least resistance." Frost disintegration of hilly ground and boulders early awakened the attention of nature observers, and was well known by the scientists in the beginning of 1700. Considerably later, the various types of phenomena that can be described under the heading of Arctic or Alpine frost—and the earth slide phenomenon—were a subject of interest to geographic-geologic investigation, and also to Tilas at the end of 1700 who, according to N. Zenzen, seems to have been acquainted

with a part of this phenomenon. Concerning highways it was observed quite early that frost heaving had done damage to culverts and bridge piers (especially during the development of stage coach traffic in the 1700s). It was known that this phenomenon was not well understood. Observations that the disturbance of soil due to freezing is damaging to roads are naturally probably as old as the roads themselves. However, this occurs at the same time that the ordinary difficulties associated with spring weather occur, such as floods, thawing of snow and ice, as well as a general softening of the roads due to the melting waters. These difficulties were so large that they covered up the real phenomenon of the melting of frozen ground. It is apparent then, that few observations could be made. It is peculiar that locally restricted frost heaving of a volcanic silt, which occurs long after the road is dried up, has long been noticed in the northern part of Sweden. However, I have found no published accounts of it. But preventives against frost heaving, such as beddings of twigs, has, according to the information from road superintendent O. Sander, however, been used in Norrbotten [in northern Sweden] at least sixty years ago.

Only recently, due to the general rebirth of highways by the tremendous growth of the automobile as a means of transportation, has the technical problem of frost action on roads become practically and economically significant. There has since then been an important demand for a direct and thorough investigation to clarify the general phenomenon of the occurrence of frozen soil, the mechanical effect of different types of soil, and the possible practical preventive measures and their relative suitabilities.

The treatment of the problem of frost action in soils has considerable practical significance in the civilized nations of the north temperate zone, where the winters are so severe that freezing occurs to considerable depths. The treatment of the problem also concerns the effects and extent of the quaternary glacial period, for this period determined the special characteristics of the soil.

^{*} Although his explanation is somewhat different. He says, "In fields where boulders have been picked out and the ground cleared, more boulders will be generated."

Glacial soils have a lack of colloidal matter, and a large amount of medium-grained sediments, this type of soil being highly susceptible to frost action.

In many countries having this climate and soil characteristics, the problem of frost action on roads reached such importance shortly before 1920 that special investigations were undertaken independently at the same time in several different countries.

Sweden was the first country to consider the problem seriously. In October 1925, under the auspices of the Swedish Institute of Roads, a conference on the frost action of soils was held in Luleå in which representatives from highways, railroads, and other technical interests were present.

The complete proceedings of the conference, published in the Transactions of the Swedish Institute of Roads (no. 2), gave a valuable comprehension of the practical knowledge of the frost heaving and soil freezing phenomena, and the use of effective preventives, the cost, and the technical procedure. These were given in the statements of the northern road and railroad technicians (Hjelm, Lundström, Sander, Skarstedt, Wedberg, Wolff). Also the theoretical progress in the frost heaving study and the physical assumptions involved were discussed (Simon Johansson).

Shortly after this conference, the Swedish Geological Survey and the Swedish Institute of Roads decided to undertake jointly a searching theoretical-practical investigation of the frost problem. This investigation, after a bit of preliminary work, began in 1927. While the investigation was in progress, the preliminary conclusions of the important results were presented in reports and papers (Beskow 1928, 1929b, 1930a-g, 1931a, 1932a, 1933a,d, 1934a,b). Parallel with the frost problem investigation, results of other geological road problem investigations have been published (Beskow 1930g, 1931b, 1932a,b, 1933b,c).

In various countries, the frost problem has been a subject of investigation with respect to road and building problems. The foremost of these are the United States, Soviet Russia, Norway and Germany. The American frost investigations began with some experiments in 1925 and 1926, and have since then been carried on chiefly by Stephen M. Taber and A. Casagrande (see Taber 1929 and 1930a,b).

The Norwegian investigations, which began a little later, were carried out chiefly by T.B. Riise, A. Dahle and H. Brudal (see Bibliography). Finland has done little with the road frost problem, but Kokkonen, however, has conducted valuable investigations of the structure formation of frozen soil in different types of soils, mainly from the geological-agricultural standpoint, but with much significance in the road-frost-action studies.

Recently Germany has become considerably inter-

ested in the road frost-action problems. These have been dealt with mainly by K. Backofen, W. Loos and E. Schönleben.

From a practical standpoint, considering the effect of the change of state of the groundwater, we can divide the effects into two headings, depending upon which way the change occurs [i.e., freezing and thawing]. Along with the freezing of the groundwater, there is a damaging effect upon all structures—roads, streets, railroads, bridges or houses—which can be attributed in all cases to the same cause, that of the raising of the ground surface associated with freezing of the soil. This phenomenon is even damaging to farming in that the expansion of the soil layers can tear apart roots, especially after the autumn planting. This was of special concern to Kokkonen (1927, 1930, 1931), Ekströ (1930) and Beskow (1934a).*

The difficulties encountered during the melting of the frozen soil consist partly of the resulting settlement of the ground surface (important for railroads and buildings), and partly of the loosening of the soil during thawing, which reduces the bearing power of the ground.† The loosening and corresponding reduction in bearing power is due to a large increase in water content of the soil. The water content is increased during freezing of frost heaving soil by the sucking up of water from underneath, and also during melting when the melted water from the upper layers and the rainwater from the surface is prevented from draining downward by the still frozen layers underneath. Thus an excess amount of water accumulates when thawing occurs, which considerably reduces the bearing value of the ground. This phenomenon is called "frost boil." It is of fundamental importance that in both frost heaving and frost boils there is a sucking up of water during freezing and the consequent rising of the ground surface, and also that during thawing the bearing value of the roadway is reduced. Both frost heaving and frost boil can then be considered as different phases of the same phenomenon. Frost boils occur only (but not always) at places with considerably strong frost heaving ground. However, the occurrence of frost boils depends upon the passive factor of the composition of the soil of the roadbed, and the active factor, the intensity and load of the road traffic.

Because the primary process of frost heaving is,

* With this appears a localized special form of the phenomenon, called hoar-frost, which often plays an important role. In general it is the repeated shallow freezing during the spring night frosts that has done the greatest damage.

† There is even a structure change during the loosening caused by freezing in fine-grained soils, which in farming is very significantly advantageous, since otherwise most of the clay soils are difficult, if not impossible, to plow.

from a technical standpoint, the fundamental cause of all frost damage, the technical and theoretical study of frost heaving is the essence of the problem. For this reason, the investigations in Sweden and the United States have first concentrated on this question.

It was early discovered that frost heaving was accompanied by an increase of volume as the water underwent the change from liquid to ice. E.O. Runeberg (1765) noticed this, after observing the freezing of soil in place. He says, "We can say for certain—that earth, stone, or houses, that rest on this (referring to the saturated soil that is freezing) must either resist or be displaced" (page 931). Later he determined the water content of a frozen piece of clay, and found it to be quite large.* "Is it not a wonder then, that a clay layer can displace a load that rests on it, when the water freezes" (page 95).

This discovery, that the cause of heaving is the expansion of the existing water in the soil due to freezing, was retained until recently. The first investigator to find that during freezing of a fine-grained soil water flows to the freezing layers, increasing the water content, was Simon Johansson, who published his experimental results in 1914, and uses this observation to explain heaving (Johansson 1914, p. 95). Here lies the key to the whole frost problem, both frost heaving and the formation of frost boils. At the Luleå conference (1925), Johansson explained how this process takes place from the physical standpoint of the soil, and by this, explained how frost heaving and frost boils occur.

Taber (1929 and 1930) has further shown that the expansion of water due to freezing is not at all the fundamental cause of heaving, for experiments in which water has been replaced by other liquids (benzene and nitrobenzene) which decrease in volume when frozen, have shown strong heaving during freezing, provided the freezing soil column has access to suck up the liquid.

While Simon Johansson assumes that the water movement is caused by capillary flow, which has later been verified by Taber's and the author's investigations, Pöpke suggests that water is transported by the flow of water vapor through the soil pores and condenses on the outside of the frozen soil (1926, 1927, 1928). This theory is, however, hardly feasible (see Beskow, 1929a), for frost heaving soil is always filled with capillary water, and also even if this is the case, the rate of diffusion would be so small that it could not account for the large increase in water content.

Gustaf Dahlberg (1924a,b) agrees with Simon Johansson that only capillary flow is responsible for the increase in water content, while Hörmell (1924) believes

the amount of water transmitted by vapor through the soil is also significant.

The author, after observations during the early investigations, agreed with Simon Johansson. The results showed that the amount deposited by evaporation and condensation was insignificant. On the basis of many field and laboratory observations, the author decided that a gas phase always exists, and that the transportation in the form of vapor has an important significance, though the quantity was very small. The vapor diffusion between the ice crystals and the soil particles occurs over a distance of $1/100$ to $1/1000$ mm. The author, however, has later given up this opinion on the basis of later experiments and of Taber's careful (summarized 1929) study. It is possible that in many cases such transportation happens, but not in all cases, and it is not a necessary step in the process.

The water supply to the freezing layers in frost heaving ground comes practically exclusively from capillary flow of the water below. The important factors of this process, which are extremely variable, are the distance to water and the soil grain size. An analysis of these questions includes the manner in which water freezes in soils of different water content and in different types of soils.

II. MECHANICS OF SOIL FREEZING

A. Structure of frozen soil

It was mentioned in the *Introduction* that in the freezing of a sufficiently saturated fine-grained soil, such as a clay, water flows to the frozen part, increasing the water content. This process was assumed by Simon Johansson (1914) to be the cause of frost heaving and frost boils. This has been verified by a large number of borings of frozen soil, especially from localities of severe frost boils on roads and railroads, where throughout the frozen earth there are clean ice layers, from hardly visible short strips of a few tenths of a millimeter in thickness, to layers several decimeters thick, and further, that this ice-banded frozen soil contains a larger water content than the soil in an ordinary state can hold.

The occurrence of such clean ice layers in frozen soils is more sporadically written by several other investigators. The oldest known published observation is an early citation of E.O. Runeberg (1875).*

This ice-layer building in soil has later been observed independently by several investigators. For example: Holmquist (1897, p. 418), Simon Johansson

* "It sucks up so much water in the dry clay layer that the proportion is $19\frac{1}{2}$ to 5, this being almost four times as much water as soil."

* On page 95 he says of his observations of frozen clays: "At the depth where freezing stops, I found ice granules, lumps and clear blue ice in large pieces. In one place I saw a matrix of hanging ice layers"...etc.

(1916, p. 73), O. Glaerum (1920) and Ekström (1926, p. 33). The first one to make a more detailed study was Kokkonen, who published in 1926 an extraordinarily detailed and well-illustrated article, explaining the structure formation of ice layers in different kinds of soils.

Kokkonen classifies, on the basis of his large amount of observed data, the structure of frozen soil into three groups: massive, porous and stratified frozen soil. Frost action appears only in very porous and loose soil. Both massive and stratified forms of frost action can, however, according to Kokkonen, occur in non-porous soils, independent of grain size, when the water content is the governing factor. That is, there is a certain water content limit, below which the soil becomes massive, and above which it becomes stratified.

Kokkonen's thorough studies are, however, on the basis of Taber's and the author's independent investigations, incorrect in three main respects:

1. Whether the frozen soil becomes massive or stratified depends not only on the water content, but above everything else on the grain-size; coarse sand never gets ice stratified. Besides this, as will be shown below, the load and the rate of freezing play a considerable part.

2. In spite of water content analysis of many frozen soil samples from apparently strong frost heaving ground, the fact that with such soil freezing, in order to have an increase in water content, there must be a supply of water to the freezing ground, was not considered or at least not mentioned.*

3. The water in the ice-layers was assumed without exception to be clean ice and the soil particles covered with frozen water. The very important fact that in an ice-banded frozen soil with a moderately low temperature, the soil between the ice bands is fully plastic and soft, therefore unfrozen, has not been considered.

My own investigations in the field as well as in the laboratory, of the conditions of ice-structure formation of frozen soil, have led to the following conceptions of the structure of frozen soil, the influence of the type of soil, and other factors.

Massive or homogeneous frozen soil. This soil type indicates that all the frozen water is frozen in the pore space of the soil. This always occurs in nature in the freezing of coarse-grained soils, medium sands and coarser.† Fine-grained soils can freeze homogeneously in the following manner:

1. Always when the water content is less than capillary saturation.

2. When the water content is greater than capillary saturation, the structure depends partly on the rate of freezing and partly upon the pressure on the soil. Quick-freezing, and an increase in pressure tends to prevent the growth of separate ice layers, and to produce homogeneous frozen soil. The finer the soil is, the less is this effect. Coarse silts often freeze homogeneously. However, with a small surface load and a high groundwater level, coarse silts can become very appreciably ice-stratified. On the other hand, silts, fine silts, and clays will almost without exception form an ice-stratified frozen soil.*

It should be noted that in the case where open cavities exist in otherwise homogeneous sandy soil, they can be totally or partially filled with ice, usually in the form of thin layers of ice crystals on the cavity walls. When these cavities occur frequently, the frozen soil is obviously not homogeneous. Nevertheless from a theoretical standpoint, it still belongs to this group. It should be emphasized, however, that there is a large difference between such sand and stratified frozen soil.

Stratified frozen soil (discontinuous, heterogeneous frozen soil). Here a part of the water is frozen outside of the soil pores, in large aggregations, i.e., banded or stratified, which may have considerably variable structure formations. The remaining water can either freeze in the pores or remain unfrozen. The lower the freezing temperature and the finer the grains, the larger will be the amount of water that remains unfrozen. In fine clays, cooled only a few degrees below freezing, all the pore water is unfrozen, leaving the clay between the ice layers plastic and soft.

Even in stratified frozen soil open cavities can exist which also become partly or fully filled with ice. Often the ice in these cavities is porous, and therefore white in color. A distinction is usually made between this type of freezing and the normal type of ice layers.

The building of clean or partly clean ice crystals or layers, which occurs during the freezing of the ground, can be separated into the following types:

1. Ice formation on the ground surface (so-called hoar frost).

2. Ice formation in cavities, cracks or holes (inside hoar frost).

3. Ice formation around objects, such as stones, pieces of wood, etc.

4. Ice layers that occur inside of homogeneous soil.

This division, however, does not mean that there is an absolute differentiation or a theoretical difference,

*Kokkonen made numerous water content determinations that were as large as three hundred and even up to 470 percent by dry weight—in spite of the fact that in an unfrozen state, the soils could not have much more than 30% water content.

† However, with the exception of sand in a saturated, closed system.

* For quick freezing, such as the laboratory tests that are explained later, silts usually freeze homogeneously or show only sporadic ice-strata.

for there is always a combination of the different types. We might find several other bases for division, for example the ice building that occurs at discontinuities, that is, in the border between two different substances such as soil–air, soil–stone, clay–sand, etc. However, for practical purposes and theoretical considerations, the following division is suitable:

1. Ice formation outside the soil (hoar frost).
2. Ice formation inside the soil (stratified soil; discontinuous frozen soil).

1. Hoar frost

This damaging phenomenon, especially significant in agriculture, has been investigated quite thoroughly by several investigators, some of whom are H. Hesselman (1907), B. Högbom (1914), A. Hamberg (1915), S. Johansson (1916) and P. Kokkonen (1926). The following remarks can be made of the observations of these investigators:

Hoar frost consists essentially of long thin needle-like ice crystals, appearing immediately below the ground surface which build or grow perpendicular to the ground surface. The stalks or needles are grown together in bunches, which can either grow into larger bunches or clusters, or can remain separate, but more or less closely spaced. The smaller bunches are often somewhat bent and crooked, but the large bunches of ice needles appear to be more straight.

The hoar frost ice itself is always dense and clean, but there is often on top of these bunches of ice needles a

thin porous soil layer. The hoar frost is often built in several “stories” or layers, the border between each layer being blended with soil. Surface leaves, lumps, stones and other objects on the ground surface are lifted up by the building of hoar frost ice (that is, it has the ability to lift). Of course the hoar frost cannot lift objects so large that the frost cannot penetrate downward and build ice under it.

Concerning the question of what kind of soil hoar frost builds on, opinion differs. Hesselman describes the typical hoar frost building on greatly decayed turf (sod) (which is quite prevalent in the flats of northern Sweden). Hamberg (1915, p. 383) says that hoar frost in mineral soil occurs mostly and more strongly in clay. This disagrees, however, with the observations of Simon Johansson, Kokkonen, the author, and several other authorities. Hoar frost can develop in all mineral soils from sand to clay, but the mineral soils that are most susceptible to the formation of hoar frost are the coarser soils, silt to fine, rather than clayey soils. In the finer soils, especially in stiff clay, the hoar frost is very thin. The cause of this, as Simon Johansson (1916, p. 73–73) says, is that in order for hoar frost to grow, the soil must be quite pervious.

A number of conditions are necessary for hoar frost to form or to grow to any large extent—a damp or wet ground, a ground surface that is not densely covered with vegetation, and certain temperature conditions, etc., which shall be dealt with in more detailed under the discussion of the frost action theory.



Figure 1. Hoar frost from Mörby, Province of Stockholm. The hoar frost is removed from part of the surface and a loose piece is shown in the foreground. Observe the different layers. Type of soil: lean clayey silt. See Table 1.

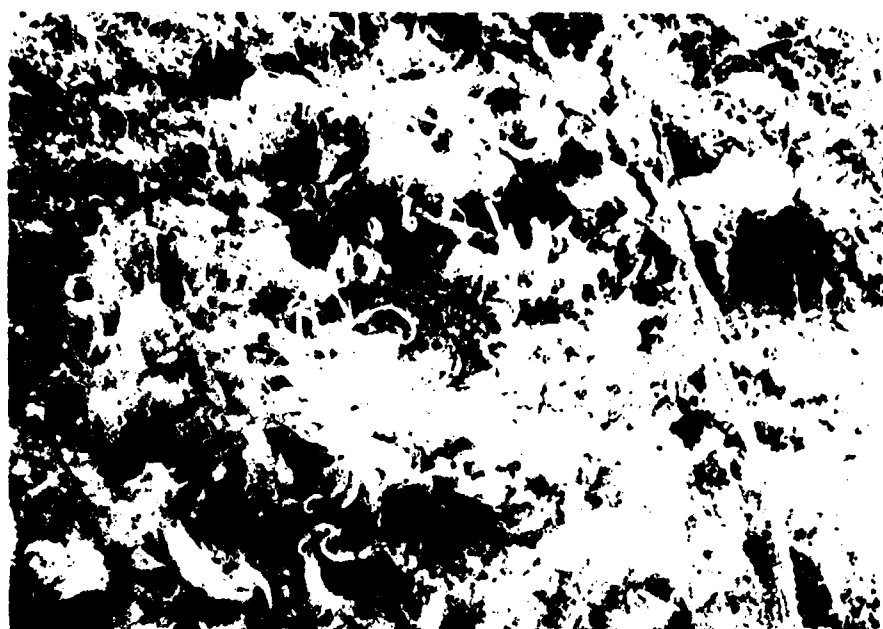


Figure 2. Undisturbed, day-old hoar frost on road cut between Örnköldsvik and Björna, Province of Västernorrland. Type of soil: Coarse silt. These single ice needles, or groups of needles (note curling) occur when the soil is relatively warm and the water supply is plentiful.

Hesselman stresses as very significant the fact that while hoar frost is forming in a sod or turf soil, the lifted sod clumps are always frozen, but the sod under the hoar frost is unfrozen. The author's conclusions from observations on mineral soil are the same, namely that the soil is unfrozen under the hoar frost. However, the frost will slowly penetrate through the soil under the hoar frost and the growth ceases, the ice needles still remaining on the surface as "dead" hoar frost.

As an example of the extent of hoar frost formation, the following observations are presented:

Locality: Vicinity of Mörby, Province of Stockholm.

Soil and Topography: Soft clayey silt, lying on the

slope at the foot of a hill, with the groundwater flowing a few decimeters under the surface. In the fall, a large amount of hoar frost occurs in lengths of $1\frac{1}{2}$ to $2\frac{1}{2}$ cm. Table 1 gives the height of the hoar frost and its conditions during a frost period that occurred 5–12 of November, 1928.

The time of observation was the same each day—9 to 10 a.m. As the table shows, the rate of increase in thickness is about 2 cm per day, but varying considerably from day to day depending on how cold it is. It also shows that while the old hoar frost layer grew 3 cm, the new layer grew 5 cm. This is because the old layer becomes isolated by the new layer, and the rate of cold

Table 1.

Date (1928)	Temperature condition	Maximum depth of hoar frost (cm)	Number of layers observable		Depth from surface to hoar frost (cm)	Average daily temperature at Stockholm (°C)
			a At place of observation	b Nearby locality		
5 Nov	Frost begins	0	—	—	—	+1.1
6 Nov	Frost	2	1	(1)	—	-0.3
7 Nov	Frost	4.5	1	(2)	—	-2.4
8 Nov	Frost	8	2	(3)	—	-3.0
9 Nov	Frost	10	3	(4)	0.0	-3.0
10 Nov	Brief thaw in morning	11.5	4	(5)	1.5	+0.6
11 Nov	Frost	—	—	—	—	-1.4
12 Nov	Frost; thaw since forenoon	13	—	—	5.0	+1.3



Figure 3. Hoar frost from a road cut, Province of Stockholm. Warm groundwater flows out on the slope near the water-filled ditch, and the hoar frost is therefore not continuous. But the patches are solid, probably because of the capillary rise of groundwater in the clusters of ice needles, which later froze (see Fig. 5). Type of soil: silty lean clay.

penetration becomes less, and therefore the rate of growth becomes less the thicker the hoar frost is.

The structure formation depends upon this fact. The upper part of a layer is usually thin with thin ice needles, which become thicker and larger with depth. At the bottom of the layer there is a plane of thick massive hoar frost impregnated with soil particles. Under this are found weak thin needles of the next layer, etc.

Kokkonen has noticed that each layer is formed in a day (24 hours) and that therefore each layer marks a day's growth. Table I also shows this. However, it must be mentioned that one often finds hoar frost of several days' growth to be homogeneous, and also there are cases of layers of less than a day's growth. One might also find cases of uniformly stratified systems, in which each stratum is subdivided into layers of a day's growth.

Naturally the formation of layers depends upon the temperature variations, variations in the water supply from underneath, and the porosity of the soil. The dominant cycle of variation is usually one day, and therefore a day's layer will be the normal cycle; but sometimes these daily rhythmic variations do not appear, and a layer can represent several days' growth.

From the clean ice at the surface, the hoar frost changes continuously downward to the ice stratified soil. In hoar frost there is always a scattering of soil in the border between layers, and also occasional lumps of soil in the border between layers, and also occasional lumps of soil in the ice layer. It seems as if the hoar frost changes gradually into ice-stratified soil with depth, the soil layers between the ice growing larger. The zone of transformation is actually so small that one can say there is a definite plane of division. Sometimes this change occurs not in a vertical direction but sideways, as in the



Figure 4. Two pieces of hoar frost from same cut as in Figure 3. 1—From approximately the center of the formation. 2—From the upper part including the boundary between hoar frost (a) and the frozen soil (b). (Notice the horizontal surface plane is at a 1:2 slope with respect to photograph.) Type of soil: silty lean clay.

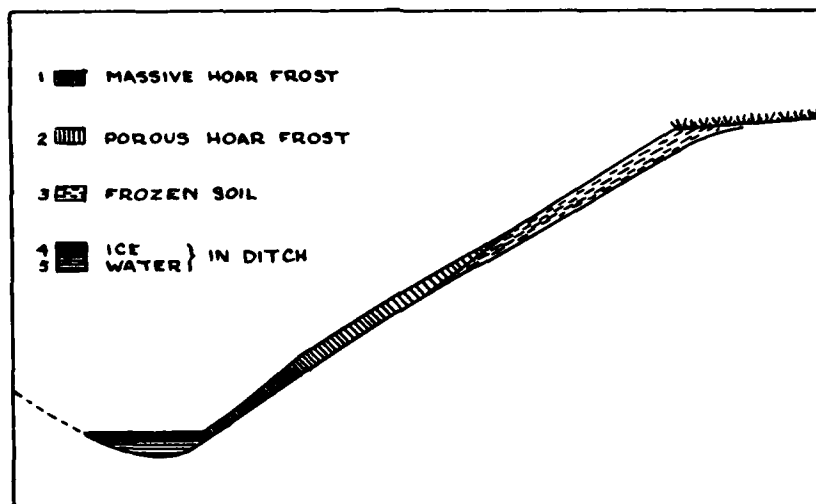


Figure 5. Schematic diagram of a hoar frost formation on a damp slope of a road cut. 1—Solid hoar frost. 2—Normal (porous) hoar frost. 3—Frozen soil. 4—Ice. 5—Water (4 and 5 in ditch).

case of a road cut, with sufficiently high and long side slopes. On such slopes it often occurs after a few days of frost weather that the bottom of the slope has a blanket of pure clean hoar frost. Advancing up the slope, the layer of hoar frost* becomes thinner and more porous and gradually disappears (see Fig. 5). Where the frost disappears the soil becomes ice-stratified. In between, there is a gradual change, the border between the hoar frost and ice-stratified soil being oblique to the slope (see Fig. 5). This division can be sharp or gradual.

As a rule, the transformation with depth between hoar frost and frozen soil is clear and distinct. The difference is less marked, the finer-grained the soil is. In pure clays, the hoar frost occurs as a thin surface ice layer attached directly to the stratified ice layers in the soil below. This is only a special case of the normally stratified soil. The coarser the soil the larger the difference is. For example, in a silt, the hoar frost ice forms a very marked distinct layer above the ice-stratified soil. In coarser soils, such as sands and turf, the ice can only form in the open pores as hoar frost, and the soil below is frozen homogeneously. Note that when pockets and discontinuities exist in sand, the hoar frost grows in these places and can fill them entirely, giving it the appearance of being ice stratified. When the porous soil near the surface is in a very loose state, such as occurs in soils with many roots, the co-called "inside" hoar

frost can build and melt together, forming a very irregular and coarse ice-banded system. In compact sand, the hoar frost forms in a regular pattern at the surface and does not branch off into "inside" hoar frost as in loose soil. There will then be a sharp division between the hoar frost and the homogeneous frozen soil.

2. Discontinuous frozen soil (stratified soil)

The structure of stratified frozen soil is extremely variable, depending upon the soil properties, especially the grain size, but also on the structure, water content, rate of freezing and the overlying load. As mentioned before, ice building readily occurs in pre-existing discontinuities: cracks, worm-holes, around stones, etc., through which the structure formation brought about differs markedly from the normal frost structure in massive soil. The upper layers of tillable farm land freeze into a very irregular, greatly porous frozen mass. In the following discussion, we are not so much concerned with the occurrence as we are with the freezing of relatively homogeneous packed soil, such as is commonly found under all top soil and which is the most important thing in highway and railroad structures.

To summarize we can say that under ordinary conditions of soil type and climate, the usual frozen soil structure of relatively compact earth is as follows: The fine grained soils in the frozen condition are composed of layers of clean ice which are essentially parallel to each other and parallel to the surface. The character of these layers depends mainly on the fineness of the soil. In clays, the layers are thick and widely spaced (Fig. 6-11), built in uniform and distinct systems, being more distinct and thicker, the finer (fatter) the clay is. The

* This difference between the top and bottom of the slope, as shall be explained more fully later, depends on the difference in distance to the groundwater table; the groundwater table often rises somewhat above the bottom of the ditch, and hoar frost lies along the line of the water seeping out, while closer to the top of the slope the soil may be quite dry.



Figure 6. Stiff clay frozen for the first time, from the bottom of a $3\frac{1}{2}$ -m-deep excavation for a building. Scale 1:1. (Photo courtesy of P. Kokkonen.)

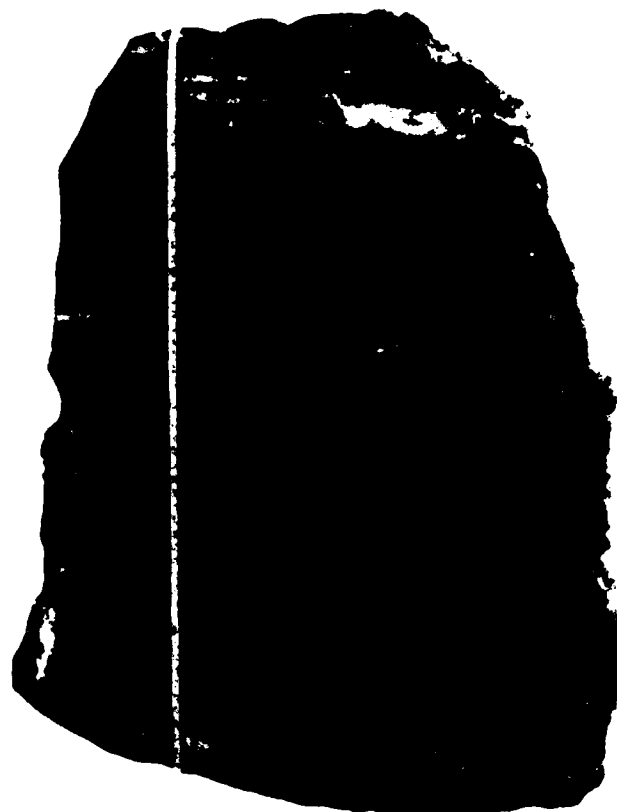


Figure 7. Natural frozen medium clay from Ramvik, Province of Västernorrland. From subbase, near edge of road. The top of the block shown is about 22 cm and the bottom about 70 cm from the surface.

coarser the soil is the less ice there will be, both in thickness and in spacing of the layers. For example, in a silt we find very thin (a few tenths of a millimeter), short (a few centimeters), separate, parallel, and well-oriented layers a few millimeters apart, which gives the mass a streaked appearance. The coarser the silt gets, the finer and more distinct becomes this structure, until finally at a certain range of grain size (0.06–0.1 mm) it disappears entirely. Soils coarser than this freeze totally homogeneous (with the exception of ice-building in pre-existing cavities in the form of "inner" hoar frost).

Aside from this regular structure of ice-banding in fine-grained soils, whose character is largely determined by the particle grain size, there exists, however, ice strata of another character in these soils. These strata consist of unusually plane beds parallel to the ground surface, the ice layers being especially tough and thick, the ice is quite porous, consisting of needle-like ice crystals perpendicular to the strata, the structure containing many air pockets which gives the ice its white color. In clay these thicker strata do not differ very markedly from the normal ice bedding, which is dark in

color. Sometimes, however, there are also air pockets in these beddings, giving similar white layers. There is thus a gradual transition between the two types, making them difficult to distinguish. As an example of this refer to Figures 7, 8, 9, and 10. In coarser soils, in which the normal ice-banding consists of finely dispersed, even, dark-colored bands, there is a marked contrast between the two types (compare Fig. 2 and 32 in Kokkonen 1926). The soil in this case is, as the analysis shows, a coarse silt.

In clays, the coarser ice layers appear only as ordinary layers which have become thicker than usual. The explanation is that at certain levels, the rate of freezing (that is, the rate of heat conduction upward) is less than normal, and the zone of freezing remains constant. The layer then has a chance to grow thicker than usual. Observation has shown this to be true. A section through the soil then will give an indication of the temperature characteristics or variations under which it has been frozen. Notice the thicker ice layers in Figures 7, 8, 9, and 10.

The coarser the soil is, the less frequently does this

Figure 8. Detailed photo of Figure 7. Measuring tape shows depth below road surface.

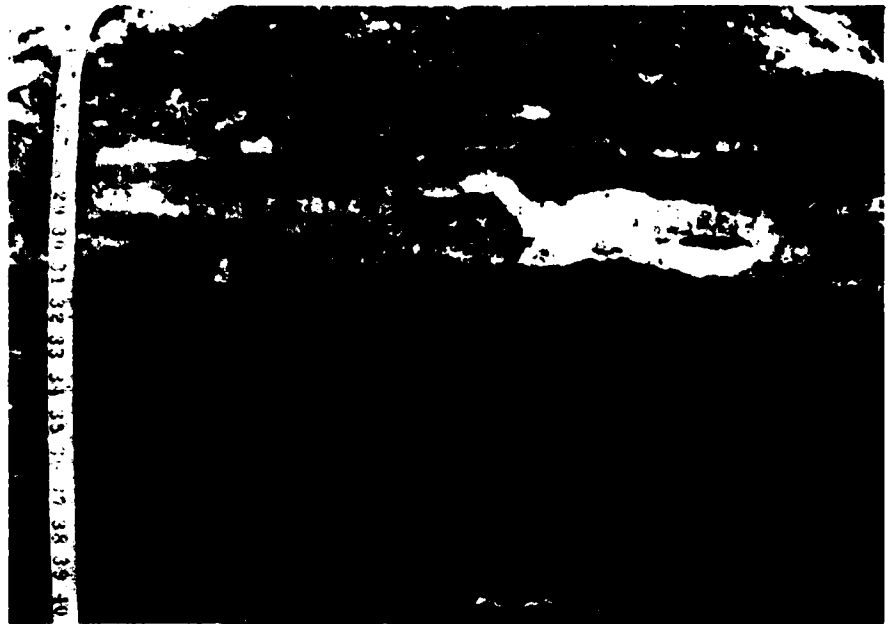


Figure 9. Medium clay from Ramvik (analysis number GB17) frozen in an insulated cylinder at the State testing grounds during the 1927–1928 winter. Observe the two especially thick ice layers (a and b) where the frost line advanced slowly and was stationary (during brief thaw periods). The marked rectangular area is shown in Figure 10.

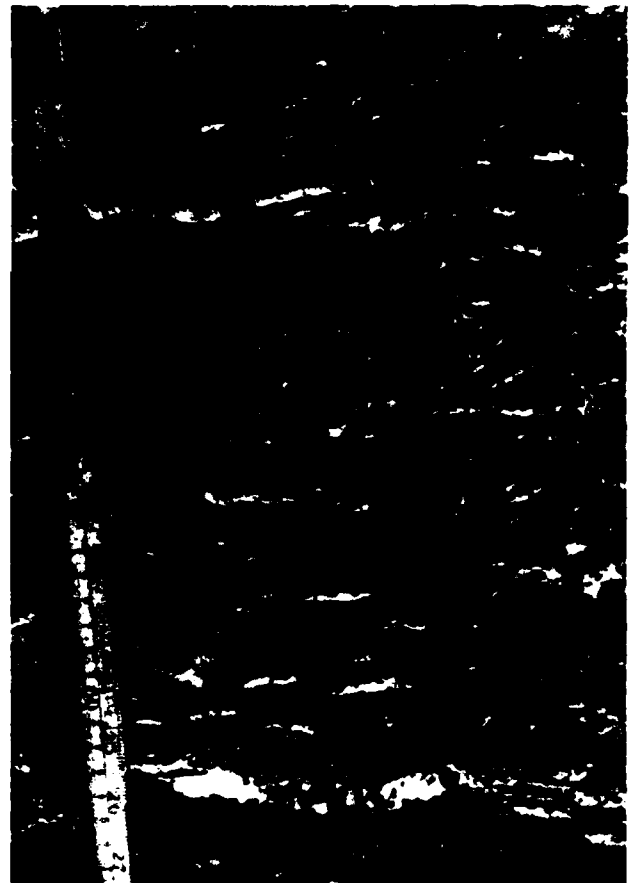


Figure 10. Detailed photograph of Figure 9. The centimeter rule shows the distance from the top of the cylinder.

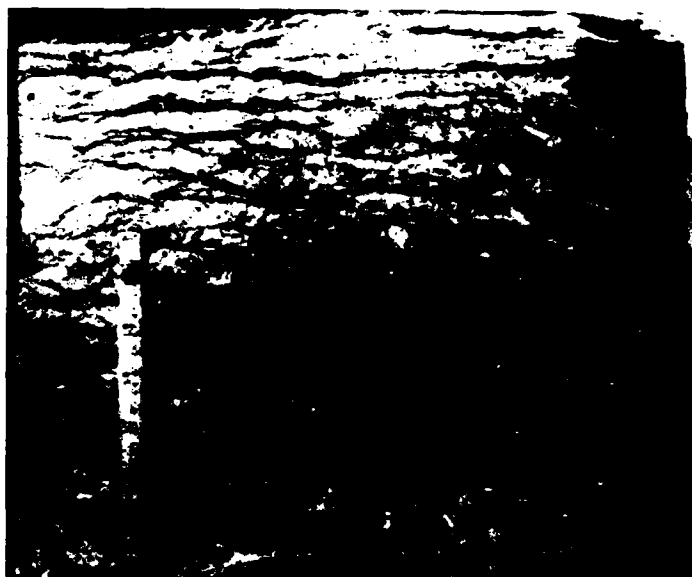


Figure 11. Medium clay from Ramvik, frozen in a cylinder in the State proving ground freezing chamber, May-June 1929. Notice the regular ice layered structure.



Figure 12. Lean clayey silt from Southern Sunderbyn (analysis number GB 20), frozen in a cylinder in Proving grounds freezing chamber, May-June 1929.

type of thick ice layer formation occur. But we may see, however, even more of the thick layers in coarser soils. We must be careful here to distinguish between two different causes. In coarser soils, these layers are caused by pre-existing discontinuities, especially at the boundary between different strata. In all the cases the author has investigated, the coarse ice layers (in coarser soils) were always formed only in pre-existing discontinuities, and this has been verified by the experiences of several road officials. This has also been verified by experiment. An experiment carried out by freezing three sample soil cylinders at the same time consisting of a medium clay, a clayey fine silt, and a somewhat clayey silt, respectively, in which the temperature of the freezing chamber was constant for a period of time, showed in clay thick ice layers throughout the cylinder, but in the silt and the fine silt merely a closer spaced normal ice stratification (see Fig. 11 and 12).

It has been already mentioned that in sandy soils, the cause of ice growth is the existence of discontinuities, usually fissures or cavities. If no fissures or cracks exist or if there are no thin layers of fine material, no ice layers can occur.

However, in sands, if an ever so thin layer of fine material, a silt, fine silt or clay seam exists, an appreciable ice layer can form under favorable circumstances. This gives the impression that the coarse sand has become ice-stratified. This occurrence may be of considerable practical importance, for while the sand may appear at the surface to be non-frost-heaving, the existence of thin layers of fine silt underneath may make the ground strongly frost-heaving.

In clays, thick ice layers also occur at the surfaces of discontinuities, caused mainly by the stratification of the soil itself. In all soils, stratification is the usual occurrence, and has therefore a large significance in the structure of the frozen mass, in that the strata plane becomes the starting place for ice crystallization. As an example of how remarkably parallel the ice strata become in a varved sediment, refer to Figure 15. Naturally in a varved clay with slow freezing, many thick and uneven ice layers can grow, because there are many places at different levels at which the ice can start to grow.

Such ice layers occur especially along the very distinct boundary between two soils, for example between a sand and clay or fine silt (Fig. 17). A few cases follow:

1. A road in Västerbotten Province. During a trip in May 1927 there was observed on a road pavement after the snow had melted away, a high (few decimeters)



Figure 13. Frozen silty soil, Iisalmi, Finland. Scale 1:1. (From Kokkonen 1926, Fig. 34; photo courtesy of P. Kokkonen.)

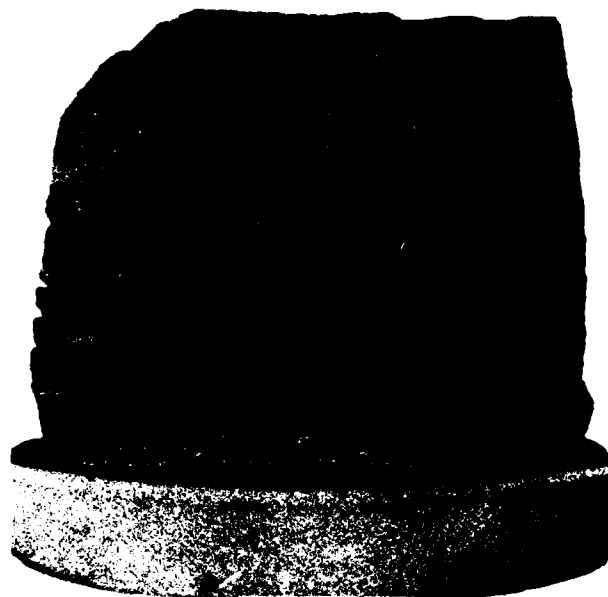


Figure 14. Varved lean clay before freezing in jacket. Arrangements: piece of clay placed as shown in photo, insulated on its sides with powdered cork, surface cleaned bare, and put out for freezing.



Figure 15. The same piece of clay as in Figure 14 after freezing. The clay froze with the free surface and the isothermal lines parallel to the stratification, both causing the ice layers to form in parallel planes (in spite of freedom for sidewise expansion).



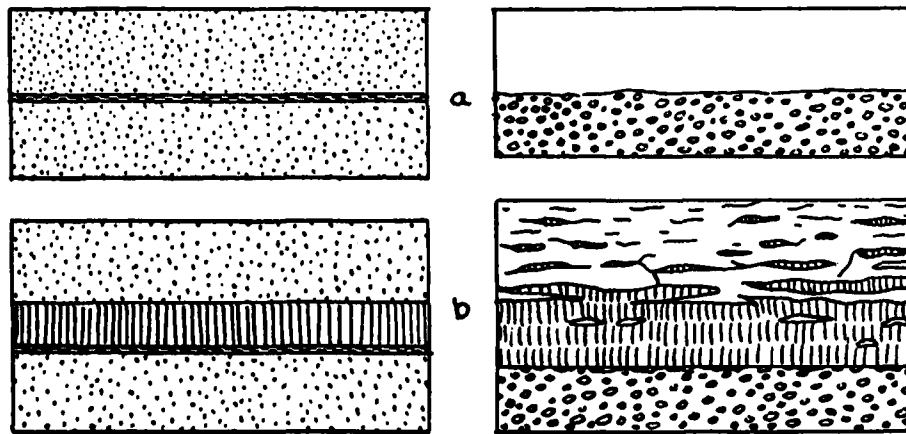
Figure 16. Another piece of the same varved lean clay as in Figures 14 and 15, frozen from the top in the position shown in the figure. The isotherms thus are almost transverse to the stratification. The ice layers then are a compromise and freeze across, along and at an angle to the stratification.

hump in the road, a few meters in diameter. After digging, a layer of clean ice about 20 cm thick was found at a depth of about 80 cm. A short distance away, a frost boil appeared.

Upon examination of the place in September of the same year, it was found that the ice layer had come up in one place, where to one side the sediment cover or moraine ended at the ground surface. The sediment consisted of fine silt and gravel, and the depth to the bottom of this moraine was 7–10 dm at the place where the heave appeared. A borehole showed the moraine material to be quite saturated.

2. At Tärana village (in Västerbotten Province) it was found that a stretch of road 3 m long had lifted 60 cm. At this place, a single ice layer, which reached the entire width of the road, had a thickness of 35 cm. The place is in a low location with fine silt on top of bedrock. The large ice layer was formed at the boundary between the fine silt and the coarse silt fill material in the 15- to 20-cm-thick road bed.

Discontinuities of another kind also influence ice structure formation. These are air discontinuities, those that occur in the cracks of the dried upper portion of varved clays, which upon freezing become filled with white ice. Discontinuities such as borders between stones, pieces of wood, worm holes, etc., are also filled with white ice. The ice layer becomes thicker on the under side of the object, and thicker the bigger the object is.



a. Single ice layer in a clay or fine silt strata in otherwise non-frost-heaving sand. (Practical importance: Because of the occurrence of such insignificant, unnoticed layers, a sandy soil can be frost heaving.)

b. Very thick ice layer at the bottom of an ice stratified clay or fine silt layer, which lies on top of a saturated gravel or sand. (Note: In both cases it is assumed that there is a high water table, so that the coarse soil next to the ice layer is saturated.)

Figure 17. Example of the origin of particularly thick ice layers, caused at the boundaries between soil layers; a = unfrozen, b = frozen condition.

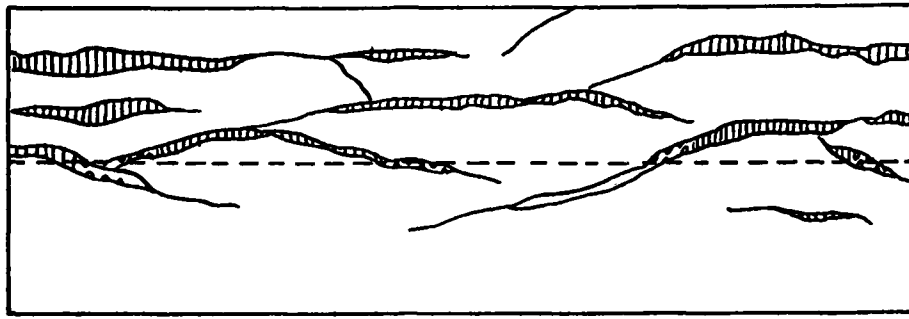


Figure 18. Schematic illustration of the usual appearance of the frost line in clay undergoing freezing. Ice crystallization occurs principally in open fissures which are progressively formed below the frost line. The lowest ice layers continue downward as air-filled cracks. Dashed line = frost line, i.e., the level where the ice layers end. Somewhat beneath this, single ice crystals usually occur. It is important to note that the clay itself is unfrozen a short distance above as well as under the frost line.

It is interesting to note that at the frost line, the freezing of ice in fissures causes the fissures to widen and work their way progressively downward (Fig. 18). At the frost line, the freezing stops but the open fissures extend a few centimeters deeper.

This phenomenon is much more frequent and has a larger tendency to occur, the more fine-grained the soil is. Under ordinary circumstances this is met in all freezing clays; in silt soils this has not been observed, and may not occur at all. This is not a necessary or characteristic phenomenon for ice-stratification or frost-heaving to occur. The author's and Taber's investigations have shown this; Taber has carried out tests on freezing under conditions that exclude all possibility of a gas phase in the freezing soil system, and has obtained entirely normal ice-stratification and heaving. The air fissure zone under the frost line, therefore, occurs only under certain circumstances, especially those which occur under natural conditions, and is of a considerable theoretical interest.

An occurrence that is of great significance is the fact that the soil between the ice layers is not completely hard and frozen. Just to what degree this occurs depends on the temperature in the soil and the grain size. Near the frost line (where the temperature is only slightly below 0°C) the soil between the ice layers is unfrozen and soft even in silty soils, while at lower temperatures (with colder weather and as the ground surface is approached) the soil very quickly becomes hard. In clays, on the contrary, not only near the frost line, but also higher up, the soil is just as soft and plastic as it is under this level. Thus in clays at moderately low temperatures, the soil itself between the ice layers is soft and unfrozen.

B. The process of soil freezing

1. The lowering of the freezing temperature in soil

It often occurs that the soil between the ice-strata is

unfrozen, or only partially frozen, being less frozen the finer the soil is. This fact (first mentioned by Holmquist [1897], but overlooked by most of the later investigators) obviously shows that the water in the soil pores has undergone a lowering of the freezing point. This lowering is in itself the key to the stratified frozen soil problem.

Some investigators have said that the freezing of water in soil at a temperature lower than 0°C is an "instability" phenomenon due to supercooling. Hamburg (1915), on the contrary, says the lowering of the freezing temperature is definite and stable, pointing out that in colloidal substances (gels) this can amount to a few degrees centigrade.

The first known systematic investigation of this water freezing phenomenon is that of G.J. Bouyoucos, who published a detailed article of his findings in 1915–1921. He used two methods for his investigations, the freezing point method and the dilatometer method. The first uses the common method of reading temperatures at frequent time intervals of a thermometer inserted in a freezing sample. The beginning of freezing can be determined by a sudden drop from the gradual lowering of the temperature (often first a slight lowering due to previous undercooling). The dilatometer method is based on another principle, that of the expansion of water upon freezing. This is determined by enclosing the sample in a graduated glass vessel, and filling it with a suitable liquid (for example, kerosene or ligroin [a petroleum ether]) and allowing the dilatometer to cool in a suitable cooling bath, and reading the change in height of the liquid surface. The increase in volume of the soil sample due to freezing (which is obtained after correcting for the change in volume of the glass container and the liquid) is taken as a measure of the quantity of frozen water.

A part of Bouyoucos' results are shown in Figure 19.

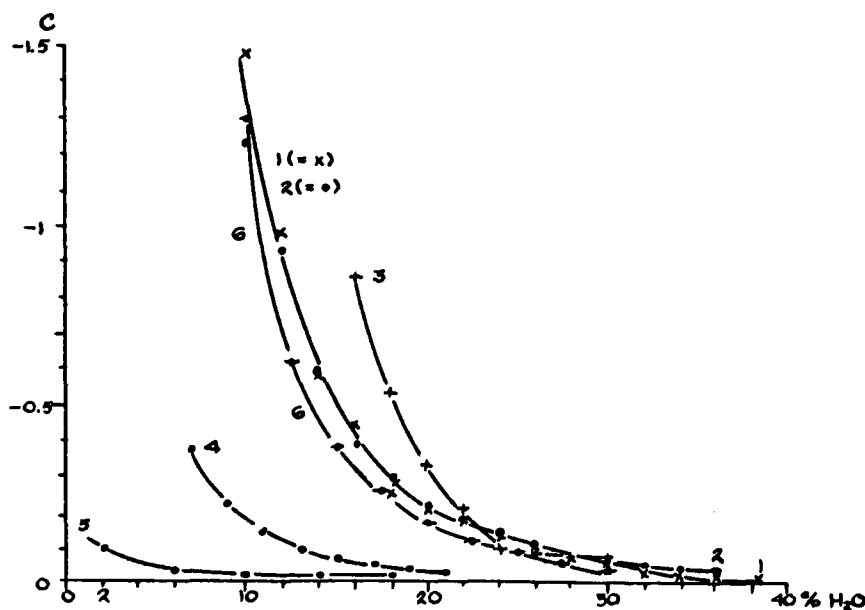


Figure 19. Curves of the lowering of the freezing temperature for different types of soils plotted from Bouyoucos' figures, obtained by the freezing point method. 1- "Humus loam," 2 and 6- "clay loam," 3- "silt loam," 4- "sandy loam," 5- "quartz sand."

His purpose was at the start to determine in this manner the percentage of dissolved salts present in the ground-water. He has in his later publications proposed that the presence of these dissolved substances was the cause of the temperature depression. His theoretical conclusions, which I shall discuss later, are based entirely on this idea. Because of other soil samples, which generally have a considerably larger content of dissolved salts than is usual in our regions, the lowering of the freezing point is naturally caused by both the salts and the effect of the particle system.

Lately E. Jung (1932)—from the theoretical discovery that the lowering of the freezing temperature is dependent upon the soil particles' adsorption power—published the results of his careful experiments by the dilatometer method, which are shown graphically in Figure 20. It is seen from the diagram at first glance, however, that the curves have little accuracy. The cause of this lies in the inherent technical difficulties, in that the error in the correction for the liquid and the sample container's volume change can be very large, and possibly also in the method itself, in that the change in volume of a low water content cannot be measured accurately.

The author's method is by direct determination of temperature readings while cooling or heating, and constructing the temperature-time curve, and determining from this the temperature at the beginning of freezing, and freezing point of the free water.

The possibilities of errors, especially in the determi-

nation of the freezing of the small last quantity of water, are certainly even in this case considerable, depending on both technical imperfections and errors in principles due to approximations, but since the accuracy is greater and the method itself is simpler, this principle is believed to be superior to the dilatometer method.

The particular apparatus used in this case is shown in Figure 21. In order to protect the stem of the thermometer during freezing, it is placed in a nickel- or silver-plated case, filled with alcohol or mercury for good conductivity. In most cases the usual thermometer graduated to $1/10^{\circ}\text{C}$ is used, but for especially accurate determinations of a very small decrease of the freezing point, a special thermometer graduated to $1/50^{\circ}\text{C}$ is used. The determination of the thermometer corrections or calibrations were accurately determined before and after each test by the determination of the freezing point in freezing and melting distilled water.

The sample is held in a tube made of a poor heat-conducting material called "isolit," which keeps the rate of cooling or heating and, therefore the temperature difference between the outer part and the inner part of the sample, small. For cooling, a eutectic mixture of a salt and ice is used; BaCl_2 is mostly used, the eutectic temperature being -21.6°C (the bath temperature being $1/2$ to 1 degree higher). For warming, water of a higher temperature is mixed in the water bath.

The measurements are made graphically, by constructing on millimeter-ruled paper the temperature vs time curve. As an example, Figure 22 shows each curve

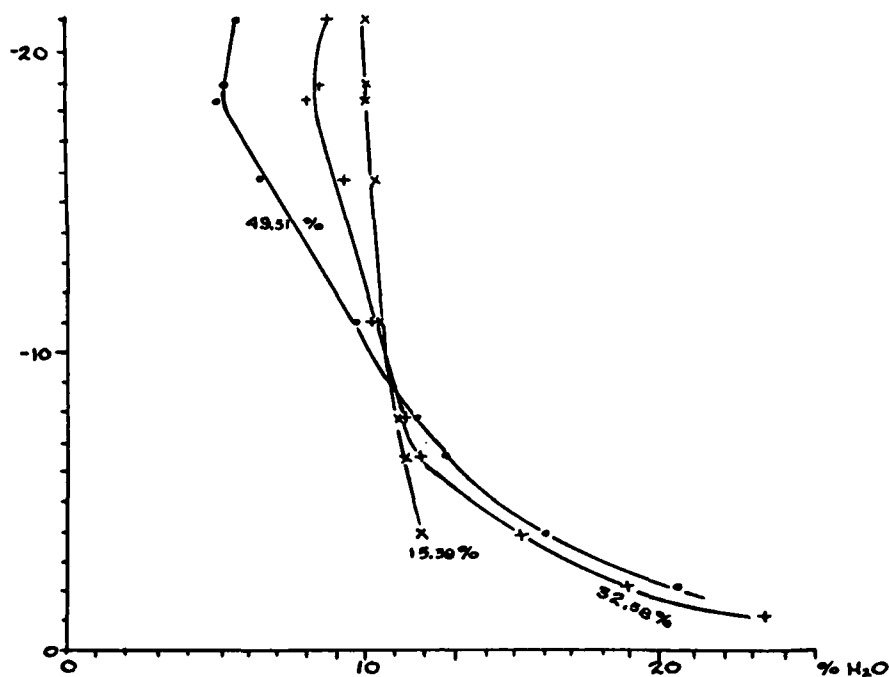
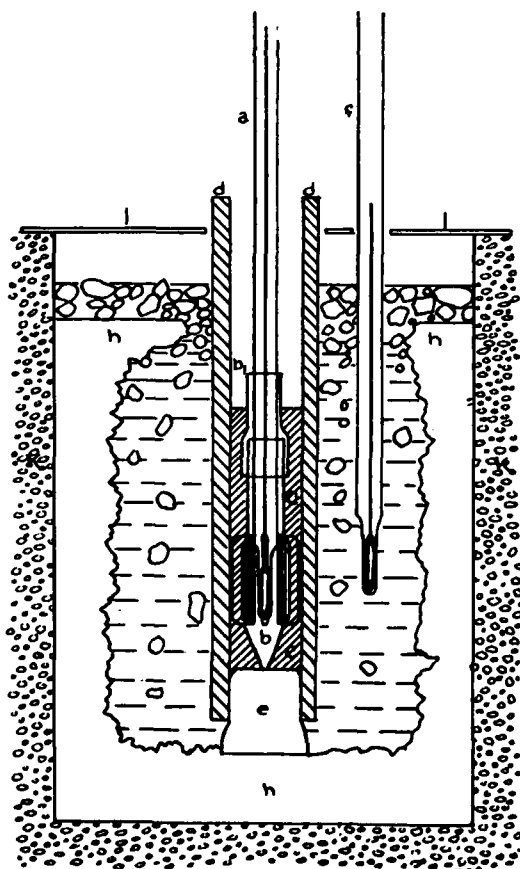


Figure 20. Freezing temperature curves for the same soil (stiff clay) for three different original water contents plotted from Jung's figures, obtained by the dilatometer method (Jung 1932). Diagram shows, by the very unlikely appearance of the curves, the inaccuracy of this method (compare Fig. 22 and 23).



consisting of an upper, steep hyperbola, which reaches a little below 0° , and then a nearly horizontal curve, corresponding to the freezing of the free water, and continues more steeply and slowly changes into a lower hyperbola, representing the cooling of the totally frozen sample.

The hyperbolic curve has the following form:

$$\log_e \frac{T-y_0}{T-y} = c(x-x_0)$$

where y = temperature

x = time (y_0 = starting temperature, x_0 = starting time)

T = the temperature asymptote ($y = T$) that represents the bath temperature

c = constant, depending upon the temperature coefficient and specific heat of the soil container and soil sample.

Figure 21. Apparatus for the determination of the freezing point of soil. a-thermometer, b-case of nickel filled with alcohol or mercury and having eight fins for better heat conductivity, b₁-rubber tube, c-soil sample, d-insulating tube of "isolit," e-rubber stopper, f-thermometer for the determination of the temperature of the cooling bath, g & h-cooling bath, consisting of g-salt solution, h-eutectic ice-salt mixture, crystallized on the walls of the container i, k-pulverized cork, l-cover.

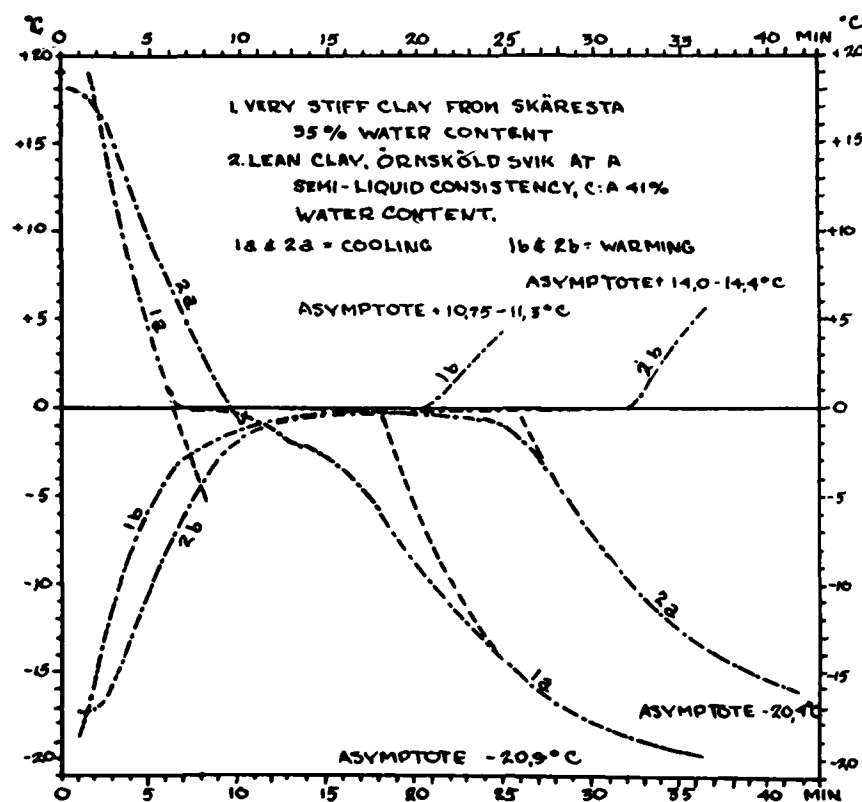


Figure 22. Temperature-time curves for two different types of soil, for cooling and warming, to determine the freezing point for different water contents. From the deformation of the harmonic hyperbolic cooling-warming curves, the curves for the freezing temperature for different water contents can be constructed (Fig. 23). 1—very stiff clay, 35.6% water content; 2—lean clay, at a loose consistency, at about 41% water content; 1a and 2a—cooling; 1b and 2b—warming

If we put $x_0 = 0$, we get the following form:

$$cx = \log_e \frac{T - y_0}{T - y}$$

which can be written as:

$$x = \frac{\log_{10} \frac{T - y_0}{T - y}}{\log_{10} e^c}$$

By the use of this formula, it is easy to construct the continuation of both hyperbolic curves (dashed line in Fig. 22). The constant c of course is different in the both branches, since the temperature coefficient and the specific heat of ice and water are different. However, these components vary in the same manner and effect as the soil sample's heat coefficient in that they partially compensate each other, and since these are small in comparison to the values of the sample container, the

properties of the container are dominant, and the difference in c for both hyperbolic curves become quite indistinct. The lower hyperbolic curve corresponds in form to a parallel continuation of the upper. The determination of the soil freezing curve—that is, the percent of water which is frozen at a certain temperature—is thereby easily obtained.

If we assume as a start that the temperature asymptote is very distant, that is to say, the temperature of the cooling or warming bath is quite low or high respectively, and if we call the distance from a certain point (for example -2°C) to the extended upper hyperbola a , and the whole distance between the extended hyperbolas ℓ , the amount of water frozen q , in relation to the total water content is:

$$q = \frac{a}{\ell}$$

In this manner enough points can be obtained to construct the desired freezing curve (Fig. 23).

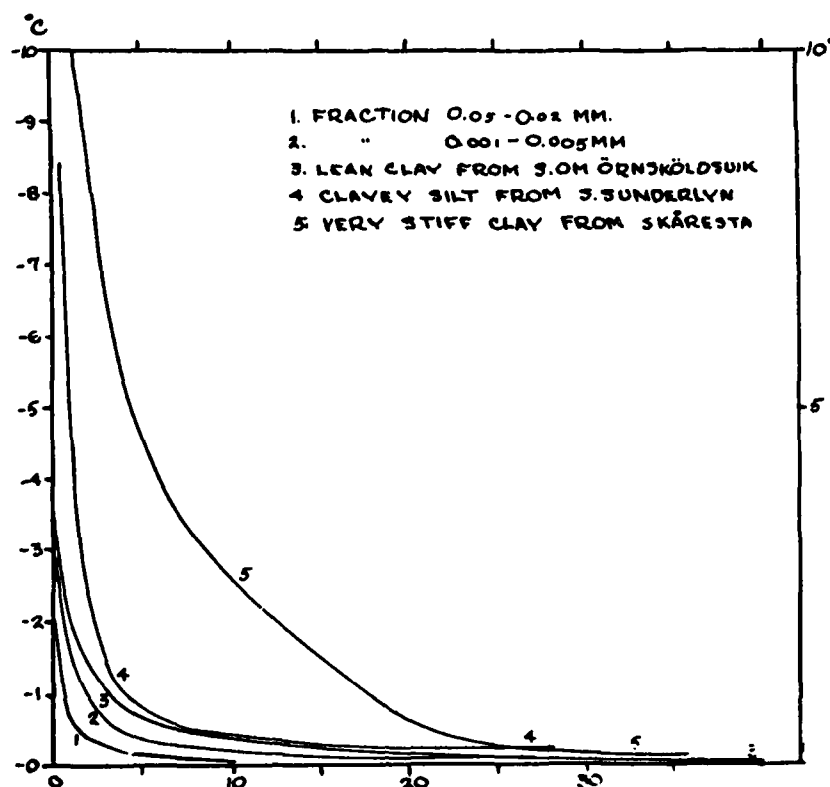


Figure 23. A number of freezing temperature curves for different types of soils, determined by the thermometer method, and obtained from the curves of Figure 22. Water-content given in % weight of dry soil. 1—Fractions 0.05–0.02 mm; 2—fractions 0.001–0.005 mm; 3—lean clay from Örnsköldsvik; 4—clayey silt from S. Sunderbyn; 5—very stiff clay from Skåresta.

Actually, the temperature asymptote distance is not infinite; at least when fine-grained soils are concerned, a suitable correction must be used.*

The obtained temperature curves, of which parts are given in Figure 23, show the following (for capillary saturated soil) similar features: The curves begin just above the 0°C line (x-axis), further from zero the finer-grained the soil is, and curves up gradually, becoming steeper and asymptotic to the y-axis.

In a capillary saturated soil, a certain amount of water freezes first with almost constant, but a very slowly lowering, temperature. The more water that is frozen, the lower the freezing temperature becomes, and becomes very low for the last remaining water. The freezing temperature is lower for the same water content (that is, the temperature decrease is greater), the finer-grained the soil is.

The explanation of the lowering of the freezing point

is obviously the attraction of the water molecules to the soil particles' "adsorption power," which is important in many other properties of fine-grained soil, hygroscopy and plasticity, etc. The water molecules are grouped together in a skin around the soil particles. This skin consists of many thousands of layers of molecules, the innermost layer being bound the strongest, and the next, next strongest, etc. During freezing an "extra-force" (lowering of freezing point) is needed to pull the water molecules from the skin and place them in the structure of the ice crystals. This "extra-force" is proportional to the strength which the molecules have in the adsorption skin; that is, how far in, or how near the water molecules are to the particle walls.

The action of the force of adsorption which is expressed as the "hygroscopy" can be explained by the following analogy: For the force of adsorption, an "excess pressure" is necessary in order to change the water meniscus into vapor, that is to say, the same thing causes a decrease of the relative vapor pressure, and the decrease is larger (that is, the water molecules hold together tighter) the less the number of molecular layers in the meniscus.

* The important fact is that the amount of heat energy given up or absorbed in unit time is directly proportional to the distance to the temperature asymptote (temperature difference between the sample and the bath).

Zunker (1928) has shown that the force of adsorption is practically directly proportional to the surface area of the particles, down to a grain size of about 0.002 mm; for smaller grain sizes there is a definite increase, but it is not as fast as the increase of the specific surface. It is therefore quite natural that the decrease in the freezing point for each water content is larger, the finer grained the soil, that is, the larger the specific surface.

The freest water being in the center of the pore space furthest from the particle walls naturally freezes first. This water is essentially what is called capillary water; but between this and the extremely tight bound water nearest the particle walls lie layers of molecules (adsorption water) which have a continuous change from the one to the other. Capillary water (or pore water) and adsorption water are not differentiated by a sharp border of different kinds of water; the radius of influence of the force of adsorption theoretically extends out an infinite distance from the particles, but rapidly decreases in magnitude. The finer grained the soil is, the larger the relative volume of adsorption water films is—a part of these related mechanical soil properties are treated in Beskow (1930c, p. 52–53 and 47). The water which is the most important in this problem is the freest water, the central pore water, which freezes first and determines the starting temperature of freezing. In order to get a quantitative measure on the influence of grain size on this, the temperature has been determined (which is quite easy to determine very accurately, even though the freezing curve is almost horizontal) for a series of samples of sorted fractions of different grain sizes, with the same water content in the samples (capillary saturated with a constant but very small capillary pressure). The samples used were based on Atterberg's original mate-

Table 2. Freezing point (temperature at beginning of freezing) in different soil fractions from Atterberg's data.

Fraction (mm)	Capillarity K (cm)	Particle size d^* (mm)	$\sqrt{1/d}$	Freezing-point (°C)
0.5–0.2	25.5	0.235	2.06	–0.009
0.2–0.1	40.7	0.147	2.60	–0.015
0.05–0.02	186.0	0.0322	5.62	–0.031
0.01–0.005	820.0	0.0073	11.7	–0.067
0.005–0.002	1700.0	0.00353	16.8	–0.092

* Calculated from K.

rial, and is the same material on which the author determined capillaries (Beskow 1930c). The results of these are given in Table 2 and Figure 24.

As shown in the figure, the values lie quite close to a straight line through the origin. We can say then that the lowering of the freezing point in the center of the pores is inversely proportional to the square root of the reciprocal of the average grain diameter, if this is called d , the formula for the freezing point depression then becomes:

$$t = c \frac{1}{\sqrt{d}}$$

where the constant $c = 0.0056$, when t is measured in °C and d in millimeters.

The expression for the relation of the freezing point depression to the specific surface (U) becomes:

$$t = c \sqrt{U}.$$

The formula is then only an empirical relation for the series of tests made for varying grain sizes.

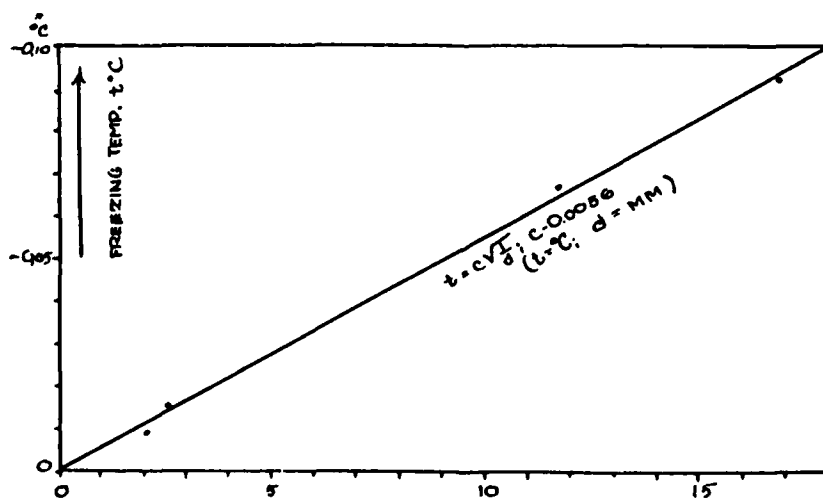


Figure 24. Graphical representation of Table 2, of the last two columns, showing the freezing temperature as a function of particle sizes (a linear function of $\sqrt{1/d}$).

A theoretical detail of interest is that these determinations also give a measure of the radius of influence of the force of adsorption. The largest thickness of the adsorbed water film that has been observed is about 3 to 4 μ (1 μ = 0.001 mm). Far beyond this boundary of measurable mechanical effect, there is, however, a continuous measurable thermal effect of the force of adsorption; in the largest fraction in which a freezing point depression was observed (0.5–0.2 mm), the average distance from the pore center to the particle walls is about 40 μ —that is, at a distance of 0.04 mm the force of adsorption still has a noticeable effect.

We can say then that the determination of the radius of influence of the force of adsorption is merely a question of the accuracy of the apparatus (here it is the thermometer); Figure 24 (on which the empirical formula is based) confirms the idea that the radius of influence of the force of adsorption is theoretically infinite, and that the force of adsorption exists even in coarse sands over the whole pore space with a continuous definite magnitude.

2. Ice crystallization in a soil

On the basis of the freezing point depression in the pores of a soil—or in more general terms the general effect of the force of adsorption, of which the freezing point depression is a part—the discontinuous freezing in fine-grained soils and the significance of grain size can be understood.

Let us consider a saturated fine-grained soil such as a clay, which has been cooled to below 0°C. Even the freest water in the center of the largest pores is within the radius of influence and has a considerable freezing point depression; the water in the clay pores can therefore begin to freeze at a temperature definitely under 0°C. If there exists, however, a discontinuity ever so small; a crack, a flaw, a contained foreign solid, etc., this discontinuity represents a surface of weaker force of attraction on the water, that is, a higher freezing point than in the clay itself. Ice crystallizations therefore begin in such places; and due to the force exerted by the crystallization, the cracks widen and grow sidewise, with the consequence that crystallization of ice continues in the newly made cracks, which in turn grow wider, etc. And similarly, if a few pre-existing discontinuities exist in a soil (in a dried or frozen clay), ice crystallization must begin somewhere (the border plane between homogeneous clay strata is naturally a discontinuity, which explains why crystallization begins here); from the resulting crystallization small cracks are immediately formed which spread as explained above.*

* On the surface of a piece of wet clay that is subjected to quick freezing, a needle-like ice structure is built; crystallization continues inward from these needles in the form of thin lenses.

This explanation has been confirmed by numerous observations of freezing clay both in the field and in the laboratory. This explains how all air-discontinuities, cracks, fissures and other void spaces, insect and rodent holes, etc., become filled with ice, and also how ice forms on contact surfaces between contained solids: stones, pieces of bark, etc. One can often see how an ice-band under a stone or bark chip continues sidewise out into the clay as a normal ice-band. But most important of all, the crack zones immediately below the frost line explain just how the ice strata can form by successive filling of cracks, which build anew by the action of the ice-crystallization in prying open new cracks.

From the above reasoning it becomes very evident why discontinuous ice formation decreases with increasing grain size and at a certain maximum size does not exist at all. The coarser the soil is, the less the difference is between the discontinuities and the normal pores. In a moderately coarse soil, such as a sand, the freezing point depression in the pores is so exceedingly minute that crystallization can occur practically as easily in the pores as in a discontinuity.

This reasoning, however, does not explain completely the structure difference between the fine ice laminae in a silt and the coarse ice strata in a clay, nor does it explain why the rate of freezing and the surface pressure have an effect. Even though it would be possible to explain by the above method of reasoning how these phenomena are effected, the phenomena are much more complicated than this simple explanation makes them appear, and a more detailed analysis is necessary.

Let us examine what happens at the frost line in a fine-grained saturated soil, a fine silt for example. The particles are surrounded by concentric films of water, the binding force rapidly diminishing as the distance from the particles increases. These "adsorption films" act partly to lower the freezing temperature of the water and have also a purely mechanical effect, which can be imagined as a pneumatic elastic rubber skin. Suppose a surface loading acts on the soil system. At the ice-soil boundaries this pressure is distributed on the contact points between the ice surface and the soil particles. But the ice and the soil particles do not touch each other directly, there being the adsorption films between them. The resistance of these films becomes larger the thinner the films are. There is then a balance for a certain loading, that is, a squeezing together of the adsorption films so that their elastic resistance corresponds to the load pressure.

What occurs now if the system is cooled by conduction of heat upwards? If there is a small temperature depression underneath an ice crystal, the water molecules in the film nearest the ice crystal enter into the crystal structure, that is, ice crystallization grows down-

ward. This makes the adsorption-water film thinner, causing an increase in the negative pressure (pressure deficiency) causing water to flow to this surface, and the adsorption film swells until it reaches the same thickness as before. Actually, this whole process occurs continuously.

Ice can grow easier out in the pores, since the force of adsorption is less here and the freezing point higher; the surface of the growing crystals, therefore, becomes somewhat uneven and protrudes down into the pores, similar to that shown in Figure 35. This building of elementary ice layers by projecting in steps can be stopped by three causes: the manner in which the formation of the crystal occurs (the slowness of growth being the causing factor), the temperature gradient, and the projection of crystals reaching down into deeper "films" (with a lower freezing point) of the zone of adsorption forces.

The factors that have been found empirically to affect the formation of ice layers can be easily understood from these theoretical considerations.

3. Factors that determine the type of frozen soil (heterogeneous or homogeneous frozen soil).

a) *Grain size.* We have above considered a relatively fine-grained, discontinuously frozen soil. If we consider a coarse-grained soil, such as a sand, the following occurs: The thickness of the adsorption water films is very small in comparison to the grain size. We will assume that the surface is loaded with the same pressure; this pressure will in the coarse soil be distributed over much fewer points of contact, which makes the pressure at the contact points larger. The adsorbed film then becomes much more "squeezed out"; there will exist an extremely thin film of only the innermost molecules. Moreover, this "shell" is stretched out much further, and the water molecules have a longer distance to travel (compare Fig. 35). The flow of water to the surface of the ice crystals at the contact points occurs by the exchange of molecules—by the molecules from the film changing into ice crystals and being replaced by water that has come up. And the nearer to the particle surface the water molecules are, the less movable they are and therefore bound much tighter, the innermost film of molecules having a rigidity in the order of solid bodies. The coarser the soil is, the more spread out and the thinner the adsorption films become, which means the molecules become less mobile and have a longer distance to travel. For a constant load and a constant rate of cooling, the possible rate of transportation of water molecules at a certain limiting grain-size becomes so small that the flow of water to the ice surface cannot keep up with the rate of cooling, and the ice crystals freeze solid on the adjacent particles—and by this the

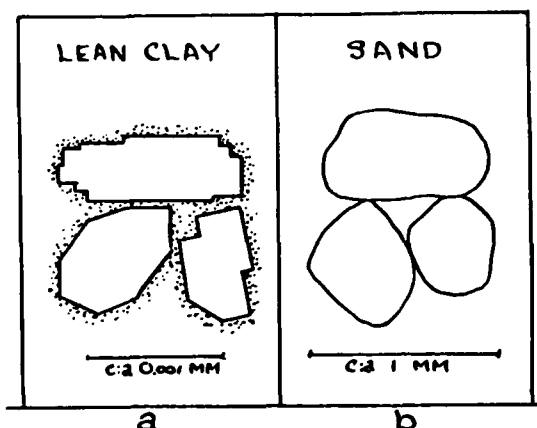


Figure 25. Schematic diagram showing the relative importance of the adsorbed water films in a fine-grained (a) or coarse-grained (b) soil. The stippling illustrates the magnitude of the force of adsorption, i.e., how much the water molecules are adhered to the particle surface. It is assumed both soils are capillary saturated, and are subjected to the same pressure. The sand grains are in direct contact, while the clay or fine silt particles are separated by the layers of adsorbed water.

possibility of ice crystals growing freely is presented. The ice then grows in the pores, surrounding the soil particles, and we have homogeneous soil freezing.

b) *Rate of freezing.* From the above, the effect of the rate of freezing is clear. For increasing rates of freezing an increase in the mobility of the adsorption-water film is necessary to allow flow of water. The heat lost in conversion into ice compensates the heat conducted away from the frost line, i.e., for an increased rate of freezing the frost line moves downward.

Jung has recently published his investigation of this (1932). He has by a special arrangement been able to study soil freezing at different rates under a microscope. The soil sample on an object glass was cooled by a blast of expanding carbon dioxide. The soil used was a stiff clay (specific surface $U = 12,000$; water content 42.5%) which was allowed to freeze at two different water contents, 51.4 and 24.8%, and also at different temperatures varying from -1° to -190°C . It was shown that in general, the lower the temperature, the greater the rate of freezing and the less the amount of water frozen as visible ice crystals and therefore the more finely divided the system becomes.

For the larger water content at -1°C , large ice strata appeared; at -2° , extended, large aggregate; at -10° , scattered large ice crystals; at -25° , finer and wide; at -78° , very fine ice crystals; at -190° , no visible ice crystals were observed* and the stiffened soil mass ap-

* The microphotographs shown were magnified 188 times.

peared homogeneous. At the lower water content there appeared no ice strata, but closely spaced short crystals which practically disappeared at -25° and were completely gone at -78° . We can see then from this how a fine-grained soil such as a stiff clay in a saturated condition can cool so quickly that all ice banding does not appear and even microscopic crystals are prevented.

It should be remembered—because of the small thickness of the sample—that even at moderate freezing temperatures the rate of freezing is very fast, and at the lowest temperatures freezing occurs practically instantaneously.

The same thing has been found by the author (Section II) where series of Atterberg grain fractions from 1–0.5 mm down to 0.002–0.001 mm were frozen at the same time under the same conditions, but with different rates of freezing. For an air-temperature of -2°C and the time required for total freezing at 42–48 hours, 0.1–0.05 mm was the largest fraction that showed noticeable frost heaving and ice-banding, while with a temperature of -14°C , and about 12 hours for freezing, the largest size that showed any ice-banding was 0.05–0.02, and the largest that showed definite heaving, 0.01–0.005 mm.

The rate of frost penetration in the apparatus shown in Figure 36 was $\frac{1}{3}$ to 1 cm per hour or 8 to 24 cm a day. This rate is approximately ten times as large as usually occurs in natural freezing. It has also been found with this apparatus that the coarser frost heaving soils—nearly all silts and also some clay free fine silts—either show no ice-stratification at all or only sporadic strata, while these same soils show definite stratification under natural freezing. The cause of this difference is undoubtedly the difference in the rates of freezing.

c) *Pressure.* The effect of increased pressure is to counteract the formation of ice strata by “squeezing” out the adsorption film, thus diminishing the supply of water to the freezing zone. Pressure has the same effect as an increase in the rate of freezing.

d) *Water conditions.* Water conditions are affected by many marked and extremely variable factors. In order to understand this, a short summary of the theoretical principles of the flow of water in soils, including the work of the most recent investigations, would be desirable; for an exhaustive treatment, the reader is referred to Zunker’s excellent presentation (1930).

All flow of liquids (or gases) depends upon the existence of a pressure difference, which causes a continuous flow from a higher to a lower pressure, in this case being the flow of water through soil.

The pressure difference and the consequent flow depend on the difference in height, therefore on the slope, and the water flows in the same manner as occurs in a river, only inside the soil pore space. In very coarse soils, crushed stone and gravel, the water flow occurs

entirely in this manner. But water flow can also occur when, for example, a dry soil column comes in contact with water, caused by capillary adsorption.

The forces of capillary pressure are known as surface stress conditions; the concave menisci (border between air and water) in the pores try to straighten themselves out, and exert therefore a “suction,” giving a pressure deficiency, whose magnitude is inversely proportional to the pore diameter and can be expressed in the usual hydrostatic pressure measure (mm mercury or water). This measure of the capillary pressure of soil can be defined statically as the maximum height to which the capillary pressure can lift a water column, but it acts also with considerable force as a dynamic effect, that is, as capillary adsorption, which exists as a pressure deficiency directly under the menisci which is equal to the capillary pressure.

In fine-grained soils the behavior becomes considerably more complicated, due to the fact that the adsorption film around the particles has a considerable thickness relative to the size of the particles. The mechanical effect of this film around a clay particle is to exert an expansive pressure effect. In a saturated clay the particles are separated from each other, surrounded by water films, which act as a pneumatic elastic rubber skin. With increasing load pressure, the film becomes thinner; with diminishing pressure, it tries to expand, thereby changing the volume of the whole system, decreasing or increasing it. But the actual soil particle volume remains the same, and therefore the volume change occurs in the voids, which are in this case filled with water. An increase in the load pressure, therefore, causes a consolidation of the soil which squeezes out water. With decreasing pressure the soil tends to expand, and even does this, if it is possible for it to suck up the water necessary for the increase in volume.*

The compressive force acting on the particles can consist partly of the actual load pressure, that is, the pressure of the overlying mass, and also—and this is the most important—of the positive capillary pressure effect on the entire capillary system.

If we have a uniformly thick horizontal tube with a water column in the center, both menisci try to draw the water column towards its end; the result is a pressure deficiency which we call capillary pressure. But at the same time, the menisci exert a compressive effect, a positive force on the tube (Fig. 26). Or if the tube stands vertically with one end in water, the capillary water column is pulled up by the meniscus to a height which

* This compression and swelling can easily be illustrated experimentally; for several reasons, however, stiff colloidal clay does not conform very well to this principle, because of its unusual tightness (very low permeability) which has a tendency towards an “irreversible aggregate structure.”

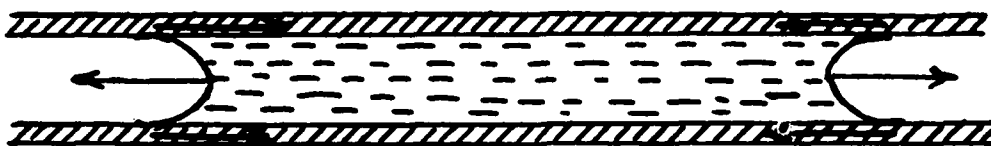


Figure 26. Partly filled, horizontal capillary tube, both menisci pull with equal force, causing a pressure deficiency in the water. The tube is subjected to an equal compressive force. A similar effect occurs in soils.

corresponds to the capillary pressure. This acts as if the meniscus is fastened to the tube holding up the weight of the column and direct compression exists in the tube equal to the negative capillary pressure.

The same occurs if instead of the single capillary tube, the capillary system consists of pores of a soil; the capillary water columns act as if they were a load pressure of the same magnitude acting on the surface of the capillaries (for experimental results see Beskow 1930b), and the menisci exert the same compression as if they had a certain compressive pressure.

If we have a piece of clay of a plastic consistency, the pores are saturated and at the boundary between the water and the surrounding air there exist menisci curved inward. These menisci exert a definite pressure deficiency in the pore water, and an equal positive compressive force on the soil. If water evaporates from the surface of the clay, the menisci curve inward, causing an increase in the capillary pressure, and the soil particles are pressed closer to one another. The pore volume is decreased and the excess water squeezed out is lost by evaporation. This process naturally occurs continuously so that during evaporation the menisci curve inward more and more, and the compression of the clay gradually increases. This process continues until the menisci are fully developed and the particles touch one another, or until the negative pressure in the pores reaches the soil capillarity and the menisci and also therefore air advances into the pores.

Conversely, the soil swells when put in water because of the expansion of the menisci and the flow of water into the pores.*

The characteristics of flow of water in a fine-grained soil is therefore, in summarizing, the following: The

* In most cases, however, the process is not completely reversible. Note also that swelling usually requires a much longer time than shrinking due to the fact that at the thickness of capillary film existing the expansive pressure is small. A piece of soft clay with an inside pressure of $1/10$ atmosphere, for example, can easily have a capillary pressure in the menisci of a few hundred atmospheres, and the water therefore—having a pressure drop in the order of 100 atmospheres/cm—can be squeezed out of the clay very fast. When the soil is put in water, the water is first sucked up at the same large pressure, but the pressure decreases very rapidly and approaches the original pressure deficiency inside the pores, i.e., about $1/10$ atmospheres. Also, due to the irreversibility of the process, the original volume cannot be attained even after swelling an infinite length of time.

elastic-adsorption films represent a stable reservoir supply which, by causing a slight decrease in pressure at a point when evaporation occurs, causes a flow of water to that point by the tendency to equalize the pressure. This pressure change is transmitted inwards, and straightens out by straightening of the films, but occurs slowly, due to the dampening effect of the films. The water thus flows slowly. The process of the rate of flow is then regulated by the porosity of the soil and the viscosity of the water.

The effect of grain size, then, is a double one. For decreasing grain size the pores become smaller; therefore the resistance to flow gets larger and the thickness of the adsorption film becomes larger, therefore also decreasing the amount of water that can flow out for a certain increase in compressive pressure. Both of these factors act together in causing a slower spreading of pressure change due to the loss of water at a certain point in a soil, which is to say that the change in potential is transmitted slower the finer grained and soil is; the amount of mobilized water becomes greater, and the mobility less.

Now in a freezing soil in which ice is forming, the water next to the ice strata changes from a liquid to a solid state and has the same effect as a change into vapor by evaporation. At the frost line a drying-out occurs, a squeezing together of the adsorption films, which spreads further and further, causing a shrinkage. If now this zone spreads to a place of contact with a free supply of water—to the free water in a submerged sample or the flowing groundwater in the fissures of a clay—the water brings to flow upward from this point, and the fundamental requirement for an appreciable volume increase and a subsequent frost heave is fulfilled.

In this phenomenon soils act quite different from one another, depending on their grain size. In a clay (assuming that it is saturated and under a small capillary pressure) the amount of water “mobilized” is large due to the pressure increase, and is larger the stiffer the clay, which explains why even without contact with a free groundwater supply considerable ice stratification can occur. In stiff clays the amount of mobilized water becomes so large and the permeability so small, that the “zone of compression” can only have very small spread (which in natural ground means it cannot reach to the groundwater table) and consequently cannot suck up

water. On such ground, therefore, there is practically no frost heaving, and the entire ice formation is caused by a local flow from the unfrozen clay nearest to the ice bands (Fig. 27, 2a and 2b).

The phenomenon is quite different in a coarser soil such as a silt. Due to the relatively small thickness of the adsorption films the amount of water that can be set free by a pressure increase is very small, whereas the permeability is quite large. If the soil is not in contact with free water, the mobile or free water is quickly used up in ice formation (thin ice bands) at the first point of freezing, and therefore no ice layer can form in the rest of the soil (Fig. 27, 1a). If, on the other hand, the soil is in contact with a free groundwater supply at not too long a distance away from the frost line (Fig. 27, 1b), the water that flows up and the consequent increase in water content at the frost line can be very large due to the relatively high permeability, which results in considerable frost heaving.

Groundwater conditions are treated further in Section IV.

4. Critical grain size between heterogeneous and homogeneous soil-freezing (frost heaving and non-frost-heaving soils)

From above we have seen in what manner various factors have an effect on whether a soil becomes ice-stratified or not, and also that the critical grain-size between heterogeneous and homogeneous freezing soil is not constant but depends on the prevailing factors. It was shown that it is possible by instantaneous freezing to have even a stiff clay freeze homogeneously. In nature, however, the variation of this factor is very limited and the so-called critical grain-size can be fixed within a relatively close interval; in earlier works of the author the critical size was first established as 0.06 mm (Beskow 1929a), and later as 0.1–0.06 mm (Beskow 1930e).

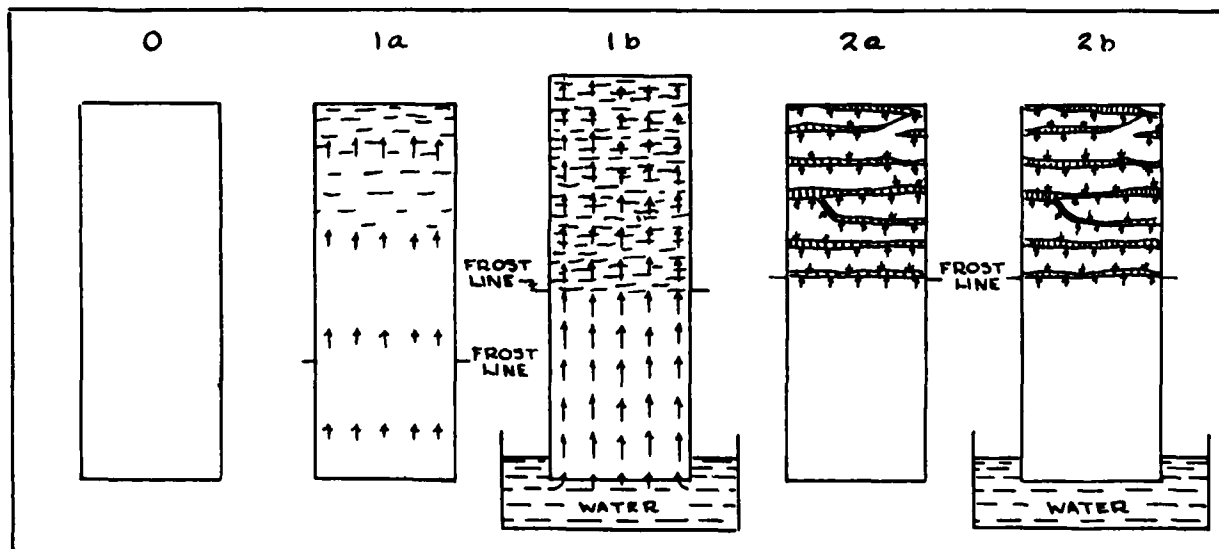


Figure 27. Diagram illustrating the difference between the freezing of a silt (1) and a fat clay (2), without (a), and with (b) access to free groundwater. The difference between the two is that soil (1) has a small amount of "mobilized water" but has a large permeability, while (a) has a large amount of "mobilized water" but a very small permeability.

In (1a) without access to free water, the small amount of available water flows quickly to the frost line and forms ice strata in the top part of the frozen soil; since it diminishes rapidly and ceases completely, the ice bands therefore quickly become thin and sparse, and finally cease all together. The total expansion becomes very small. When there is free access to water (1b) however, water is continuously sucked up, and the entire frozen soil becomes ice stratified, accompanied by a considerable expansion (frost heave).

When the fat clay freezes, the large amount of available water flows from the adjacent parts of the soil cylinder to the ice bands; due to the enormous tightness [structure], the zone of suction force does not go far, and the flow becomes entirely local. Also, when the soil column is in contact with free water (2b), no water is sucked up; 2a and 2b therefore are alike. The expansion in both cases is small. Fat clays become richly ice banded, but are hardly frost heaving.

The medium soils (fine silts, and lean clays) behave intermediately: they become noticeably ice-stratified even without access to free water, and with free water, become considerably ice enriched, and appreciably frost heaving.

It should be noted that in all cases it is assumed that the soil cylinders are capillary saturated from the start to the same degree that is usual in natural soil when freezing begins under our climatic conditions. The more water clays have from the start, the larger becomes the resulting amount of ice; when soft clays freeze, the ice stratification is especially large.

In natural soils it has been found that definite ice stratification is always necessary for a soil to be appreciably frost heaving. This has been observed in so many cases that it was concluded in preliminary work that the formation of separate ice layers is fundamental to the process of frost heaving. Laboratory experiments have shown that this is by no means the case: a soil can with rather quick freezing show a large amount of heaving without showing any ice strata under the most minute observation. As we have seen above, at a rate of freezing of about $\frac{1}{3}$ to 1 cm per hour (approximately ten times the rate in ordinary ground) almost all silt soils and even some clay free fine silts freeze homogeneously or show only a small amount of ice-banding near the surface. In these cases the heaving consists practically entirely of the additive effect of small increments of expansion between the particles.

The difference between this and natural freezing is due primarily to the lower rate of freezing, but also to the fact that the structure is less homogeneous, thus providing fewer discontinuities in which ice layers can start. In natural freezing it is then true that as a rule frost-heaving soil is practically always ice-stratified and we can say that the critical size between homogeneous and heterogeneous frozen soil is therefore the same as the critical size between frost-heaving and non-frost-heaving soil.

The value of the critical grain size has been determined by two different methods. One method consisted of a laboratory investigation of a series of soil samples of different grain sizes, and the other method consisted of gathering samples of the coarsest frost-heaving soils from different localities (either ice-stratified frozen soil or soil from places of frost-heaving) and making a grain size analysis of these samples.

The freezing test was carried out in the following manner. Nine of Atterberg's pure fractions were used as the sample material, with grain sizes of 1.0–0.5, 0.5–0.2, 0.2–0.1, 0.1–0.05, 0.05–0.02, 0.02–0.01, 0.01–0.005, 0.005–0.002, and 0.002–0.001 mm. The samples saturated to a thick consistency were packed in paper cylinders impregnated with paraffin, 5 cm high and 3.5 cm in diameter, and were tied at the bottom with linen cloth through which water could flow. The cylinders were placed in a metal tray filled with a sand bed covered with filter paper and which was filled with distilled water to a few centimeters above this. The test cylinders were then weighed, placed on the tray into a box of cork dust, packed with cork dust so that only the top of the cylinders protruded, and the box was then put in the refrigerator. The starting temperature of the apparatus was the room temperature, about 20°C. The temperature of the refrigerator was thermostatically regulated and could be held at a very nearly constant temperature. For tests 1 and 3 the temperature was held at –2°

(variation between –1.4 and –2.6°), and for the second test at about –14° (variation between –12.5 and –15.5°).

In both of the first tests the paper cylinders remained whole. However, the samples showed a tendency to freeze fast to the walls, causing a large pressure in the freezing cylinder which was noticed by an expansion of the walls and a bulging of the cloth bottoms. In order to eliminate this resisting pressure, the paper cylinders were cut from the top edge to about 1 cm from the bottom (see Fig. 28). Before freezing the stiff cylinders held together as before, but only a very small pressure was necessary to open the cylinders. The cylinders opened up in a conical form as the frost-heaving occurred freely. The amount of ice formed in this case became larger.

After the soil samples were frozen all the way through, the ice underneath was melted and the water dried off and the samples weighed again. The appearance of the samples was recorded (form, size, noticeable ice stratification, etc.) and where necessary (for the whole series except the last test), the frozen samples were cut in half to observe the frost structure.

The results are given in Table 3 and Figure 29. Notice that the considerable evaporation from the surface of the samples occurred after freezing started, which therefore was not caused by being sucked from below. The evaporation was shown by the fact that soil was all dried out for a few millimeters below the surface. Weighing the samples showed that the coarser sample had a definite loss of weight. For the three largest fractions, which showed no ice-strata at all, the loss in each case was practically the same: in the diagram the curve was straight and nearly horizontal. In order to get the actual increase in weight due to frost heaving, it is necessary to lower the base line to the place where the lines start to curve upward.

The following is a summary of the tests:

The three coarsest fractions, 0.2–0.1 mm, showed either no ice-strata or relatively little; these are essentially "safe from heave." On the other hand the fourth fraction, 0.1–0.05 mm, for slow freezing showed a small but definite increase in weight. This is seen in Figure 29 where this fraction shows the first rise from the horizontal. For tests 1 and 3 the point of tangency lies about at the 0.1-mm size.

The fraction 0.1–0.05 had a discontinuous structure (tests 1 and 3). The stratification was of course very thin; actually the ice was built from needles of ice crystals in the soil which grew downward in the freezing column. They did not extend down more than a centimeter; lower down, no traces of ice-stratification were noticed.

The next fraction, 0.05–0.02, on the other hand, showed stratification throughout the whole column. This soil column was, therefore, entirely heterogeneous.



Figure 28. The nine paper cylinders with soil after freezing of test number 3 (Table 3 and Fig. 29). The split sides of the cylinders are turned to the front; the amount of expansion can be seen in the picture.

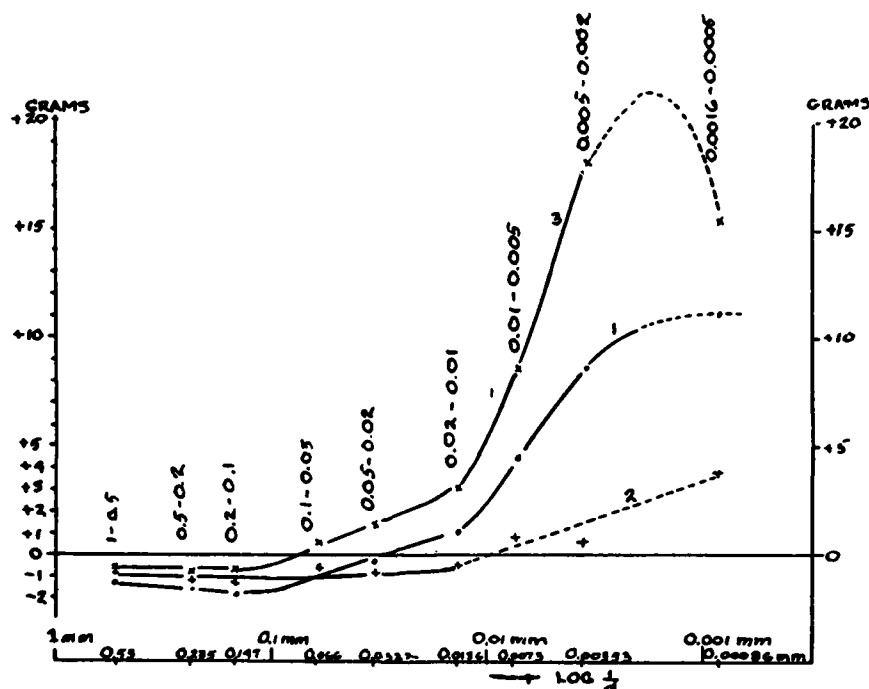


Figure 29. Relation between frost heave (amount of water sucked up during freezing) and grain size for pure fractions (Atterberg's classification). Curves give amount of water sucked up in grams for different fractions vs grain size in millimeters (graphical representation of Table 3).

The following fractions showed with a gradually increasing magnitude an increasing stratification. It should be noted that horizontally oriented ice stratification was first noticed for the 0.01–0.005 fraction; at almost this point the curve becomes much steeper, starting at the 0.01-mm size.

For fast freezing (test 2) the curve does not show any definite rise until 0.01–0.005 mm; before this there is only a small gradual rise. Fraction 0.1–0.05 showed no ice strata at all; fraction 0.05–0.02 was the first to show ice needles on the surface and very sparse stratification below. Therefore, we can conclude that the critical size for ice stratification has shifted for quick freezing from between the third and fourth fractions to between the

fourth and fifth or in other words from about 0.1 mm to about 0.05 mm. Thereafter a considerable amount of frost heaving occurred, starting with fraction 0.01 to 0.005.

Similar tests have been performed on natural soils with comparable results.

We can conclude from these tests that above an average diameter of 0.1 mm practically no stratification (discontinuous freezing) occurs; for more rapid freezing, the size is reduced to 0.05 mm.

The author has previously found the critical size between frost-heaving and non-frost-heaving soil to be at the separation size between coarse silt and silt, 0.06 mm (Beskow 1929a), and has further found that in special

Table 3.

No.	Fraction			Change in weight (g)			Remarks (for no. 3)
	Grain size limits (mm)	Capillarity K (cm)	I/d computed from cap. (1/mm)	Exp. no. 1	Exp. no. 2	Exp. no. 3	
				Freezing temperature			
				-2.2°	-14°	-2.0°C	
1	1-0.5	11.3	1.88	-1.4	-0.9	-0.6	Non-ice-banding
2	0.5-0.2	25.5	4.25	-1.65	-1.2	-0.8	Non-ice-banding
3	0.2-0.1	41	6.79	-1.9	-1.3	-0.7	Non-ice-banding
4	0.1-0.05	99	16.5	-0.55	-0.7	+0.5	Small but visible banding at surface (to 7 mm depth)
5	0.05-0.02	186	31.6	-0.3	-0.85	+1.35	Sharp thin ice bands throughout the whole sample
6	0.02-0.01	440	73.4	+1.0	-0.5	+3.05	Ditto, but more prominent
7	0.01-0.005	820	137	+4.45	+0.8	+8.6	Extreme ice-banding
8	0.005-0.002	1700	283	+8.6	+0.5	+18.05	Numerous and thick ice-banding
9	0.002-0.001 (= 0.0016-0.0005)	7000	1170	+11.1	+3.75	+15.4	Numerous and thick ice-banding

cases it could occur for larger sizes, and gave the interval to be about 0.1-0.06 mm (Beskow 1930 e). We can say that these results agree substantially with the above.

From the other method of determining the critical size, mechanical analysis of natural soils, the results have likewise agreed. For these results, see the conclusions in Section III.

5. The structure of stratified frozen soil

a) *The dimensions of normal ice-strata.* The dimensions of the ice-strata structure, and the quantitative differences between clay and coarser soils can easily be explained from the above considerations; of essential significance in this connection is super-cooling.

The cause of this troublesome ice-stratification which forces its way down through the soil and hardly seems to end with continuous cold and whose growth is determined by the heat conducted away, is principally that the water is not sucked up fast enough, so that the amount of heat conducted upward from the frostline is larger than the heat which is released by the freezing of the water sucked up; due to this there is a super-cooling of the system, an isotherm below, which causes freezing to begin sooner or later below the previously formed ice layer.*

As has been said before, it is noticed that in fine soils, such as clays, an open pore system of cracks is formed along with the ice stratification, which grows continu-

ously downward as the ice forms in the cracks and opens up more cracks ahead. Another contributing cause is the consolidation due to the drying out below the frost line.

In such a case the dimensions of the network of cracks depend on the mechanical properties of the clay, of its cohesion, which determines the tightness of the system of cracks formed by drying out; the larger the cohesion is, or the "fatter" the clay is, the coarser the network of cracks will become.

For a coarser frost-heaving soil such as a fine silt, the phenomenon is the following: when crystallization occurs in the center of one of the larger pores just under the surface, an ice-layer is started which through the force exerted has a tendency to form a little crack. The occurrence of this crack, however, is not facilitated by any shrinking of the surrounding soil; the cracks then only extend sidewise a very short distance. In this manner a bi-convex lens shape is formed in the ice strata; as the soil has very little plastic deformation, the mechanical pressure exerted on such an ice lens becomes considerable. But an increased load pressure has the effect of deterring ice crystallization, and the ice strata soon cease to grow. The coarser the soil is, the less it can shrink and therefore the less plastic it is, and the condition for the formation of such bi-convex lenses becomes more favorable; likewise the effect of a load increase becomes more effective in deterring crystallization. For increasing grain sizes the size of the ice lenses becomes smaller and smaller, until they become as small as the pores.

It is worth noting this detail: the local pressure increase accompanied by the growth of such ice lenses occurs as a local pressure zone (rapidly dispersing) above and below the lens. The next center of crystallization has therefore a tendency to form at the side of or directly under the preceding. Thus each ice lens lies immediately below the one above it.

The dependence of the structure formation on the freezing point depression can be stated as: the finer

* In certain cases, when the water sucked up can keep up with the freezing, a single very large ice layer is built, which theoretically can grow to an unlimited thickness; practically in our locality it can only reach a magnitude of a few decimeters. This process (Fig. 17) is facilitated by the occurrence of definite discontinuities, such as boundary surfaces between different layers. Possibly—and perhaps actually—the ice of Northern Siberia was formed in this manner, over a very long period of time; thicknesses of up to tens of meters are the largest known on record.

grained the soil is, the more difficult it is for crystallization to start at a new place, that is to say, the more continuous the ice layers become, therefore the distance between them will be larger. This rule holds whether the ice system is continuous or not. As a matter of fact in the latter case the following criterion can also apply. With decreasing grain size a larger temperature depression is required for crystallization to begin at a new center, that is the larger the distance becomes between a new layer and the preceding one. There are other influencing factors in the phenomena but subordinate in importance.

The principal reason for the occurrence of successive ice strata, and not merely a single growing ice layer, has been assumed to be that the rate of flow of water upward to the freezing zone is not large enough to compensate for the conduction of heat away. In case the rate of upward water flow is sufficient, a single ice layer should then occur with a stationary frost line. Numerous experiments have shown that such is the case. However, it is probable that exceptions to this exist, depending on another process, which in certain cases has a tendency to prevent the continuous growth of the lowest ice layer.

The cause is the following: at the under surface of a growing ice layer, which presses against the underlying soil with its entire weight of the overlying soil, a temperature somewhat below the freezing temperature of free water exists, let us say -0.2°C . Let us assume that the temperature gradient is not too high, for example 0.05° per cm. This means that the 0° isotherm exists at a depth of 4 cm under the frost line; in this zone we are concerned with minus temperatures. In the pores, free from a load pressure, the freezing point depression is considerably less, for example 0.1°C . This implies that between the frost line and the -0.1° isotherm, a zone of 2 cm immediately below the frost line, undercooling exists. Now it is quite probable that in one or several large pores some distance below the frost line, for example $\frac{1}{2}$ cm, where the temperature is -0.175°C , undercooling occurs spontaneously, and new centers of crystallization are started, each one forming a small ice stratum. This acts to prevent the upward flow of water to the ice layer existing above, which causes it to diminish it more, until the flow of water no longer balances the thermal gradient, and the isotherm progresses downward. The new ice crystal takes the role of the old one, and the old becomes isolated or "dead." This process is again repeated in the same manner.

The accuracy of this explanation is confirmed by direct empirical observations. Below the lowest definite ice layer, under what is loosely called the frost line, small ice-crystallizations are often seen in small cracks up to a distance of a few centimeters under the frost line.

The above process is naturally primarily dependent

on the magnitude of the temperature gradient. If it approaches zero, that is if the cooling occurs almost uniformly, the temperature is nearly the same at different levels. The isotherm system is "thinned out" and a sharp frost line does not exist. Crystallization can just as well occur a little further down as a little higher up; the whole soil column cooled just below 0° , then, consists of closely spaced thin ice strata built at different levels approximately simultaneously; thus, the possibility of upward water flow is rapidly diminished and shut off completely from the soil under the lowest ice strata, and there is no further ice-layer growth.

The stiff homogeneous clays are an exception to the above discussion. In a clay of this kind, no, or hardly any, large discontinuities exist, and the amount of undercooling in the "tight" pore system is quite large, and is the reason why the ice formation in the soil cannot occur spontaneously. In this case a network of cracks is formed in front of the first crystallization which are successively filled and are successively built downward. The crystallization is tied to this discontinuous system and the rate of growth downward is restricted by the rate of crystallization sideways. This structure can also occur if the temperature gradient is very small.

But in general the rule is that the larger the temperature gradient is, the more distinct the frost line becomes and the larger the water content of the frozen soil (other things being constant). Not only is the absolute value of the rate of cooling important, that is, the difference at any given level between the heat conducted upward and the quantity of heat energy conducted from below, but the total heat energy transfer is also important, which quantity is directly proportional to the temperature gradient.

The larger the temperature gradient, the greater is the likelihood of the occurrence of the continuous extreme ice layers that were explained before; the other temperature factor that facilitates this which has already been mentioned is a slow rate of freezing (a small difference between the release and receipt of heat energy). These factors can at least in clay soils cause large extensive ice layers, which sometimes reach a thickness greater than a few centimeters. The process is facilitated, however, by marked soil boundaries or discontinuities where crystallization can occur so easily that no new crystallization can form near it (and thus encourage large single ice layer growth).

b. Factors determining the direction of stratification. The factors which influence the arrangement of ice layers, or frozen soil structure are the following:

1. Pressure, especially its direction and relative magnitude.
2. Temperature, especially direction of isotherms, and also temperature gradient and rate of cooling.

3. The soil structure: existence of a jointed or discontinuous structure.

a. stratification.

b. discontinuities such as fissures.

In general the most important of these factors, namely direction of pressure, direction of the isotherm and stratification, cause a system of parallel ice strata oriented parallel to the ground surface; but in particular on road and railroad structures there can be a considerable divergence for the usual conditions.

Probably the most important factor is the pressure direction. It is essentially the pressure that occurs due to the increase in volume during freezing and not the pre-existing pressure in the soil. If we have a freezing soil column in a stiff container shut tight or a natural ground that freezes over the entire width, it is noticed that the small expansion sideways causes an enormous pressure increase, since no space exists for this expansion. Above, however, there is unlimited space and as soon as ice crystallization starts, it grows perpendicular to the ground surface, the direction of least pressure or of least resistance to expansion.

The action of a vertical pressure on crystallization is obvious. Vertical pressure decreases the rate of freezing and reduces the freezing point by pressing thinner the adsorption water in contact with the soil particles. If in a homogeneous soil the pressure in a given direction is a minimum, crystallization occurs in that direction. It is obvious from this that the successively formed ice-filled cracks usually occur perpendicular to this direction.

In the most general cases, the freezing of natural ground, crystallization and also expansion occur in an upward direction, perpendicular to the ground surface, and the ice stratification becomes parallel to it; the same occurs in a closed strong container.

In the freezing of a free mass of clay the direction of least resistance is much less distinct, and is different in different parts of the clay mass, which has a corresponding effect on the frozen soil structures.

The qualitative importance of the direction of the isotherms is obvious: though in a given soil crystallization starts close to a definite temperature, it is clear that the ice strata have a definite tendency to grow parallel to the surface along which this temperature exists; an ice stratum can not grow more than a very short distance below the surface before the temperature becomes too high. Particularly in the case of thick, large ice layers, such as are formed by a frost line remaining relatively stationary at one level, the direction of the isotherm is of the utmost importance. Generally, in nature, the direction of the isotherm corresponds to the ground surface, that is to say the direction of the temperature drop corresponds to the direction of least resistance, of which

both factors act to orient the ice stratification parallel to the ground surface. Therefore the effect of direction of the isotherm can be seen when it differs from this, which can easily be investigated experimentally.

It is, however, obvious that the magnitude of the temperature gradient, that is, the fall in temperature per unit length, is of greatest importance. The smaller the gradient is, that is, the more sparsely spaced the isotherms are, the smaller is the quantitative importance of definite divergence from a given isotherm; when the gradient approaches zero the isothermal surfaces of course disappear.

The importance of soil structure is most marked at contact surfaces between different layers. It has already been explained how large ice layers can grow on the contact surfaces between a coarser and a finer soil. When the change of layers is repetitious as in varved clays, the entire structure is completely dominated by this: stratification becomes extremely parallel and distinct, with the ice formed in the boundary surfaces.

Under natural conditions the above three main factors usually work together to cause stratification to occur parallel to the ground surface. An example of how the structure is a compromise when there is a divergence of the factors is shown in Figure 16 (compare with Fig. 15).

III. THE PROCESS OF FROST HEAVING

A. To what extent may heaving be caused by expansion through freezing of the water originally in the soil?

It is usually accepted as an axiom that a freezing soil must expand at least as much as the increase in volume due to expansion of water originally in the soil when it freezes, or in round numbers, about $1/10$ of the volume of water in the soil. Practical experiments and simple theoretical considerations have shown that such is not the case. Taber first pointed out (1930) emphatically that this presupposes a closed system, such as a completely closed container. In such a container the water would expand during freezing about 10%. But actually, freezing soil is always in an open system where surplus water can flow out of the pores.

1. By the freezing of water-saturated soils

Assume that freezing takes place in a saturated coarse soil such as a natural sand (open system), where the pore water can flow downward freely. As shown below (part B, par. 1), theoretically and experimentally, in such a coarse soil the water molecules cannot wedge themselves between the particle surface and the ice surface on the frost line, which is why no expansion of

the particle system takes place. Therefore the solid particle system does not change and the expansion due to the freezing of water is taken care of by the forcing out of a corresponding amount of water from the pores. The water content of the saturated sand therefore diminishes during freezing about $1/10$ of the original.

But if the sand is in a closed system, i.e., the water is prevented from flowing away, an expansion occurs analogous to the freezing of water in a closed container, and its magnitude is about $1/10$ of the volume of the water.*

The theoretical explanation is easily understood from the experiments of which the results are summarized in Table 4.

A number of glass tubes 20 cm long and 2.0 cm in diameter having a piston of perforated paraffined cork which is easily displaced on one end, are filled with saturated soil of different grain sizes, from medium sand to stiff clay. After weighing and length measurements, they are placed vertically in pulverized cork in a refrigerator and are frozen with a constant temperature from above. In the first test the tubes were placed on a metal tray which was cooled very fast. The samples were then frozen quickly from the bottom, and thus have a closed system. In the other test, they are fully insulated below and the samples froze only from above, i.e., as an open system. For comparison, a closed container of the coarsest sand with a tight stopper in the bottom was also included. From this, it was shown, amongst other things, that the open sand samples forced out a considerable amount of water, and that they had very little or no expansion.

From the tests shown in the table we can thus conclude the following: Coarse soils—medium sands and coarse silts—act as an open system and do not expand upon freezing; instead the water is forced out and the amount is about $1/10$ of the quantity of frozen water. But as a closed system these sands have an expansion upon freezing an amount up to $1/10$ of the contained water.†

Under ordinary conditions, however, such closed systems are seldom found, and therefore of no importance for road beds.

* Where a gas phase exists, air in the pores can compress considerably under pressure, offsetting the volume increase. In other cases the water can release dissolved gases when it freezes and have an opposite effect (increase the amount of expansion).

† In one test the expansion in one direction (upwards) was about 2% or 2 cm per meter of frost depth. This corresponds to a water content of about 20% by volume, while the actual water content was between 36 and 42% by volume. The expansion was therefore only about $1/20$ of the total volume of water. Part of this discrepancy is due to the fact that some of the water (absorbed films) remains unfrozen, but also due to the existence of air-filled voids.

On the other hand, fine grained soils which freeze discontinuously and which in an open system suck up water if available undergo a very small expansion when there is no supply of water (whether open or closed system). For a given temperature the amount of expansion depends upon the amount of water frozen, which in turn is less the finer grained the soil is.

The temperatures used for the tests (an air temperature of -6°C , and temperatures of the samples at the end of the test between -5°C and -1°C or an average of about -3°C) correspond quite well with the conditions of natural freezing. The surface temperature of course varies with the local climatic conditions, but one can say that the temperature of the ground varies approximately with the surface temperature but evened out (dampening of variation). There is a large fluctuation where the snow covering is small, such as on roads which are plowed during the winter.

In addition, the author has carried out detailed determinations which are explained later in detail (Fig. 36), on soil samples packed in segmented glass tubes which have practically no wall friction (Fig. 30b) and the amount of heave measured for small time intervals. From this a curve of heaving versus time is plotted (Fig. 30a).

The soil used was a fine silt of very liquid consistency designated as BW 19, and had a capillarity of 9.6 m. The water content at the start was 21.1% by weight (always measured on basis of dry substance). The material was very thick but became liquid when shaken.

As the curve in Figure 30 shows, there is a large amount of frost heaving at the start of freezing, having the same rate of heave as determined for capillary saturation under the same load pressure. Then a very small amount of water is sucked up to the frost line, under hardly any capillary pressure. As more water is drawn up, the capillary pressure becomes greater and the rate of heaving diminishes, and the curve bends over. Finally, the rate becomes constant and the curve is a straight line. The rate is then 0.087 mm/hr. It is interesting to note that this is the same rate of heaving as if there was full capillary flow, but with a pressure acting equal to the value of the maximum capillary of the soil (about 1000 gr/cm³). Thus what happens is the following: the existing excess capillary water—that is, the water you get by squeezing the soil—has flowed to the frost line, while the pressure gradually increases. But when the pressure becomes equal to what is called for short “the capillarity of the soil”—the maximum capillary pressure—it has reached its limit and does not increase any more but remains constant, and the rate of frost heaving also becomes constant. The water flow to the frost line is therefore rapidly reduced, and the pressure approaches the maximum pressure existing at the frost line.

Table 4. Results of freezing tests of water-saturated specimens, completely frozen specimens and moist, unsaturated soil samples.

	<i>Water-saturated</i>					<i>Completely frozen</i>				
	<i>Dimensions of specimens</i>		<i>Water content before freezing</i>			<i>Change of length</i>		<i>Weight reduction</i>		
	<i>Length before freezing</i>	<i>Diam (cm)</i>				<i>Shrinkage before freezing (cm)</i>	<i>Heave</i>	<i>With reduction for evaporation, etc. (= water squeezed out)</i>		
			<i>Total (gr)</i>	<i>% by weight</i>	<i>% by vol</i>			<i>Total (gr)</i>	<i>(gr)</i>	<i>% of total water</i>
I. Closed system										
1. Medium sand	2.02	19.1	—	—	ca 36	0	+0.4 cm = 2%	-0.6	0	—
2. Fine sand	2.02	19.2	—	—	ca 42	0	+0.4 cm = 2%	-0.7	0	—
3. Fine silt	2.02	19.45	24.89	26.0	39.9	-0.15	+0.23 cm = 1.2%	-0.35	0	—
4. Stiff clay	2.03	19.25	34.80	49.5	55.9	-0.15	+0.25 cm = 1.3%	-0.5	0	—
II. Closed system.										
1.a. Medium sand	2.03	18.0	21.40	22.6	36.8	0	+0.4 cm = 2%	-0.55	0	—
Open system.										
1 b.	2.02	19.1	21.45	21.9	35.0	0	+0.1 cm = 0.5%	-1.6	-1.05	4.9
2. Fine sand	2.02	18.8	24.80	28.1	41.2	0	+0.0 cm = 0 %	-1.2	-0.7	2.8
3. Silt	2.04	18.8	23.55	25.6	38.4	0	+0.05 cm = 0.3%	-0.35	0	0
4. Fine silt	2.02	19.2	27.44	25.5	39.8	-0.08	+0.2 cm = 1 %	-0.45	0	0
5. Stiff clay	2.03	17.75	30.15	42.8	56.2	-0.08	+0.07 cm = 0.4%	-0.45	0	0

<i>Moist soil samples (not saturated).</i>				
<i>Dimensions and type of soil column</i>				
	<i>No. 1 fine sand, capillarity 54 cm, height 75 mm, diam 32.5 mm</i>		<i>No. 2. coarse sand, capillarity 10 cm, height 65 mm, diam 32.5 mm</i>	
	<i>Test no.</i>			
	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>
a. Temperature gradient (°C/cm)	1-0.6	0.1-0.05(?)	0.2	c/a 0.1-0.0
b. Freezing time	2 hr 30 min	c/a 6ft/hr	6 hr	2 hr 10 min
c. Load (gr/cm ²)	25	a) 25; b) 50	a) 50; b) 110	25
d. Initial water content (%)	10.7	10.9	10.2	9.8
e. Final frost depth (mm)	11	75	38	29
f. Increase (%) of water in upper frost cm of frozen soil by weight (%)	+9.2	+4.2	+8.6	—
g. Total heave (mm)	0.80	0.25	0.13	0.12
h. Water frozen in sample (ht) (mm)	3.17	11.21	8.35	4.4
i. Normal expansion of above in freezing (0.1 h) (mm)	0.32	1.12	0.84	0.44
k. Frost heave quotient (steps g + i)	2.5	0.21	0.16	0.27

This description also explains the significance of the diagram showing the distribution of water after freezing (Fig. 30b).

The experiment shows that in freezing of a saturated lean clay without access to free groundwater, but having a water content greater than that existing under maximum capillary pressure, the excess water flows to the frost line and the soil first frozen attains a large water content. Simon Johansson has already shown this in 1914. The rate of freezing, as the description has shown,

is first large but diminishes rapidly and becomes almost negligible; in the case at hand about 0.25 mm heave for a depth of 3 cm, that is less than 1% of the frost depth. Controlling factors are, of course, the percent of excess water and the porosity of the soil. The relatively porous, coarse silty clays as a rule have very little water excess; in fact in early winter after the fall rains, silty soils loaded with ballast, as on roads and railroads, seldom have more than 3 or 4% excess. At some depth there can be considerable water excess in clays, but due to the

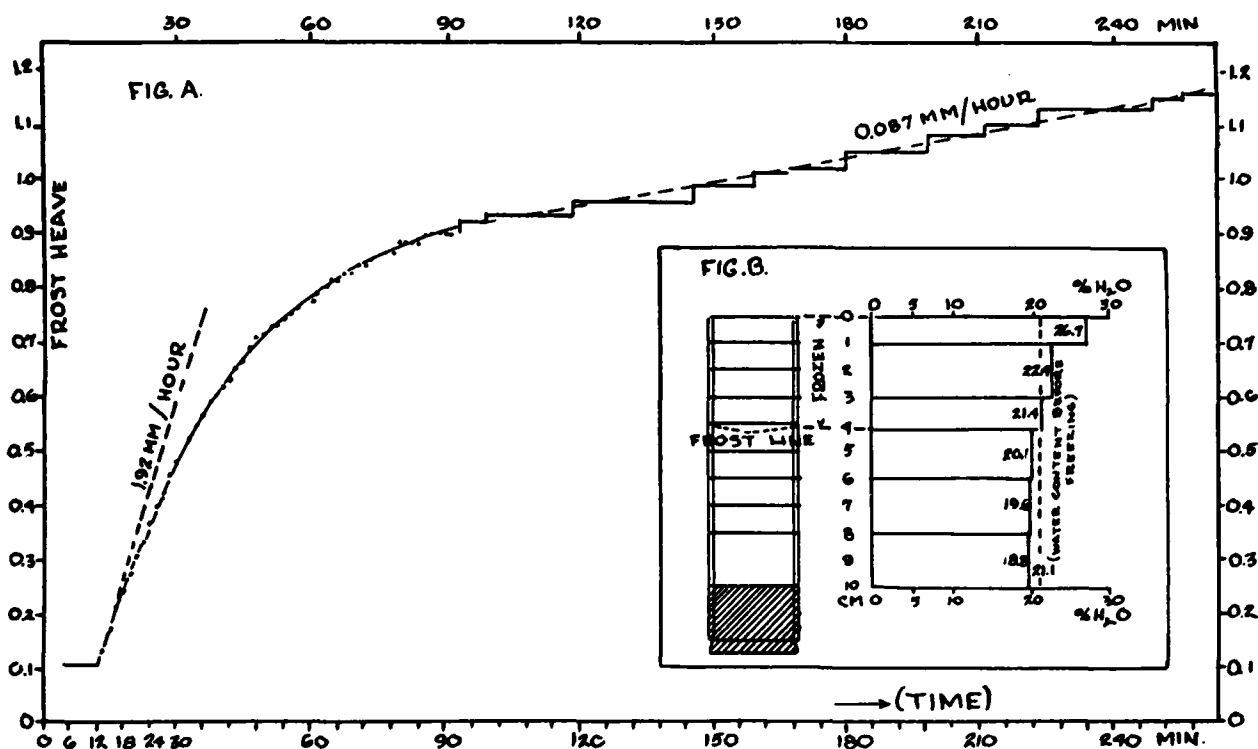


Figure 30.

a. Frost heaving curve obtained for freezing of a capillary saturated silt (BW 190), packed in at the liquid limit, and therefore at a water content greater than natural saturation. The excess water flows at the start of freezing to the frost line (see water content diagram, Figure B). The rate of heave is large at the start, but decreases after the excess water is consumed. The capillary pressure increases until it becomes constant, and the heave basic constant rate ($= 0.087 \text{ mm/hr}$) which agrees with the rate of frost heave determined from the maximum capillary pressure ($= \text{capillarity of soil, } K \pm 9.6 \text{ m}$). Surcharge load during freezing $= 50 \text{ gr/cm}^2$.

b. Soil column (height 10 cm, packed in a glass tube, consisting of eight loose 1-cm rings) and the water content distribution after freezing. (Original water content uniform throughout column $= 21.1\%$.)

tightness of the soil, the excess water can only flow from right close to the frost line.

We can say then that in ordinary cases freezing of soil without capillary connection with free ground is of no significance in road and railroad problems. Exceptions are possible, however. Freezing of a detached spot or a small strip, such as a stretch of road insulated by a snow-covering sometimes has access to a very large surplus of water which can flow in from the sides. Or there might be a very strong frost heaving soil such as a fine silt, for example, on top of the coarse soil (sand) which, due to its large capillarity, can "suck out" a large amount of water from the coarser soil below, even though it does not have access to free groundwater. It is of course possible that both of these possibilities can occur in combination.

2. By the freezing of non-saturated soils

The case of the freezing of partially saturated soil, i.e., one in which the pores are filled with air and the

water exists only as films or menisci known as "Porenwinkelwasser" [Funicular water] is of some theoretical interest. It would seem off-hand that there would be no expansion of the whole system since the air pores would provide ample room for the free expansion of the water when it crystallizes. But apriori it would certainly seem that it is possible the water would not use this possibility of expansion (amongst other reasons the temperature condition at the contact points of the particles) and that there could be an expansion due to crystallization at the contact points (see Fig. 31a).

The method used is the same as above and is explained below (Fig. 36). The soil sample is packed in a glass tube and heaving is measured at equal intervals of time.

Two kinds of sand were used as samples, 1) a fine sand of a capillarity $K = 54 \text{ cm}$ and 2) a coarse sand of $K = 10 \text{ cm}$. The sample, after being moistened homogeneously to a moisture content of about 10% by weight, was packed with uniform density into a glass tube,

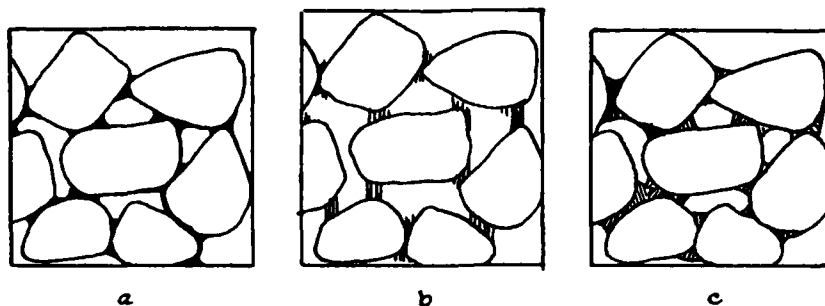


Figure 31. Diagram showing two different possibilities of freezing of non-saturated soil (moist sand) a = the sand before freezing. Its funicular water ("Porenwinkelwasser") at freezing may form either expanding ice needles between the particles (b), causing the soil to expand much greater than the 10% increase due to water freezing, or the ice crystals may grow in the free spaces of the pores (c), and the soil specimen not expand. Between these extremes, all kinds of transitions may occur. The experiments have shown that the phenomenon depends on the pressure. For no pressure [surcharge load], noticeable expansion can occur, but even a small pressure will reduce this expansion practically to zero.

consisting of a bottom tube 5 cm long, half of which was occupied by a rubber stopper, and with five 1-cm-high glass rings. The height of the soil columns were then 7.5 and 6.5 cm respectively. The tubes were placed vertically in pulverized cork and placed in a refrigerator; the bottom was placed in a refrigerator; the bottom was placed in about 2 cm of water (level with the stopper) and a thermometer placed in this, so that the temperature at the bottom could be read (see Fig. 36). Thus it was possible to compute the temperature gradient approximately.

Sample 1 was tested in three different series (different temperature gradients, different rates of freezing, and different loading). The heave and temperature were measured as usual at intervals of a few minutes and the heave vs time curve drawn similar to that in Figure 30.

Part 2 of Table 4 gives the principal results. From this and other observations we can give the following conclusion:

In freezing of moist sand there is an appreciable expansion (heave). This magnitude, however, is quite variable, and is particularly affected by the surface loading. If we consider the expansion on the basis of the percent of water in the soil we find that for a small load (25 gr/cm²) the heave is several times the ordinary expansion of water when it freezes (10%); for increased loading (50–100 gr/cm²) the heave only becomes a fraction of this amount. An increased rate of freezing also tends to diminish the percent expansion.

This implies that for small pressures on the sand the ice forms columnar crystals between the sand grains (similar to the illustration in Fig. 31b) while for larger pressures the expansion of the water occurs essentially in the air spaces of the pores (Fig. 31c). The tests did not determine any quantitative values of the effect of pres-

sure and other factors, but they definitely showed that the magnitude of frost heave in a moist sand is insignificant from a technical standpoint. For a moist fine sand (10–15%) (under no load) of a few decimeters in thickness, the maximum heave will be at the most 1–3 cm per decimeter of frost depth. For increasing depths (which means increased pressure) there is a rapid diminishing of heaving and for $\frac{1}{2}$ -m depth there is only a few tenths of a millimeter heave per decimeter. As an example, the possible displacement of a rail due to the heaving of the ballast sand of the railroad bed can be as much as $\frac{1}{2}$ cm and at the very most 1 cm, which is certainly an insignificant amount.

The very marked increase in water content in the frozen portion of the soil is of interest (Fig. 32). It is probable that this is due to a condensation phenomenon in the air-filled pores, that is, as water transported as vapor and condensed against the cold surfaces of the frost line. The quantitative determination of this is given below.

In such a fine-grained sand as this, it is not possible that convection flow could play a large part. The gaseous water transportation must occur by diffusion, i.e., a wandering of water molecules through the air caused by differences in the partial water vapor pressure. Zunker has determined experimentally the rate of diffusion for sands of different sizes, in which he got very similar values (Zunker 1930, p. 205), on the average 0.0045 gr/cm² for 24 hours and a vapor pressure gradient of 1 cm of water per cm distance. The experiment was started at an average temperature of 19.7°C, and with relatively dry sand (only hygroscopic moisture). When the temperature was nearer zero with more moist sand the rate of diffusion became less. Without using the com-

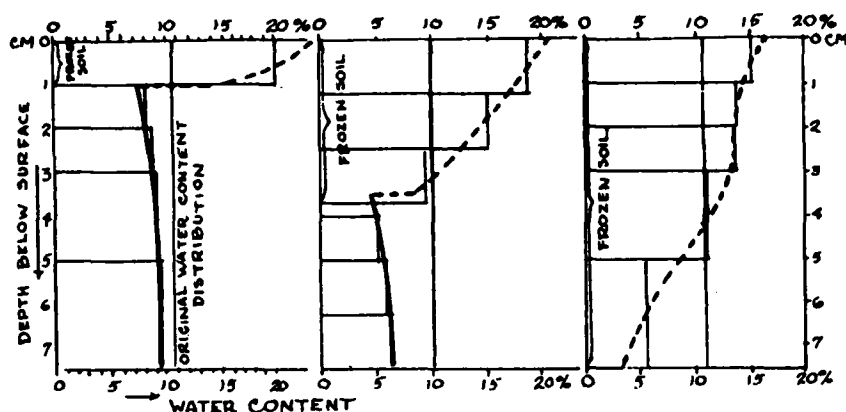


Figure 32. Water distribution after freezing of a column of moist (10-11% water) non-saturated sand (see Part 2 of Table 4). Soil: fine sand fr Bygdsiljum, $K = 54$ cm [capillarity] (test no. 1, 2, 3 in Part 2 of Table 4).

plex general formula given by Zunker, we can temporarily use his given values as absolute maximum.

In the foregoing case, the vapor pressure differences are caused by the temperature. The magnitude of the pressure drop can be found by tables when the temperature is known. When the temperature is near zero the pressure difference is about 0.5 gr/cm^2 per $^{\circ}\text{C}$. For a temperature gradient of $1-0.6^{\circ}/\text{cm}$ the rate of diffusion then becomes $0.0023-0.0014 \text{ gr/cm}^2$ per day. But from test 1, Part 2 of Table 4, the water flow to the freezing soil was 0.18 gr/cm^2 for a length of $2\frac{1}{2}$ hours, or, in round numbers, a thousand times larger than computed by diffusion. For various other tests—though the temperature gradient was considerably less—the difference was still larger.

It is therefore obvious that the transportation of water cannot be explained by diffusion. The cause is probably, instead, the following: In a freezing moist sand, the adsorption-water membranes enclosing the surface of the particle also enclose (or try to enclose) the lower surface of the ice crystals in the frost line. The ice crystals grow in such a manner that the water molecules next to the ice enter the atom structure of the ice. This implies that the membranes or films which are in contact with the ice become thinner. But there is a tendency for the meniscii to have the same stress all over (i.e., same thickness). The decrease in thickness at a point then causes a flow of water to that point. Numerous tests have

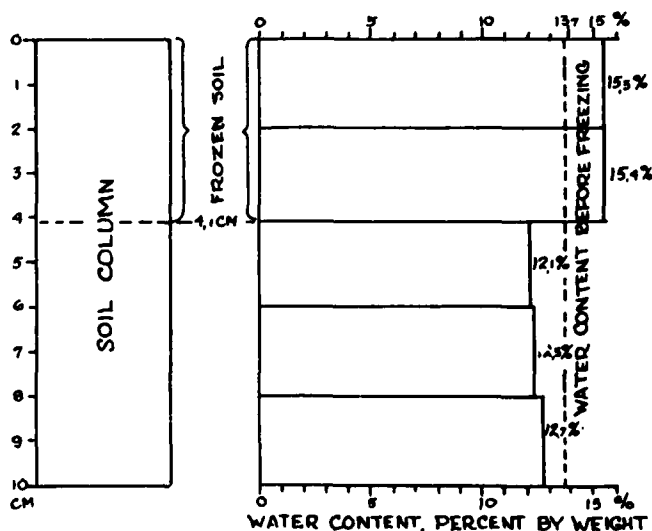
shown (see Holden 1926) that in soils which are drier at one place than at another there is a flow of water toward the dry place due to "migration of the absorption films." The cause of such a process could be that water is taken from a surface by evaporation; i.e., in exactly the same manner the water can be transported in the freezing process by changing into a solid state as well as a gaseous state.

Later in this work a test was carried out on the same fine silt (BW 19) with the same dimensions as in Figure 30, the results of which are given in Figure 33. The initial water content was 13.7% which was about 6% less than capillary saturation. No heaving at all occurred when frozen under the same load as in Figure 30 (50 gr/cm^2), but the height remained constant (within ± 0.002 mm). No spreading of the particles occurred due to crystallization, all the expansion occurring in the pore space (likewise there was a considerable amount of water increase in the zone of freezing as Figure 33 shows).

3. Summary

Under natural conditions it can be said that heaving of ground caused by the expansion of the existing water,

Figure 33. Water distribution after freezing of a column of moist (13.7%) silt (BW 19). Surcharge load = 50 gr/cm^2 ; freezing time $3\frac{1}{2}$ hours. No trace of heaving. Compare with Figure 32.



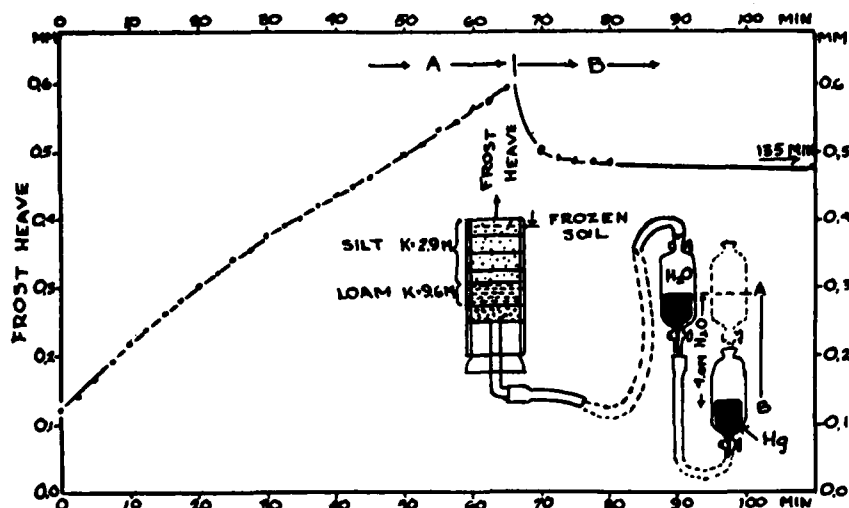


Figure 34. Frost heaving curve obtained with the arrangement shown. The soil is coarse silt with a capillarity of 2.9 m, and there is a layer of fine silt (BW 19) with a capillarity of 9.6 m underneath the silt and on top of saturated sand. At the start of freezing, the capillary pressure was only a few centimeters (positive A of the mercury-fill vessel), and the frost heaving curve A was obtained. Then the vessel was lowered to position B, corresponding to a pressure deficiency of 4.0 m of water. Since the coarse silt had a capillarity of 2.9 m, the capillary surface dropped in the soil column to the top surface of the fine silt, where it remained stationary. This is reflected by a sudden drop of the frost heaving curve, which approaches a horizontal line, i.e., no more heaving occurred although the freezing test was continued for 135 minutes.

independent of any water supply, is under all circumstances very small.

For fine grained soils, which freeze discontinuously, this heave cannot amount to 1% and at most 2% of the frost depth. The largest heave occurs on fine silts, lean clays, and also on stiffer clays of a very loose consistency (surplus water). For stiffer clays of a harder consistency, which are usually found on the upper first meter, the heave is very small, a few tenths percent or a few millimeters per meter depth. However, under natural conditions, fine grained soils are never found in such a condition that no water at all can be sucked up. The above figures are then only for the part of the heave due to the pre-existing water. Even for only moderate frost heaves of one or two decimeters, the part due to this is very small and is only from a few hundredths to a few tenths of a total heave.

* Sometimes natural sands can be in a closed system if the sand lies on an impermeable base: bedrock, clay without cracks, etc., and this base forms a sort of shell or basin. Such a basin in the fall season is naturally filled with water, and the sand in this will heave about 2 or 3 cm per meter depth. Practically this is not important for sands used as a foundation material can hardly even be in such a condition. It should, however, be emphasized that sands in such a situation can be partially ice stratified—in the tests ice stratification occurred in the lower part which froze last, where the water was pressed out from the upper part. If the load pressure is very small, as at the ground surface, such ice-stratification may occur there.

For sandy soils the frost heaving is of no importance at all (except for certain exceptions*). On unloaded soil it can reach 1 cm maximum heave; for a load of about 100 gr/cm² or more it is reduced to zero.

All frost heaving of any practical significance depends upon water flow to the freezing zone. This occurs almost always by capillary flow (capillary suction) (usually from below but can of course occur from the sides depending on local conditions). A good illustration of the significance of capillary flow in frost heaving is shown in Figure 34.

B. Frost heaving caused by water suction to the frost line

1. Theoretical considerations of frost heaving

Since the solid material particles in a soil do not themselves expand when cooled (in fact there is an extremely small contraction) it must be implied that the solid particles separate from each other within the soil mass either all over or in certain places, or at certain surfaces. Studies of the structure of frozen stratified soil show that such is the case, that is along certain places where the soil is forced apart by the growing ice layers which orient themselves generally perpendicular to the direction of expansion.

From these considerations we can say that in order

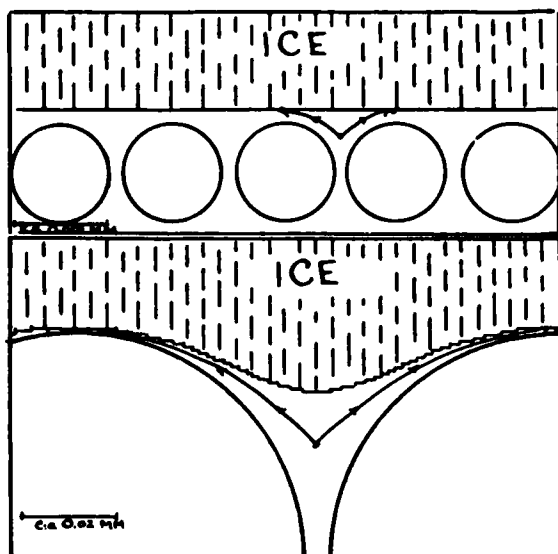


Figure 35. Schematic diagram of the frost line, or the contact between growing ice crystals and the underlying soil particles (drawn as spheres), in two different types of soils, the upper a clay, and the lower a coarse silt. Note that the scale of the upper diagram is about 20 times that of the lower. The points and the arrows show the maximum distance that the water must travel in order to have frost heaving occur.

that growing ice crystals can expand they must push the particles in front, which can only happen if new water molecules can come in between the ice crystals and the particle surfaces against which they rest. Figure 35 illustrates this further.

The diagram shows schematically what occurs at the frost line during freezing for a coarse and a fine grained soil. The outer surfaces of the ice crystals do not rest directly against the particles but lie between the adsorption films. Assuming now that the ice crystals grow from below, displacing a layer of molecules taken from the adsorption films, these films become thinner and they in turn get back their thickness by attracting water molecules from the side (this agrees with the mechanical properties of the films discussed previously).

In a coarse soil the meniscii are very wide and thin. Free passage of water is thus small because it concerns the innermost, tightly bound molecules which are the least mobile. In a fine-grained soil, on the other hand, the meniscii or films are relatively short and thick, and the flow of molecules can occur much more rapidly.

In order to have frost heaving there must be a certain rate of diffusion, or molecular mobility, in the films between the soil particles and the ice. In coarse soils this is small and for practical purposes is zero (in sands); no heaving occurs and the ice grows into the pores and surrounds the particles. On the other hand, in a fine-grained soil such as a clay or fine silt water molecules flow

quickly through the adsorption films to the surface of the growing ice crystals, which thus pushes the soil particles ahead, or actually lifts itself and the overlying mass upwards. In this manner ice strata may be formed which theoretically can reach infinite thickness. If the rate of cooling is greater than the flow of water, new layers of ice crystallization are started further down which cut off the supply of water to the upper layer, etc.

In this manner we can explain the difference in freezing of different sizes of coarser soil. However, the mobility of the adsorption films is dependent on the pressure. For increasing pressure the films are pressed tightly and the mobility is reduced; we should therefore expect that the rate of heaving would be correspondingly decreased. This is actually the case, as Taber has shown experimentally (Taber 1929). In natural ground we can conclude then that for coarse soils (coarse silt) on an unloaded surface (example, surface of a cut) there would be considerable heave, while the soil under the road bed, loaded by the ballast, would not heave very much.

Quantitative values of this phenomenon can only be found experimentally. The experiments carried out by Taber on the effect of pressure were not performed so that quantitative results could be obtained. To evaluate this, more tests were performed, which are explained below.

2. Experimental tests—apparatus and method

The biggest difficulty in performing freezing tests in tubes is the effect of friction between the soil column and the tube walls. When the freezing column lengthens, there is a sliding of part of the column in the tube, and to overcome the frictional resistance an extra pressure occurs which has an effect on the freezing. This pressure is considerable and is usually very uneven. For coarse soils greatly affected by pressure inconsistent results were obtained in earlier tests, and this was caused by the frictional effect. Of course, this effect does not exist under natural conditions.

One way to get around this difficulty would be to do away with the tube, and remove the tube from the packed soil column before freezing. But for certain soils (clayey soils) this would be impossible because of their tendency for flowage sidewise. Also the soil could expand sidewise without any resistance, while in natural ground the sidewise expansion is prevented by a considerable soil pressure which causes the entire expansion to occur upward.

The problem thus resolved itself into retaining the tube container, but reducing the sliding resistance to the least possible. After a few trials, the following arrangement was found to be the most suitable. The tubes consisted of a series of short sections, each placed on top of

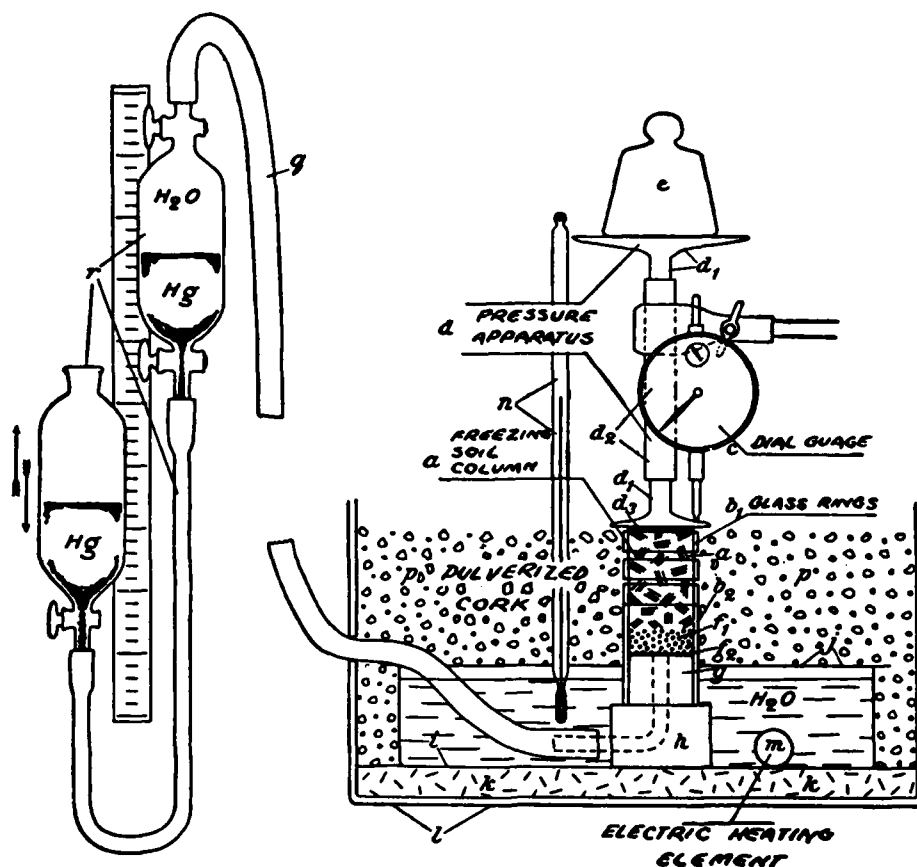


Figure 36. Apparatus for freezing tests on small soil samples, for determination of frost-heaving rate at different pressures (load pressures or capillary pressures). The right part is placed in a small freezing chamber with double window and internal electric light. The soil column (a) is placed in a segmented glass tube (b) built up of glass rings (b_1) (32.5 mm diam, and 10 mm height), and a 50- to 100-mm long tube (b_2) underneath; the soil specimen rests on a buffer (f) of sand (f_1), a copper screen (f_2) and a rubber stopper (g). The soil column is covered with the nickel plate (d_3), in contact with the lower disc of the pressure apparatus (d). On the upper disc (d_1), the load (e) is applied. The heaving is measured on the dial indicator (c), graduated in $\frac{1}{100}$ mm. The soil tube is placed in the water-filled pan (i), where the water can be heated by means of the electric heating element (m), and its temperature measured on the thermometer (n). The whole is carefully insulated by pulverized cork (p). From the water-saturated soil column a water conduit (q), consisting of a glass tube and a rubber vacuum tube, leads through the wall outside the freezing chamber, where it is connected with two partly mercury filled vessels (r), by which various subpressures in the water can be produced, resulting in a capillary pressure in the soil column. The rubber tube may also be connected with a single water container as shown in Figure 37a.

each other to form one tube (Fig. 36). This friction only occurred over a short section, less than the height of one ring; the friction was thus reduced to a fraction of what it was before. In detail the system functioned as follows: when freezing started and the topmost ring froze fast, at the upper edge the soil column under expansion was pressed out from the ring (relative to the ring). This upper ring then separates from the one below, and the distance is proportional to the expansion, until the frost reaches to the upper edge of the second ring, which freezes tight and the procedure repeats, etc. Though the whole system rests on a fixed bottom the freezing soil

lifts itself up and with it the frozen-fast tube sections.

To further reduce the remaining friction the inside of the rings are smeared with a thin layer of Vaseline. Thus the effect of friction was reduced to practically zero.

The only remaining pressure acting on the soil was the surface load and the capillary pressure. In order to have different surface loads the pressure apparatus shown in Figure 36 was used, on which different weights can be placed. Of course the weight of the dial and the soil itself was included. When small pressures were desired the pressure device was removed and the dial was applied directly on the cover plate.

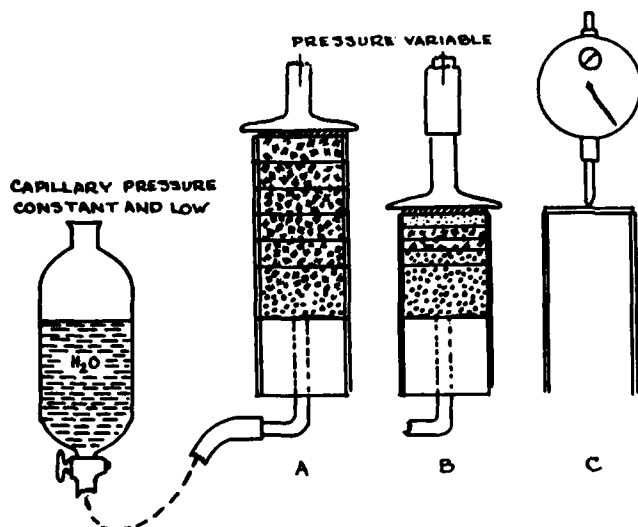


Figure 37. Example of various arrangements of test cylinders and vessels (a and r in Fig. 36). A—Only the surface load is varied, while the capillary pressure remains constant and of small magnitude by means of the water-filled tube kept only a few centimeters below the sample. B—For very impermeable soils, only a thin layer of soil is used which is covered by and lies on top of saturated sand. There the capillary pressure caused by resistance to flow is reduced to a minimum. C—For very low surcharge loads, the pressure apparatus is removed and the dial indicator spindle lies directly on a thin metal plate which covers the sample.

Increased capillary pressure was achieved by a tube leading from the bottom of the sample out from the refrigerator to an ordinary pressure apparatus (Fig. 36r). By this apparatus the pressure at the bottom of the soil column could be measured. To determine the water pressure at the frost line it is necessary to add the pressure difference and the flow of water through the soil, which can be determined exactly from the rate of flow ($2/10$ of the observed rate of heaving) if the permeability is known. Usually only the load pressure varied and the capillary pressure remained constant and small. The soil was kept in contact with an ample water reservoir with the water surface a few centimeters below the sample (Fig. 37).

The magnitude of heave was determined by the use of an ordinary dial, graduated to $1/100$ mm, and on which the change in height can be read to an accuracy of about ± 0.001 mm. To make continuous observation possible without opening the refrigerator, a double glass window (the air between the glass being kept dry by CaCl_2) was placed in the refrigerator.

The refrigerator used was supplied with a Frigidaire cooling unit. In order to get very accurate constant temperature the standard temperature regulator was cut off, and was replaced by a mercury thermostat. A temperature accurate to -0.3°C was obtained. The soil samples

were put in the refrigerator vertically, insulated on the sides by powdered cork, which was filled up to the surface of the sample. The bottoms of the samples were immersed a few centimeters in a water container, the temperature of the water of which was regulated by an electric heating unit. The water temperature was read from a long thermometer stuck through the cork insulation.

The tests were carried out by reading the time at definite intervals (usually 1–5 minutes), recording the reading on a time–height diagram so that the curve for the heave vs time was continuously recorded during the test. The air temperature was also recorded every now and then (very slow change). The load and the capillary pressure, of course, were varied, but were held constant for certain definite periods.

Usually it was required to determine the variation of the rate of heaving with pressure. In these cases a certain load was applied, freezing allowed to progress, and the heave measured until the curve remained a straight line over a long period, when the pressure was changed, the effect of the change observed, and freezing continued under the new load until the heave again became constant, when another load was again added, etc. (Fig. 41).

In the other case, when it was required to determine how the rate of freezing affected the rate of heaving, the

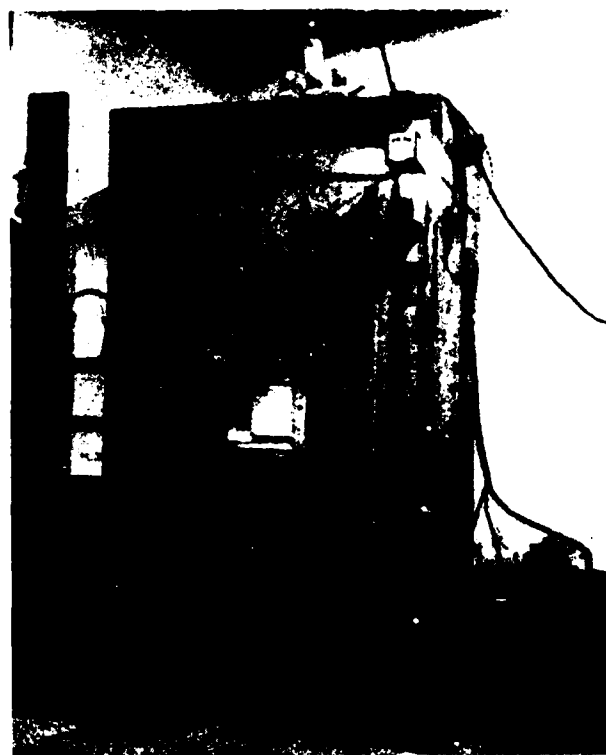


Figure 38. Exterior view of the freezing chamber.

pressure was kept constant during the entire test, and the temperature was varied.

3. Frost heaving independent of the rate of freezing

Observations of nature and of experiments (see Beskow 1928 and 1929a), previously showed that the rate of frost heaving must be at least approximately independent of the rate of freezing (the rate at which the frost line moves downward).*

From the recent tests this fact has been proven much more accurately. Tests were performed with different soils with a constant load and capillary pressure, but with greatly varying air temperatures. The temperature changes were made after quite long periods, for example -2°C for two hours, then -10°C for the same time, etc., so that a definite change in the rate of freezing would be allowed to occur. The temperature of the bottom was held nearly constant very near zero.

Curves for two of the tests are shown in Figures 39–40. As shown, the curves are practically straight during freezing, independent of the large differences in temperature and therefore independent of the rate of freezing. The small irregularities that exist occur without any relation to the temperature variations; that is due to irregularities in the materials. Even when a natural soil is carefully packed, the soil is obviously not perfectly uniform, but has layers of more fine material than others.

Frost heaving is then entirely independent of the rate of freezing. It is interesting to note that when the test was finished and the temperature raised above 0° , the curve bent over with a smooth curve approaching the horizontal, and when continued bent downward. The latter phase means melting is taking place. The important thing, however, is how suddenly the bend starts, which shows the heaving was continuous at a constant rate until the instant that the above-mentioned zone (previous footnote) is passed, when the heat condition cannot any longer keep up with the sucking up of water, i.e., cannot freeze all the water sucked up; after this the heaving becomes directly proportional to the cooling, which in this case rapidly diminishes. (The fact that the

* This implies also that the rate of increase of height (the amount of water freezing in unit time) is independent of the rate of freezing, assuming that the rate of cooling is large enough to allow all the water that can be sucked up to freeze. Neglecting this possibility, this means the frost moves downward and its rate is independent of the rate of heaving. But when the freezing just balances the water sucked up, the frost line remains stationary, while the ice layer keeps forming; if the rate of freezing is decreased the smallest amount, the frost line still remains stationary, but then the amount of water that is sucked up and freezing is proportional to the rate of cooling. For a rate of cooling under the minimum (therefore a stationary frost line) the rate of heaving is directly proportional to the rate of freezing or cooling (heat loss in unit time at the frost line).

bend does not occur immediately when the temperature passes the zero mark is due to the fact that the apparatus and the soil have a certain heat capacity and thus it takes a short time to absorb this heat.)

The tests have then fully proven, as the previous tests did not fully show (Beskow 1928, 1929a), that for a constant pressure the rate of heaving is independent of the rate of freezing.

It should be noted however that this is completely valid only for relatively permeable soil. Due to the expansion of the water during freezing an amount of water of almost 10% is released, which contributes to the heave and can therefore be subtracted from the water that must be sucked up to maintain a certain frost heave. This water, which does not have to be sucked up, is proportional to the rate of freezing and in very impermeable soils, therefore, the rate of heaving is partially dependent on the rate of freezing; the rule that frost heave is independent of the rate of freezing assumes a constant total pressure at the frost line. Concerning strong frost heaving soils then, the effect of the rate of freezing is under all conditions small and quite insignificant.

This fact is of great practical importance. While with little approximation we can say that the measure of frost heave is a measure of the amount of water sucked up, we can also state the rule as: the amount of water sucked up in unit time to the frost line is independent of the rate of penetration of frost. (Of course, we always assume that the rate of cooling is greater than the minimum mentioned above.) If freezing occurs rapidly, the water sucked up is divided up (small layers), and if it occurs slowly the layers are large. Therefore the water content of the frozen soil (a measure of the amount of ice stratification) registers the variations in the rate of freezing (in natural ground, the variations in the air temperature and the snow covering); thick ice layers are found during slow freezing and thin during fast.* Because of the reduction in bearing capacity during thawing in the frozen soil the rate of frost heaving is very important in highway problems (see Beskow 1928).

4. Frost heaving as a function of the grain-size and pressure

The fact that frost-heaving is independent of the rate of freezing greatly simplifies the determination of the rate of heaving as a function of other factors, since any rate of cooling can be used. Earlier tests have shown that grain size and pressure are the most important of these other factors. The essential purpose of the experiments is to determine quantitatively the rate of heave of different soils under fixed pressures.

* Assuming perfectly homogeneous soil; at points of change of soil composition there is a great divergence in the ice content of the soil.

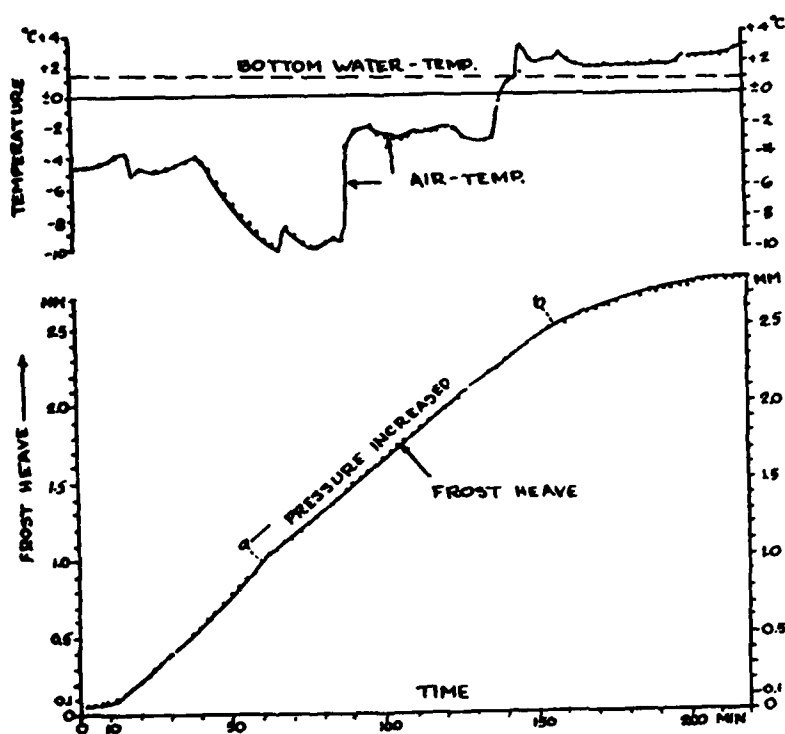


Figure 39. Diagram showing that frost heaving rate is independent of the rate of freezing. The large variations in air temperature, causing corresponding variations in rate of freezing, have no effect on the slope of the frost heaving curve until the temperature goes above 0°C , when the heave curve bends over and becomes horizontal, then drops off during thawing (not shown on diagram). The bend in the curve at a is due to an increase in friction of the dial indicator. See Figure 40 where this does not occur. Soil: M102, medium silt. Load pressure about $50\text{--}60\text{ gr/cm}^2$.

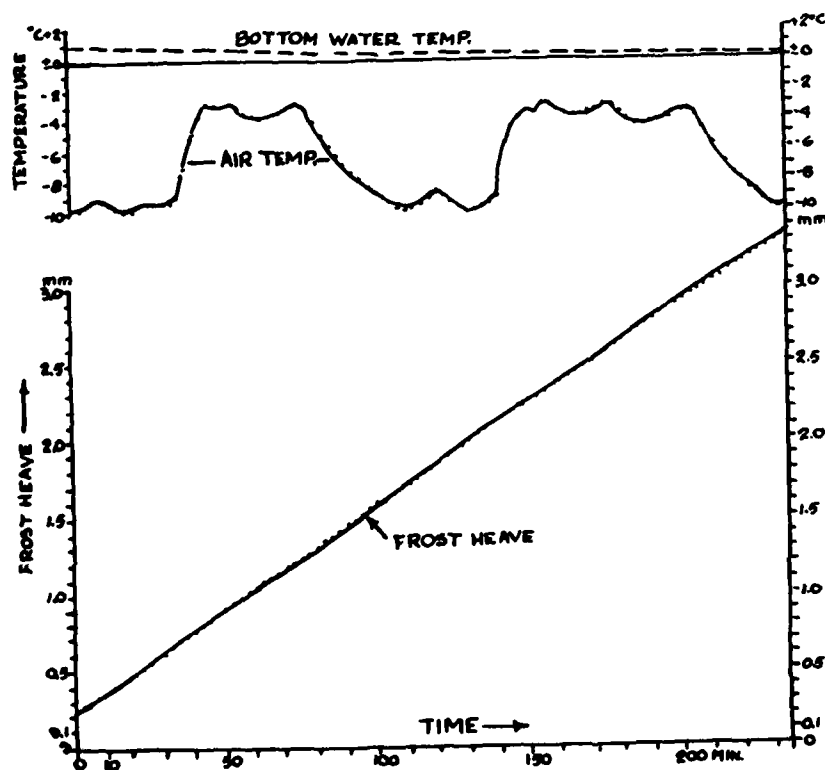


Figure 40. Another example showing that frost heave is independent of rate of freezing. Exactly the same as Figure 39, except that the end phase is left out. Soil: Bw 19, very fine silt-pressure, 410 gr/cm^2 .

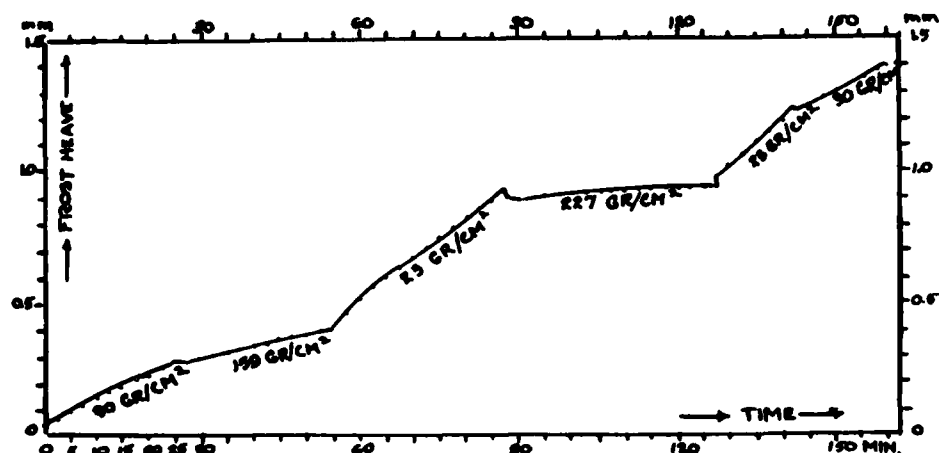


Figure 41. Diagram showing frost heaving rate for different pressures, surface load or capillary pressure. For any given pressure, there is a certain slope of the curve, i.e., a definite rate of heave. For each slope the corresponding pressure is given in gr/cm^2 . Soil: pure fractions, 0.01–0.005 mm. From diagrams of this type, the frost heaving rates for different pressures are obtained, from which the curves of types shown in Figures 42–44 are constructed.

In each series of tests the type of soil was kept constant and the pressure varied, either the loading or the capillary pressure or both. As in the above tests freezing was allowed to progress until the rate of freezing became constant over a reasonable length of time, i.e., a straight line heaving–time curve. For purposes of proper control in the series of varying pressure intervals, a constant “comparison pressure” was used, for which the amount of frost heaving has the same value. Figure 41 shows the graphical results on a heave–time curve. For each load, the curve begins at a definite slope but has a slight parabolic curve which becomes straight, each having a different slope. The slope is, of course, a measure of the rate of heave. This rate was determined graphically by drawing a line to the straight portion and reading the tangent of the angle with the x -axis. From the scale constant and this tangent the rates can be computed. For each pressure there is therefore a definite value of rate of heave.*

When a sufficient number of rates have been determined we can construct the rate of heave–pressure curve (Fig. 42–44).

a) *Frost-heave as a function of pressure.* The pressure acting at the frost line consists of two components, the external load and the capillary pressure. The capillary pressure in the pore water is, of course, negative,

but the effect of this pressure is positive since it tends to compress the soil, and it is therefore the positive value of this pressure that we want.

To determine how each of these two pressures acted, two tests were run on each soil sample, one time freezing under a varying load (and constant, very small capillary pressure), and the other time freezing under a varying capillary pressure (a constant very small load). Parallel tests carried out on different soils showed that when the pressures were reduced to the same units (gr/cm^2), the values were identical for capillary pressure and surface loading.

Capillary and load pressure then have a similar effect and the frost heave can be stated in relation to the total pressure acting at the frost line, which is the sum of the two.

As the results show (Fig. 42–44), the curves for the rate of freezing versus pressure are hyperbolic. It was then required to find a formula that would fit the curves, accurate enough (within the accuracy of the variation of the points on the curve) by computation. This can only be done graphically.

As a trial the formula for the hyperbola $X^n Y = a$ was tried, or in this case

$$p^n Q = a$$

where Q (cm/hr) = rate of heaving and p (gr/cm^2) = pressure. One can tell by inspection of the form of the curve that the exponent must be greater than $n = 1$; a value of $n = 2$ would look probable. For a start, then, n was assumed equal to 2.

* When the same pressure exists but the circumstances of freezing are different, the rates are not always identical, depending on the irregularities in the test factors (different packing, different frictional effects, etc.). These variations are not usually large but nevertheless significant; therefore several determinations were carried out for each pressure.

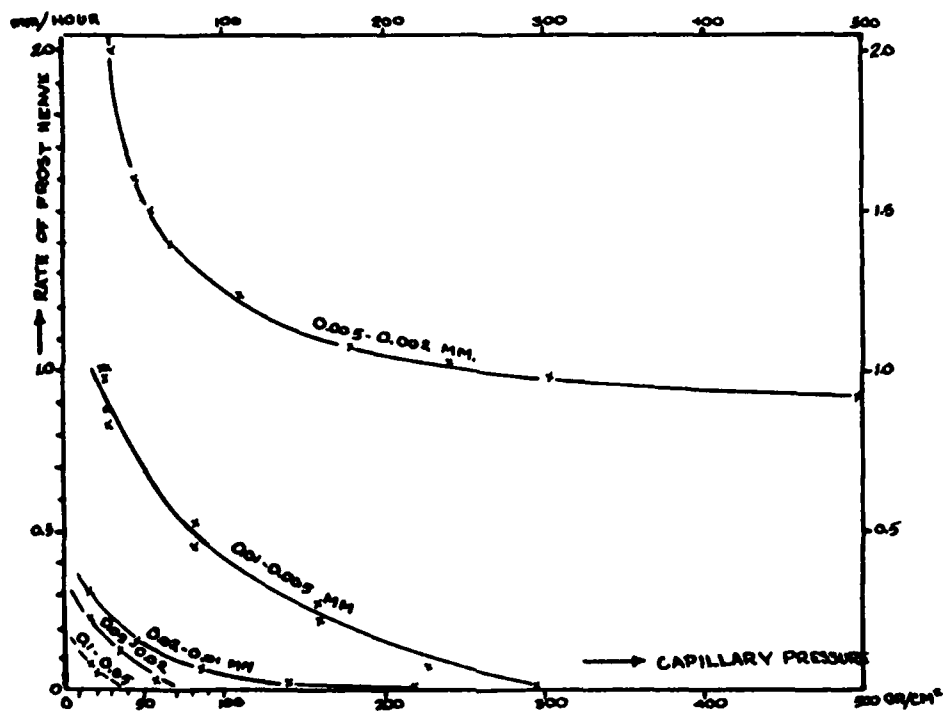


Figure 42. Diagram of frost heaving rate as a function of pressure, obtained from diagrams similar to Figure 41. Here the pressure is mostly capillary pressure (the only load pressure is that of the metal plate and the dial indicator spindle, about 16 gr/cm²). Soils: Atterberg's pure fractions.

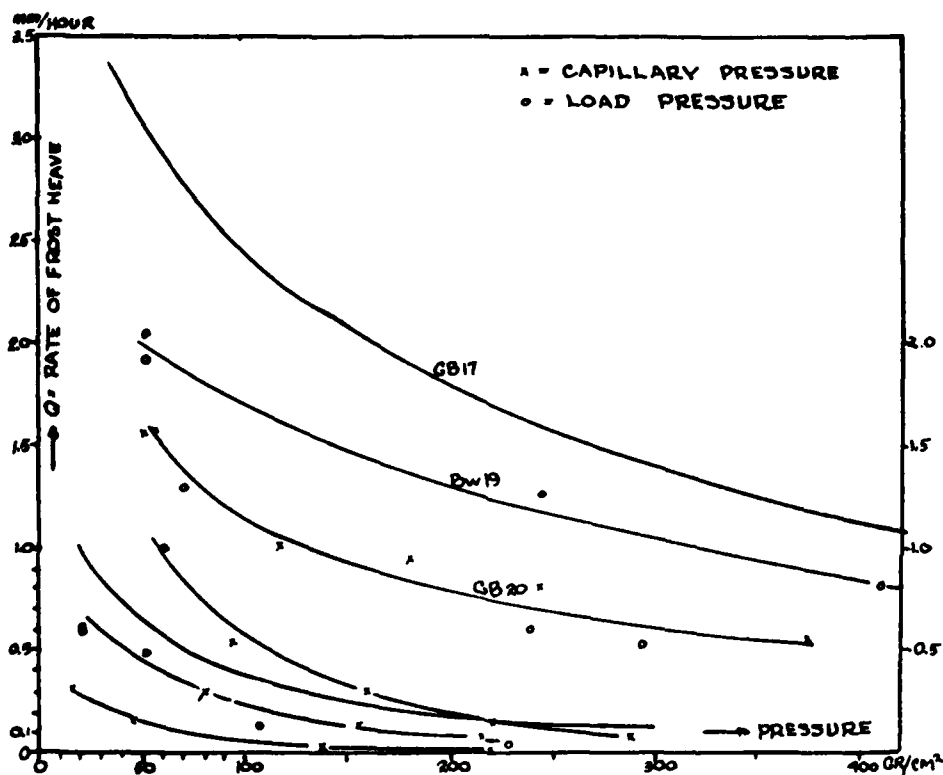


Figure 43. Same as Figure 42, but for a number of different coarse natural soils (equivalent to Atterberg's 0.02-0.01 mm fractions).

The simplest way to try out an empirical formula graphically is to choose the coordinates so that it reduces to a formula of the first degree, that is a straight line. $P^2Q = a$ can then be rewritten either as

$$a \frac{1}{p^2} = Q, \text{ or}$$

$$p = b \frac{1}{Q}; (b = \sqrt{a}).$$

In the first case $1/p^2$ is plotted along the x-axis and Q along the y-axis; in the other case $x = p$ and $y = 1/\sqrt{Q}$.

Which of these methods is preferable depends upon which is most suitable. Figures 45 and 46 show a number of diagrams for various soils obtained in this manner.

The experimental points for each soil fall quite well on a straight line, especially when there are numerous points on the curve. For the Atterberg fractions 0.01–0.005 mm, the curve would be parabolic if straight plotting were used. But with this plotting there are two points which do not fall close to the line; however the percent error in the amount of heave is small, and also since we plot the reciprocal of the heave, the error seems greater on the plot.

We can say from the diagram that the points fall with fair accuracy on a straight line. The line does not go through the origin but cuts the y-axis at a distance c from the origin; then the formula for this line becomes:

$$p = b \left(\frac{1}{\sqrt{Q}} - c \right);$$

$$\frac{1}{\sqrt{Q}} = \frac{p}{b} + c; \sqrt{Q} = \frac{1}{\frac{p}{b} + c}$$

$$Q = \frac{1}{\left(\frac{p}{b} + c \right)^2} = \frac{b^2}{(p + cb)^2}.$$

For pure fractions, the value of c is practically constant for silts, and varies between 1 and 0.7. However, it happens in general that c decreases with decreasing grain size. It is of course impossible, due to such uncertainty, to state any definite rule on the basis of such little material; it seems, however, that for coarse soils without any clay (silts) c is relatively constant (about 1 or less) and decreases with decreasing grain size and reaches practically zero for lean clays.

Or expressed in words: the rate of frost heaving for a given soil is inversely proportional to the square of the pressure (= sum of load and capillary pressure), after the pressure exceeds a certain but not large value. The reason for this "initial pressure increment" is that when the pressure approaches zero, the theoretical rate of heave does not approach infinity but approaches a limiting (but quite large) value.

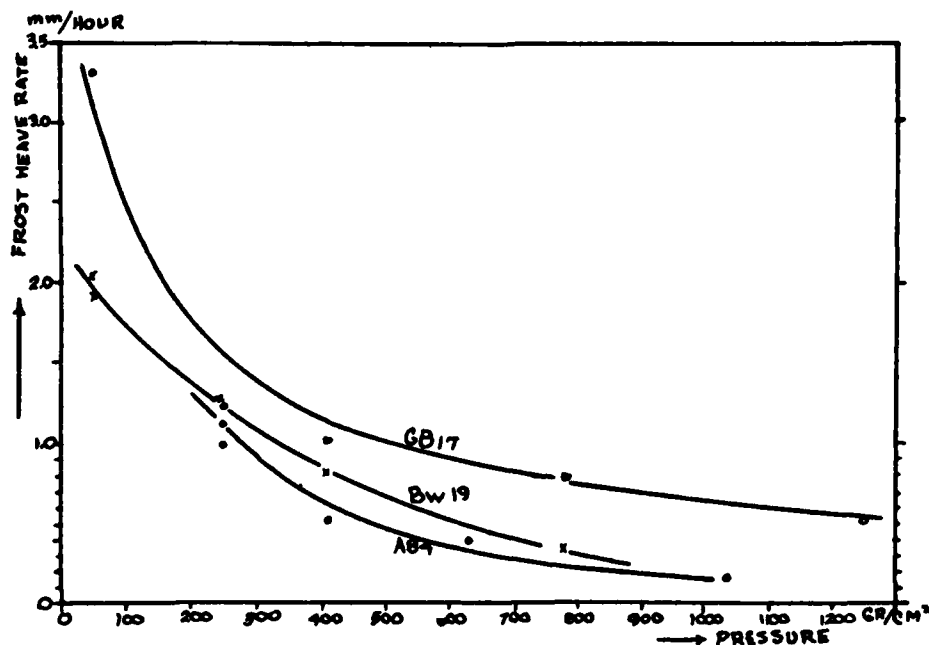


Figure 44. Diagram of frost heaving rate as a function of pressure for a number of fine-grained soils, which are considerably frost heaving even for large pressures. Analogous to Figure 43 but to a different scale.

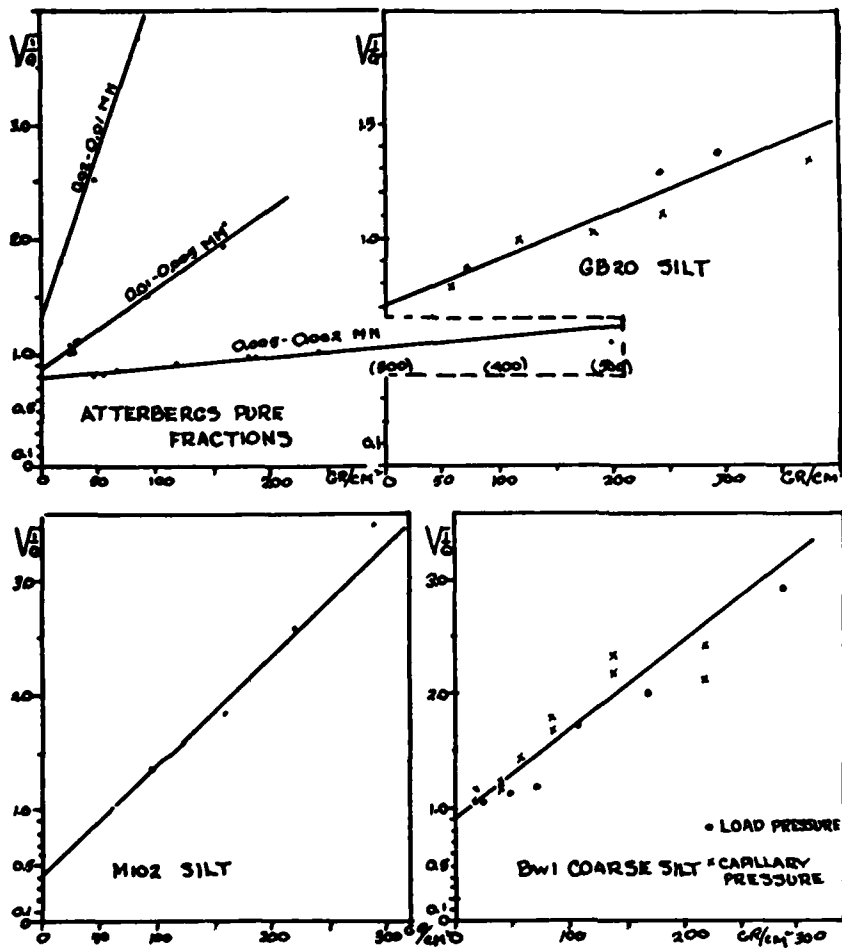


Figure 45. Diagram showing graphically the relation between rate of frost heave and pressure. On the y-axis: $\sqrt{1/Q} = 1/\sqrt{Q}$ = reciprocal of the square root of the frost heaving rate Q in mm/hour; on the x-axis, pressure p at the frost line. As seen the observed points lie on straight lines which intercept the origin between 0.4 and 1. Hence the formula for rate of frost heave as a function of pressure:

$$Q = \frac{1}{(p/b+c)^2} = \frac{b^2}{(p+cb)^2}$$

The relation between frost heave and pressure can then be given by the formula

$$Q = \frac{1}{\left(\frac{p}{b} + c\right)^2} = \frac{b^2}{(p + cb)^2}$$

where b is a definite exponent for each soil, and c is a factor which for silt is about = 1, and for decreasing grain size approaches zero.

b) Frost heave as a function of grain-size. In the formula

$$Q = \frac{1}{\left(\frac{p}{b} + c\right)^2}$$

b is an exponent depending on the type of soil. From tests on different coarse natural soils and on different Atterberg fractions, it was found that b increases for decreasing grain size.

It was found by trial that the relation of b to grain size is: the square of b is proportional to the inverted cube of the particle diameter, or,

$$b^2 = e_d \left(\frac{1}{d}\right)^3$$

This was found graphically by plotting the points on a straight line through the origin.

This relation is shown in Figure 47 for both natural sediments and pure fractions, except that the reciprocal of the particle diameter is substituted by the capillarity K .*

$$b^2 = e_k \cdot K^3$$

As the diagram shows, the points for natural soils fall on a straight line intersecting the origin as given by the formula ($b^2 = e_k \cdot K^3$; $e_k = 425$, see Fig. 47). Corresponding results were found for the pure fractions (however, for 0.005–0.002 mm, $K = 17$, the point falls above the line), but the exponent e_k has a much smaller value. In general then, b is a function of the capillarity (or grain size) but the actual relations are different, depending

* To simplify the scale, the formula $b^2 = e_k \cdot K^3$ was further reduced to $\sqrt[3]{b^2} = \sqrt[3]{e_k} \cdot K$.

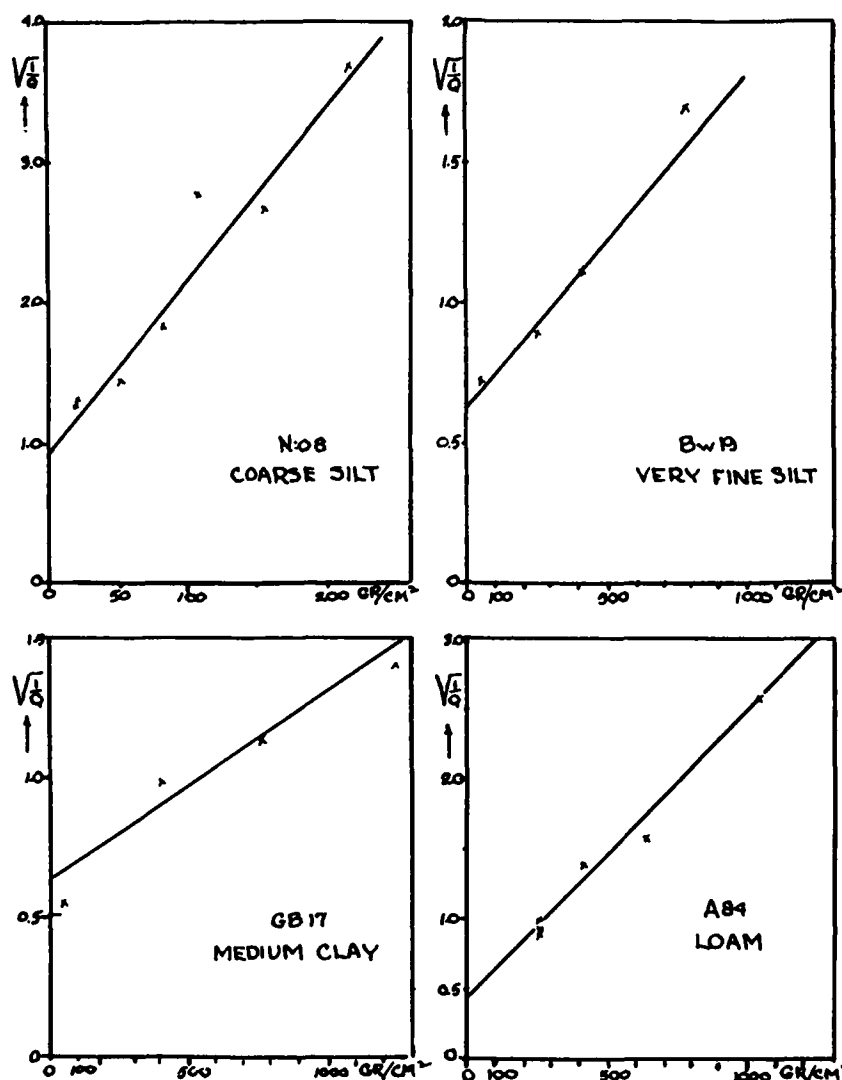


Figure 46. Analogous to Figure 45.

largely upon the grading of the soil—which is, of course, what one would expect.

For example, if a sand is mixed with a small amount of colloidal material, it has practically an infinite capillarity, but is quite different in frost heaving properties; it may become strongly frost heaving, while the clean sand might have been non-frost-heaving (see Table 10).

We can say on the whole that the exponent e_k in the formula $b^2 = e_k \cdot K^3$ has varying values, depending on the grading. Generally this value rises for a better grading distribution. For such natural well-sorted sediments as found in the northern sections, the value of e_k lies anywhere between $e_k = 100$ and 1000 , but usually is found to be $e_k = 200-600$, when K is given in meters, p in gr/cm² and Q in mm/hr.

The formula for the rate of heave (Q in mm/hr) expressed as a function of the soil properties (particle diameter d mm or capillarity K meters) and the pressure

(p gr/cm²) is then:

$$Q = \frac{1}{\left(\frac{P}{\sqrt{e_d \cdot \frac{1}{d^3}}} + c \right)^2} = \frac{1}{\left(\frac{P}{\sqrt{e_k \cdot k^3}} + c \right)^2}$$

where for natural sediments $e_k = 100-1000$, but generally lies between 200 and 600, and where $c = 1$ for silts but decreases towards zero for decreasing grain size.

If we make an approximation by neglecting c —which for very large pressures is a very slight error—the formula becomes:

$$Q = e_d \frac{1}{p^2 d^3} \text{ (approx.)}$$

i.e., Q is nearly inversely proportional to the cube of the

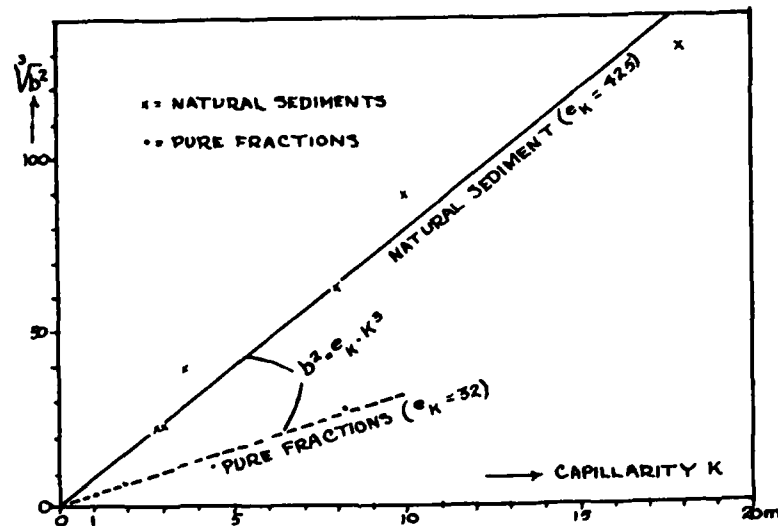


Figure 47. Diagram showing the relation between the capillarity K and the exponent b in the formula for the rate of heave (Q) as a function of pressure (p):

$$Q = \frac{1}{(p/b+c)^2}.$$

It is seen that for natural sediments, the points are scattered but fall close to the solid straight line given by the formula $b^2 = e_k \cdot K^3$ ($e_k = 425$ in the figure). This relation, however, only holds for soils of the same grading characteristics. Other differences such as the chemical and mineralogical properties also affect the relationship. Generally, then, this relationship is a good average for the same types of soils, for example, the Norrland [Northern Sweden] valley sediments.

grain size ($1/d^3$). Let us see what this means practically.

Coarse frost heaving soils have such large permeability in comparison with their frost heaving rate that the pressure difference (= capillary pressure increase at the frost line) needed to suck up the water is nearly zero. The pressure at the frost line then depends only on the loading and the distance to groundwater.

If we imagine the grain size to be variable and the surface load and distance to groundwater to be constant, the rate of frost heaving varies inversely as the cube of the grain size. It is then obvious that the rate of heaving is very sensitive to variations of grain size so that a small increase in grain size causes a large decrease in heave. Doubling the particle diameter decreases the rate of heaving eight times; an increase of grain size by $\sqrt{10}$ (corresponding to the usual grouping of grain sizes, for example, from 0.063 to 0.2 mm, etc.) decreases the rate of flow of water (heave) to $1/32$. But differences of this order of grain size are practically the difference between frost-heaving and non-frost-heaving soils. As an example, a road bed of medium silt of average diameter 0.063 mm showed a heave of about 16 cm, while a coarse silt of average diameter 0.125 mm had a heave of about 2 cm. This is why a close zone can be designated between frost-heaving and non-frost-heaving soil.

This zone is only a dividing zone for practical purposes because theoretically, all coarse sands are frost-heaving, although the magnitude is negligibly small, as experiments have proved.

Let us assume an ordinary silt and imagine the grain size to decrease. If the diameter is halved, the frost-heaving is increased eight times, and if for example the diameter changes from 0.063 to 0.02 mm (a change from the coarse silt-silt boundary to the silt-fine silt boundary), the heave is 32 times larger. If the rate of heaving varies thus as the cube of the reciprocal of the particle diameter, the rate would become practically infinite for smaller diameters. A change from 0.063 to 0.0063 mm corresponds to an increase of a thousand times. The reason why this tremendous increase does not actually occur is because the pressure at the frost line increases rapidly due to the increased resistance to flow as the grain size is diminished.

For the finer silty clays the permeability is so small, that is, the resistance is so large, that the pressure at the frost line consists essentially of the very large capillary pressure; that is to say, the formula

$$p = a + k = a + l + lQ.$$

$1/p$ can be simplified to

$$p = Q \times \ell \times \frac{1}{P} \text{ (approx.)}$$

(see par. 5 following for explanation of terms). But the permeability P is proportional to the square of the diameter ($P = c_d \cdot d^2$; see section IV).

Therefore the rate of upward flow of water as a function of particle size is

$$Q = e_d \cdot \frac{1}{p^2 \cdot d^3} = e_d \cdot \frac{1}{\left(\ell \frac{Q}{c_d \cdot d^2} \right)^2 \cdot d^3} =$$

$$c \frac{d^4}{\ell^2 \cdot Q^2 \cdot d^3}; \quad Q^3 = \frac{d}{\ell^2}; \quad Q = \sqrt[3]{\frac{d}{\ell^2}} \text{ (approx.).}$$

Expressed in words, for fine-grained soils under constant load and depth to groundwater table, the rate of upward flow of water is approximately proportional to the cube root of the diameter, i.e., decreases slowly with decreasing grain size.

Therefore we can say that for coarse silty clays frost heaving decreases with increasing grain size, and for fine it decreases with decreasing grain size. We can say then that the curve of rate of heave vs grain size must reach a maximum. But since in the former case Q is proportional to the cube of $1/d$, and in the latter it is proportional to the cube root of d , the curve will drop very rapidly from the maximum on the coarse side, while on the fine side it will drop very slowly. (The curve will have about the same shape as shown in Fig. 73.) Also on the coarse side the frost heaving soils are bounded by a rather sharp boundary, while towards the clay side there is no such zone, since the curve drops so gradually.

Both screened laboratory samples and field samples have shown that lean clays are usually always very strongly frost heaving, while medium clays are always definitely notably frost heaving (i.e., several centimeters up to a decimeter). If we define a zone of "essentially non-frost-heaving" as that which has less than 3 to 4 cm heave during a winter, the stiffer clays would fall in this zone.

Bear in mind that it is important how we define "essentially non-frost-heaving." For certain structures the tolerance might be of a smaller magnitude, and stiff clays might even be considered as dangerous; and also note, for clays, that heave is practically independent of the load acting. However, it is important to note that drainage has a big effect and is easy to perform on clays.

5. Calculation of the rate of frost-heaving for varying grain size, load pressure, and distance to the groundwater

From the preceding formulas, the rate of frost heave can be computed for a given soil for varying loads and depths to groundwater, or for a constant load and depth to groundwater but varying grain size.

The first requires the capillary pressure at the frost line be given a function of grain size (or capillarity) and distance to groundwater.

The pressure (p) at the frost line consists of the load pressure (a) and the capillary pressure k . k may be given by the formula

$$k = \ell + \ell \cdot \frac{\ell}{P} Q_v$$

where ℓ = distance to the groundwater table, P = permeability, Q_v = rate of upward flow of water.* The total pressure p then becomes

$$p = a + \ell + \frac{\ell}{P} \cdot Q_v$$

But the permeability P is a function of the grain size or capillarity (Section IV).

$$P = C_{pk} \cdot \frac{1}{K^2}$$

therefore the preceding formula becomes

$$p = a + \ell + \frac{\ell}{C_{pk}} \cdot K^2 \cdot Q_v$$

If we put this value of p in the above given formula for the rate of frost heaving Q , we get an expression for Q as a function of grain size, distance to groundwater, and surface loading. Since this expression is a complicated third degree formula, which has no general solution, and since solutions for the specific cases are too tedious, and especially since C cannot even be determined with sufficient accuracy as a function of grain size, a solution would be meaningless.

The rate of frost heave as a function of grain size for a given depth to groundwater and load pressure cannot then be calculated.

For certain soils, on the other hand (namely those soils on which freezing tests have been performed and C evaluated empirically), the frost heave can be calculated as a function of groundwater depth and loading.

* The rate of upward flow of water is not quite as large as the corresponding rate of heaving (before called Q) due to the expansion of water during freezing. If, considering the ice, we call Q_i , the relation becomes $Q_v = 0.090 Q_i$. If we assume a small error there is no difference and $Q_v = Q_i$.

It is quite simple to use the empirically found curve for frost heave as a function of pressure; if we use p and $\sqrt{1/Q}$ the curve is a straight line. From the curve we can take various different values of p and the corresponding Q -values. If a = load pressure, k = capillary pressure at the frost line, the total pressure is $p = k + a$. Then solving for ℓ (distance from the frost line to the groundwater table) from the equation

$$p - a = k = \ell + \frac{\ell \cdot Q}{P}$$

$$\ell = \frac{p - a}{1 + \frac{Q}{P}}; (p - a = k \leq K).$$

The permeability P must of course be determined empirically. This value must naturally be reduced to a temperature of 0°C (see Table 15). Also Q and P must be converted into the same units: centimeters or millimeters per unit time, and both should be computed as water or ice; in the former case the value of frost heave

is reduced by $1/10$, in the latter the permeability is increased by $1/10$.

Figures 48 and 49 give the curves for frost heaving rate and groundwater depth for a few different soils.

Figure 49 is a summary of the soils giving the greatest heave under normal conditions. The diagram shows only five different soils, but as illustrated in the included grading-curves, they are evenly divided over grain sizes from medium silt to very fine clayey silt. BW 1 and GB 20 show the strongest heave, both of which lie in the middle of the grading diagram, while the coarsest (no. 8) and the finest (A 84) show considerably less heaving. *The worst frost heaving soils are therefore the silty soils.*

The diagram is of special interest in that it shows graphically the effect of lowering the groundwater, as for example by culverts.

6. The effect of lowering the groundwater on the rate of frost heaving

This effect can also be approximately calculated arithmetically. If we proceed as above and start first

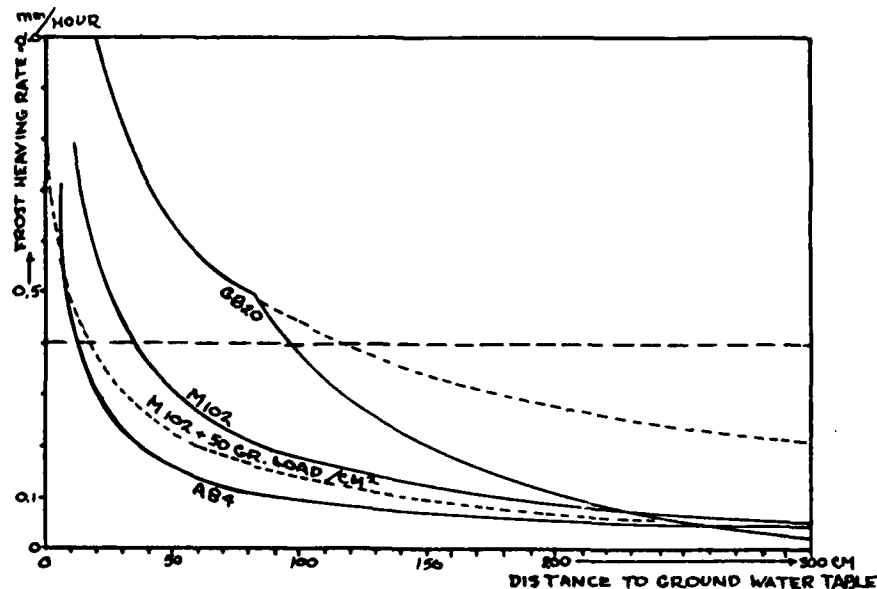


Figure 48. Curves of rate of frost heave as a function of distance to the groundwater table for a number of different types of soils. The curves are constructed on the basis of the empirical curves of rate of frost heave as a function of pressure (Fig. 42-44) redrawn as in Figures 45 and 46, and the measured permeabilities of the soils (from which, then, the capillary pressure required for a given rate of upward flow and a given distance can be computed). Observe the break in curve GB20, which is due to the fact that for this depth to groundwater (32 cm) the maximum capillary pressure at the frost line is reached ($= K$, the capillarity of the soil) and cannot increase further. For increasing depth to groundwater, therefore, the rate of heave decreases very rapidly. The dashed curve shows the relation on the assumption that the maximum capillarity is not reached. The other curves have similar breaks but fall outside the diagram (i.e., for larger depths). The break in GB20 occurs at the small depth because it is a clayey coarse silt, having a small capillarity ($K = -3.2$ m) but a large amount of frost heaving. Such a soil is therefore much more sensitive to changes in groundwater level.

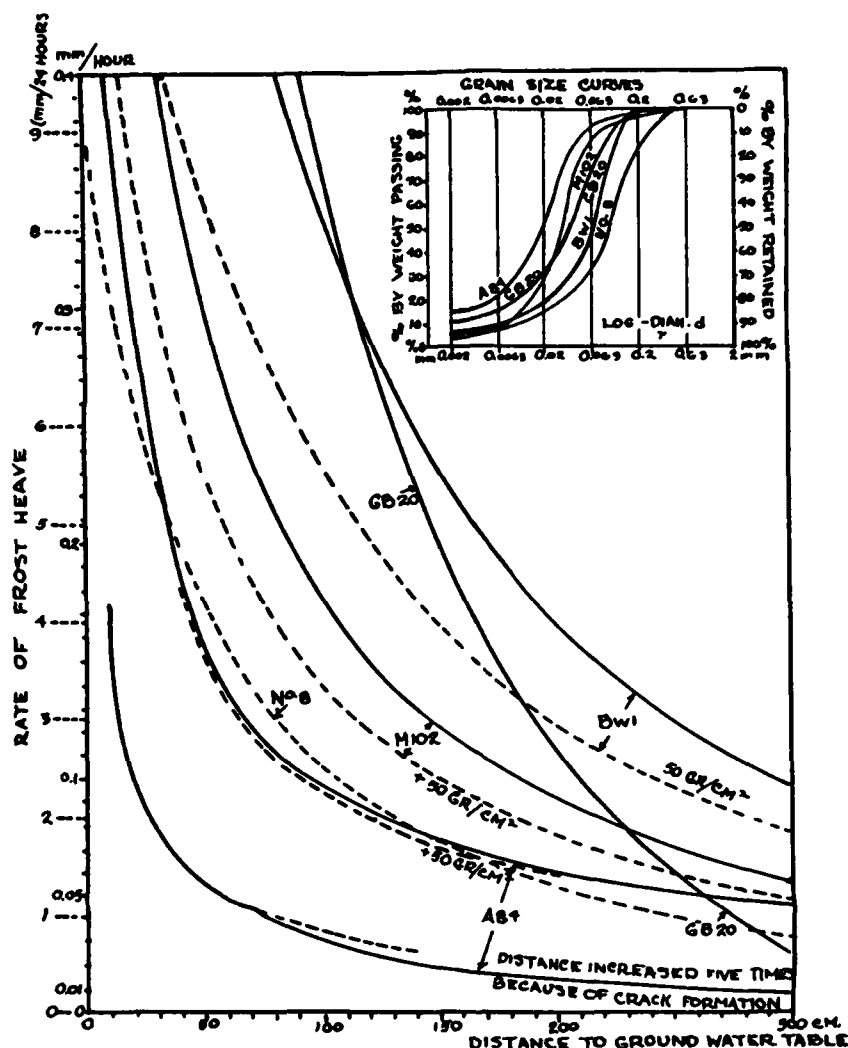


Figure 49. Same as Figure 48, except to a larger frost heaving rate scale and with additional soils. The solid lines are for no surface load and the dashed lines are for a surface load of 50 gr/cm². The effect of load is greater the coarser the soil is. For A84, there is an additional effect of crack formation (lowest curve—notice there is a break similar to GB20 in Fig. 48). Inset is a grain size distribution curve for the soils in question.

with such a coarse permeable soil that the dynamic capillary pressure can be neglected, the pressure at the frost line becomes $p = k + a = \ell + a$; where ℓ = distance to groundwater.

$$Q = e_d \cdot \frac{1}{p^2 \cdot d^3} = e_d \cdot \frac{1}{(\ell + a)^2 \cdot d^3} \quad .$$

If the load is assumed very small, so that a can be neglected, the rate of heaving becomes inversely proportional to the square of the depth to groundwater, which means that variations in groundwater have a very large effect.

For the other case of an impermeable soil such as a very fine silt, the formula is

$$Q = \frac{\sqrt[3]{d}}{\sqrt[3]{\ell^2}} \quad .$$

Since Q is inversely proportional to the $3/2$ -root of ℓ , the effect of lowering the groundwater in this case is much less than in the former case. However, there we assumed that the surface loading pressure could be neglected; since coarse soils are very sensitive to pressure (i.e., pressures of the magnitude which are common on roadbeds have a considerable effect), this relation between depth to groundwater and frost heave is not valid practically.

For cases in between, the values fall between these. Figure 49 shows the case of a silt which for an increase in groundwater depth from 1 to 2 m, the frost heave was diminished to about half, that is to say, for depths of this order of magnitude the frost heave for silty soils is very nearly inversely proportional to the distance to groundwater. For fine silts the reduction in frost heave becomes less and approaches

$$Q = C^{2/3} \sqrt{\frac{1}{\ell}}.$$

However, it is important here to note: this assumes that the capillary pressure at the frost line (k) does not reach the magnitude of the capillarity of the soil (K). But since this is not always so, the capillary pressure is often quite different.

For the first case, the capillarity K has a value which for the given groundwater depth can prevent upward water flow and the consequent frost heave; when $\ell = K$ the frost heave is zero.* The capillarity of the frost heaving sediments is hardly ever below 2 to 2½ m, which is important to keep in mind when one is concerned with lowering the groundwater by culverts and other means. As soon as the resistance becomes large enough to cause a considerable extra capillary pressure drop in the upward flow, the picture is different. Then the zone where the capillary pressure at the frost line k , is equal to the capillarity K , occurs at a much smaller distance to groundwater; after this k can no longer grow larger† and from here on the formula for the maximum amount of capillary water that can be obtained for frost heaving is:

$$Q = \frac{K - \ell}{\ell} \cdot P.$$

or, if we use the resistance (m) instead of the permeability,

$$Q = \frac{K - \ell}{\ell \cdot m}.$$

The distance to groundwater can be obtained from the equation

$$\ell = \frac{p - a}{1 + \frac{Q}{P}} \text{ where } p - a = K.$$

However, for well sorted sediments ℓ has quite high

* Actually there is a very small amount of heave, when above the capillarity, but for practical cases this can be completely neglected.

† The frost heaving curve has at this point a definite break; see the diagram.

values, larger than 3 m. We can see from the equation

$$\ell = \frac{K}{1 + \frac{Q}{P}}$$

that ℓ becomes small when for a given capillarity the permeability is very small. As will be shown later (Section IV), this is the case for unsorted soils: the less sorted a soil is, the smaller is the value of this "relative permeability" (in relation to the capillarity).

We can see from Figure 49 that silt GB 20 is considerably less sorted than the other silts (flatter curves, more fine silt and clay content), and is the reason why it also has a low relative permeability, as direct determinations have shown. The full capillary pressure also occurs at a small ℓ value ($\ell = 82$ cm); after this point (breaking point) the frost heave curve drops very fast for increasing groundwater distance (breakdown in Fig. 48); an increase from 1 to 2 m decreases the rate from 0.375 mm/hr to 0.113. The same thing occurs for moraine soils, where however, the "breaking point" (for capillarity) occurs for still smaller ℓ values.

For less graded soils—moraines and other poorly sorted sediments—the amount of water sucked up and therefore the frost heave for small groundwater distances is determined by the formula for maximum amount of capillary water that can flow up. Thus a lowering of groundwater has a very large effect.

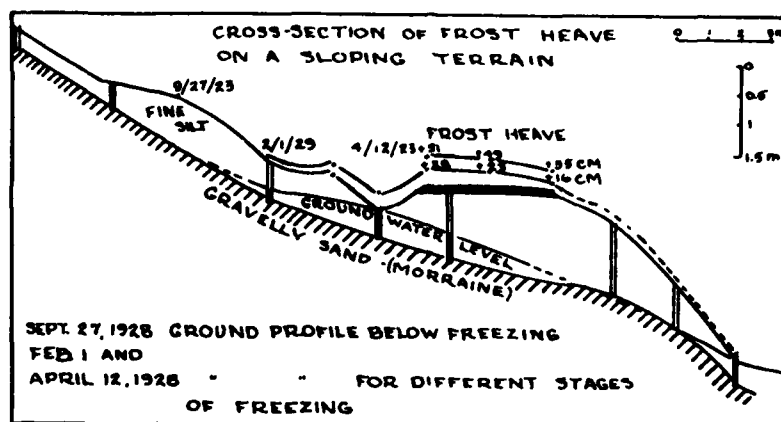
The resistance to "water suction" can also be increased by crack formations (see Section IV). This occurs mostly in clays and clayey soils; thus the capillarity is much less and the formula for maximum capillary flow becomes applicable for small depths to groundwater. In Figure 49, the curve for A 84 (lean clayey fine silt) is constructed assuming that the resistance to flow by cracks is increased five times as shown, the "breaking point" occurs at $\ell = 55$ cm.

The occurrence of these cracks in clay is quite common. Besides clays are generally less graded than the average fine silts. The "breaking point" must then occur at very small ℓ values. *For clays (as well as moraine soils) the formula for heaving for distances to groundwater greater than about 1½ m, is the same as the formula for the maximum capillary flow.*

$$Q = \frac{K - \ell}{\ell \cdot m}.$$

Where K is very large the formula becomes approximately $Q = K/\ell \cdot m$, i.e., the rate of heave is inversely proportional to the distance to groundwater.

Thus the effect of groundwater lowering is greatest for moraine and coarser soils on the one hand and for clays on the other hand. While on the contrary it is small



for the soils in between well-sorted silts and loams. This also agrees with practical experiences.

To summarize then, we can say: *To calculate the effect of lowering the groundwater on the heaving rate, assume the heaving rate to be inversely proportional to the distance to groundwater* (compare Fig. 50). This is very nearly exact in most cases; greatest deviations occur for clean fine silt, unsorted silts, and silty moraine material.

The depth for drainage that has been found to be effective practically in preventing frost heave on roads is about 1.7–1.8 m under the road surface, which means that when the roadbed begins to freeze, the distance to groundwater is about 1.5 m and the load is nearly 50 gr/cm².

From Figure 49 we see that for a depth of 1.5 m to groundwater the magnitude of the heave is between 5 and 2 mm per day (not considering the lowest curve). This is a considerable heave and seems therefore to be in contradiction to the above-mentioned value of proper groundwater depth.

We must however take the following into consideration: Only in special cases does such drainage necessarily make the soil completely non-frost-heaving; as has been pointed out several times before (Beskow 1929b, 1930d, 1933d and 1934b) for most cases, *such drainage is only a means of preventing damage due to thawing of the frozen soil, to frost boils*. What happens is that the amount of water sucked up is diminished so much that the excess water content is no longer large enough to make the soil unstable during thawing when subject to the weight of the road and the traffic.

Samples taken from northern roads, which although

considerably frost heaving, do not have frost boils, can have a frost heave in certain cases up to 10 to 15 cm. An example of this is a drained roadway shown in Beskow (1934b, Fig. 100). This drainage remedied frost boils, but there was still a frost heave of about this magnitude after the drainage was installed. It was also found that the frost heave is greatest at the beginning of winter and later decreases rapidly; this is because after the ground freezes, the groundwater is lowered so fast that the distance to groundwater (in spite of the lowering of the frost line) is increased, and besides the pressure due to load above is increased. An average heave of say 20 cm under normal conditions has a heaving rate of about 4–5 mm/day at the start of freezing (see Fig. 49), and for 10 cm total heave, a rate of about 2–3 mm/day. And when we consider the thawing of frozen soil, it is the excess water in the upper-most part that is most important, and this is dependent on the starting rate of heaving; 3 mm/day is the value of the largest tolerable starting rate of heaving,* which for the soils in Figure 49 corresponds to a groundwater depth of 0.65–1.9 m.

The excess water at the frost line, however, depends on the rate of frost penetration; the above therefore refers only to the normal cases of freezing on northern roads with machine snow plowing. In these cases the rate of freezing at the start of winter is quite constant and is about 2 mm/day. For slower freezing then, the value for the tolerable rate of heave must be lower.

From the above it can be said that lowering the groundwater to $1\frac{1}{2}$ m under the surface of the road fill

* We, of course, consider only roadbeds of frost heaving soil and not non-frost-heaving beds on top of frost heaving ground.

will prevent frost boils on most northern roads, but certain very poor soils (the worst fine silts) seem to require a distance of almost 2 m, and for slower freezing even as much as $2\frac{1}{2}$ m. Therefore for these special cases considerably more drainage is needed.

However, as will be shown later, high groundwater can only exist in silts if it is underlain at small depths by more permeable layers. If then the coarse soil touches a culvert, the effect of drainage becomes greater than computed assuming a homogeneous fine silt layer.

7. Influence of dissolved substances in the soil water on frost heaving

From the discussion given previously, the rate of heaving depends upon the thickness of the adsorption films and various other properties at the boundaries between the growing ice crystals and the underlying soil particles surrounded by water films. Increased pressure squeezes together the films and decreases the rate of heave. Decreased pressure has the opposite effect.

Other factors which affect the adsorption films should have effects correspondingly. Many years of experience and numerous tests have shown that dissolved substances in the groundwater have a very marked effect on the adsorption films. One should expect then that they should have an influence on the rate of heave in such a manner that by adding substances that are hydrating, i.e., increasing the films, they would increase the heave, and conversely the addition of dehydrating substances which would make the adsorption films thinner would decrease the heave.

Freezing tests on soils where the pure water was replaced by dilute salt solutions (NaCl and CaCl_2) showed a definite effect, but also showed that it was quite complicated, sometimes increasing and sometimes decreasing the heave, depending on the concentration of the salt solution and other factors. Further tests (fall 1933) had to be postponed on account of lack of time, but were resumed in February 1935 and are now, at the time of this writing, still in progress.

The freezing tests were as a rule carried out in series, where the influence of several different substances was tried on different soils, each at several different concentrations. The dissolved substances used were NaCl, CaCl_2 and H_2SO_4 . The tests were summarized in the following paragraphs—some of which are presented graphically in Figures 51–55.

NaCl has a most complicated effect. In weak concentrations the salt solution increases the frost heave over ordinary distilled water (notice [Fig. 51b] that the results are quite different when the soil is first put in HCl and washed). At a certain concentration a maximum is reached; and then for increasing concentrations the rate of heaving decreases.

These observations agree with the experiments of Sante Mattson on the influence of electrolytes on films, measured by the variation in volume of fine colloids. According to Mattson (1932) the osmotic phenomenon is very important. The difference in ion concentration of the films and the surrounding pore water determines the thickness of the films. For certain colloids there is a maximum thickness of the adsorption films for a certain concentration of a hydrating salt. For example, with NaCl, the particles absorb the sodium ions which then form part of the films, swelling them up, increasing the force of adsorption, when the ion concentration of the pore water is very small. This effect is interpreted by Mattson as an osmotic phenomenon, a sort of swelling of the films due to the difference in ion concentration between the films and the pore water. However, a further increase in the total concentration causes an increase in the ion concentration of the pore water and therefore a decrease of the difference in concentration, and thus the osmotic pressure difference between the films and pore water is diminished, which results in a decrease in the size of the adsorption films.

The results of the tests carried out here seem to be in agreement with Mattson's theory.

The effect of the various substances used is to decrease the rate of heaving, noticeable at very weak concentrations, and generally an increase in this effect for stronger concentrations. For CaCl_2 and sulphite leach [sulphite waste liquor from the sulphite paper process], the decrease in rate of heaving for increasing concentrations seems to continue indefinitely, but for H_2SO_4 the decrease is quickly stopped and then remains constant. The coarser the soil is, the quicker the constant rate is reached with increasing concentrations. For the silt U296 and N102 the minimum value is reached at a concentration of about $\frac{1}{2}$ to $\frac{1}{10}$ of normal, and for the clayey fine silt H 28 at about $\frac{1}{2}$ normal.

The magnitude of this decrease in the rate of heaving can be seen from the diagram. For sulfuric acid the magnitude is about $\frac{1}{2}$ to $\frac{1}{3}$ —in favorable cases such as soil U296 it is about $\frac{1}{5}$ of the rate of heaving with pure water. These differences could be accounted for by differences in the soils' original acidity, so that the smallest decrease would correspond to the soil which had the largest hydrogen ion concentration. But determinations of the original pH values of the soils did not show any agreement with this. Surely, however, the differences depend upon the original ion adsorption, but this seems to be quite a complicated phenomenon. For this reason the tests have not yet been completed.

The action of CaCl_2 is practically zero at small concentration but increases very rapidly. For the rock flour H28 the amount of frost heave for a 1-normal solution was reduced to less than a tenth.

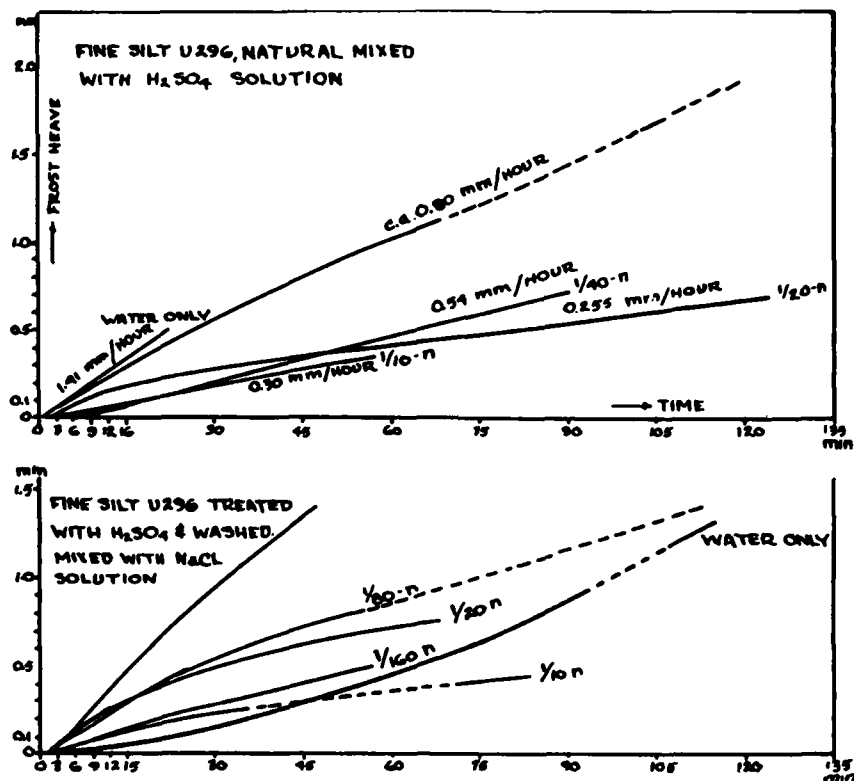


Figure 51. Curves showing different rates of heave caused by using solutions of different concentrations as the ground water. Upper curves, Silt U 296, undisturbed, frozen with distilled water (curve marked " H_2O "), and with H_2SO_4 of different concentrations. The concentration for each curve is given in normality (1 normal has 49.04 gr/L, or about 5% by weight; $1/10$ normal about 1/2%, etc.) As shown, the effect of dilute sulfuric acid in decreasing frost heave is very large for this soil. Lower curves: The same soil, but prepared with HCl and carefully washed and dried at 105°C, then frozen with NaCl solution in distilled water of different concentrations (given in normality: 1-n NaCl = 58.46 gr/L = about 6% by weight). The frost heaving rate shows a maximum for a given, rather small NaCl content. It is believed that the crooked curves are due to changes in ion concentration as the soil freezes.

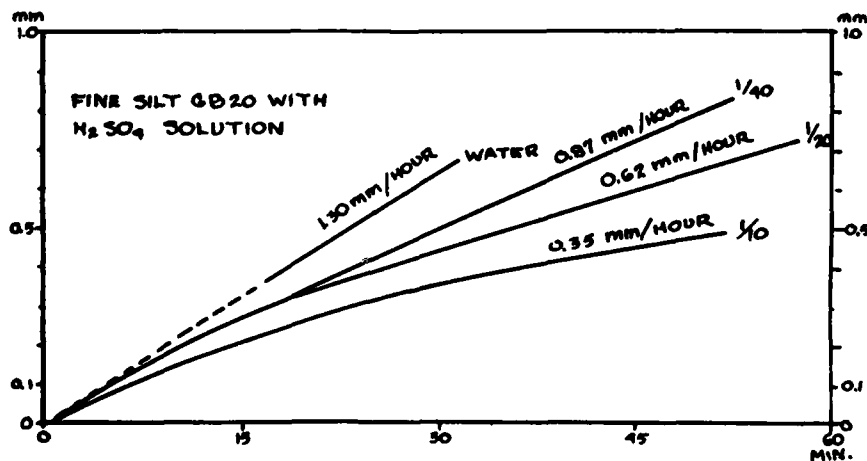


Figure 52. Analogous to Figure 51: frost heaving curves for silt GB20 with different concentrations of sulfuric acid in distilled water.

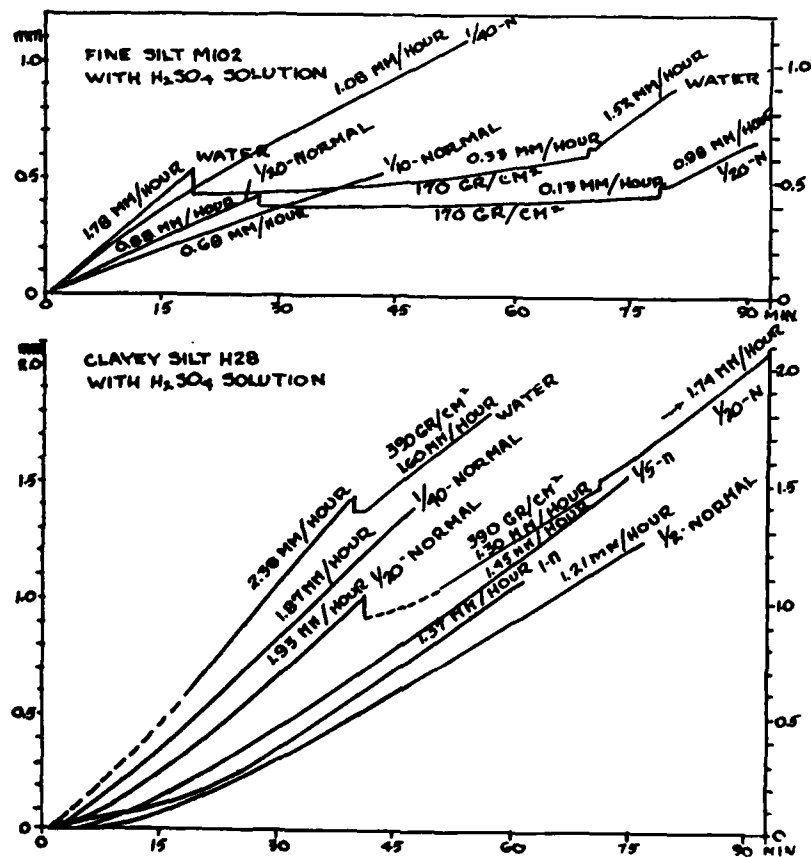


Figure 53. Analogous to Figures 51-52: frost heaving curves of silt M102 and clayey fine silt H28, with different concentrations of sulfuric acid in distilled water.

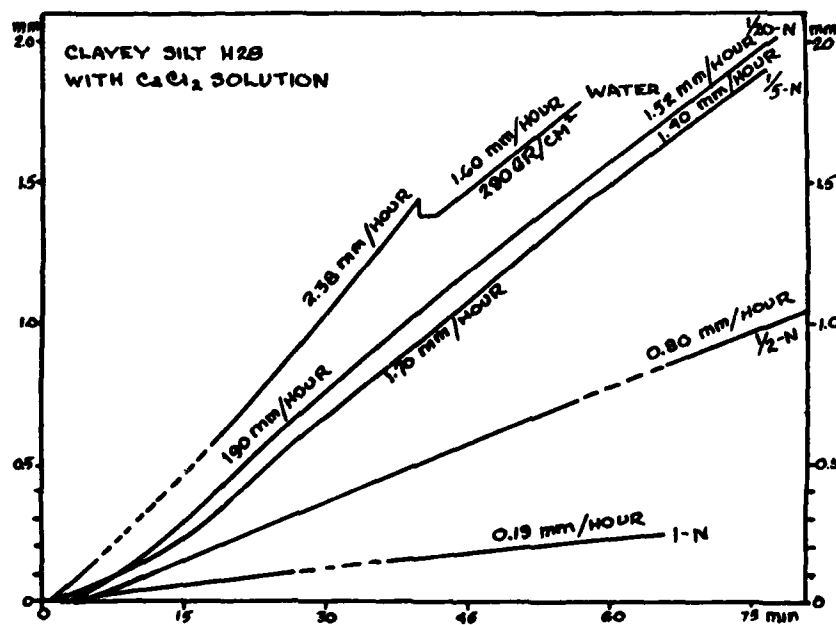


Figure 54. Analogous to Figures 51-53: frost heaving curves for clayey fine silt H28 with different concentrations of calcium chloride solution. Concentration for each curve is given in normality: 1-normal CaCl₂ has 55.50 gr/L, or about 5 1/2% by weight.

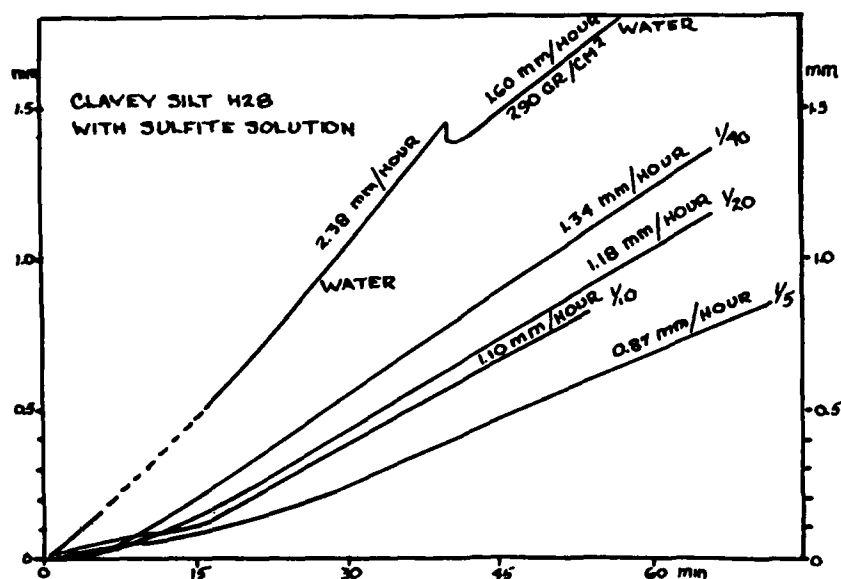


Figure 55. Analogous to Figures 51–54: frost heaving curves for clayey fine silt H28 with different concentration of sulphite leach solution [a by-product of the sulphite process of paper pulp mills]. Concentration for each curve is given in parts of syrup-thick leach, having 60% dry substance. For example, "1/20" means a sulphite solution with 3.0% dry substance, "1/5" with 12%.

The effect of sulphite leach is quite interesting. Sulphite leach intended for the purpose of keeping down dust was used for the tests. It was moistened to a syrupy consistency with 40% water. The values of concentrations in the diagram of Figure 55 refers to this moistened leach; 1/10 therefore corresponds to 6% dry substance. The effect of this substance is very large; for a concentration of 1:20 the rate of heave has already been decreased to one-half, for 1:5 to about 37%.

Since considerable decrease in frost heaving can be obtained by the use of cheap substances in small concentrations, it does not seem unlikely that the application can have practical use, as a substitute for other substances.

As an example, the following determinations are considered: A 1/20 to 1/10 normal sulfuric acid solution (about 1/4 to 1/2%) causes a decrease of 1/2 or more. If we assume a penetration of 1/2 m of depth, and the water content of the soil to be 30% by volume, this requires 0.38 to 0.75 kg of sulfuric acid per square meter. Even if the quantity is doubled several times to allow for spilling and washing out, the cost of the material is relatively small. The same applies for the sulphite leach. A quantity of about 5 kg dry material per square meter is sufficient in most cases with a wide margin of safety to reduce heave 1/2 or more; the cost of the material is on the average about 50 öre per square meter or less [about 12¢/m²]. In many districts waste leach can be obtained free. Acid unneutralized leach is most suitable and can usually be obtained.

An important technical question is how fast the material will be washed out under different conditions. Various tests are in progress at this time, and others are planned.

8. Relations between frost-heaving and increase in water content of the frozen soil

The question of how much increase in water is caused by a certain amount of frost heave depends first upon whether the ice in the frozen soil is massive or porous, which in turn depends upon several conditions, most important being the degree of saturation of the soil, its structure, and the magnitude of the load pressure.

For practical purposes the ice is usually always massive or nearly massive at least to some depth underneath the surface when the soil is capillary saturated. For this case, which is the only important one from the practical standpoint, the amount of water increase (Q_v) = $9/10$ of the frost heave (Q_i), minus the heave due to the expansion of the existing water in the soil (Q_o):

$$Q_v = 9/10 (Q_i - Q_o).$$

If $Q_o = Q_i$, no increase in water content occurs, and if Q_o is greater than Q_i , the increase is a negative one, i.e., the water is pressed out of the system, which is the case with the freezing of sand (Section III). However the magnitude of Q_o is always very small, the very most being one or two centimeters per meter frost depth. For an appreciable heave of the order of several decimeters

for a frost depth of one or two meters, Q_o is then very small in comparison with Q_i , and can be omitted; for a close approximation then, the amount of water sucked up is $9/10$ of the frost heave.

Considerably larger heave for the same amount of water content increase occurs when the ice is porous, as is often the case when the ice forms on the surface of the ground (hoar frost). The heaving ice layer consists of more or less thin vertical needles, and often the volume of ice is only a fraction of the total volume. The unloaded ground surface can actually heave a few centimeters up to a decimeter or even more in a few days by the formation of hoar frost, in which the actual water increase (measured in relation to the thickness of the ice layer) is only a small part.

Such thin, heaving ice layers can even occur at depths and under considerable load, especially in pre-existing cracks in clays, as well as at the boundary between two different soil layers ("inside hoar frost"). The cases where such occurrences are significant are rare; as a rule one can usually assume that as soon as there is an appreciable amount of frost heave to be important for roads and railroads, the amount of "water sucked up to the frost line is equal to or very slightly less than $9/10$ of the amount of the frost heave."

9. The magnitude of frost heaving and increase in water content of natural frozen soil

a) *Magnitude of heave.* There is a vast amount of observational data existing of the total amount of frost heave that occurs under natural conditions. In many cases the relative differences in elevations between frost heaving and non-frost-heaving ground can be directly determined.

Such cases are seen at culverts on highways built on frost heaving soil, where the culvert remains stationary, but the roadbed is heaved on each side. The magnitude of the frost heave can be measured at such depressions directly, either by levels or by laying a long stiff stick transversely across the depression and measuring the

distance to the bottom (Fig. 56 and 57). Depths of such depressions quite often exceed 15 to 20 cm and sometimes 30 to 40 cm. A similar but special case is shown in Figure 58. Another example is the case where a large stone remains stationary while the ground around it heaves (Fig. 59)

On railroads it is the relative magnitude of heave, that is, the difference between adjacent unevenly heaved parts of the rails that is noticeable. The magnitude can easily be measured, and is usually done by recording the required thickness of the wedges necessary to bring the various points back to level. Wedge thicknesses usually vary between one and four inches, but are sometimes even larger.

However, wedge thicknesses only give a measure of unevenness, that is, a detailed profile of the heaved surface, and is not a measure of the total magnitude of heave, that is, often only a fraction of the total.

Absolute magnitudes of heave can only be obtained by taking levels of the unfrozen and frozen ground. Such levels have been taken over long stretches by the State Railways especially of the newly built lines, and have recently also been taken over various highways. Simon Johansson has plotted this voluminous material for various successive years graphically on a continuous profile. Figure 60 is a sample of such a diagram. In order to determine the occurrence of frequency of a certain magnitude of frost heave, these values have been computed from two of such diagrams. This is given in Table 5, and shows the total lengths of roadbed having various values of heave. These values are for two different railroads over a length of 20 km and represent about the average type of sections found along these railroads.

It is seen from the table that the extreme top values are about 40 cm, and that heaves over 30 cm are already quite extensive (about 1% of the length), magnitudes over 20 cm occur over 3% of the total length. Just where the line can be drawn for the magnitude between frost-heaving and non-frost-heaving ground depends of course



Figure 56. Unusual example of a depression over a culvert in frost heaving ground, due to heaving of the ground on both sides of culvert but not over it (from A. Casagrande).

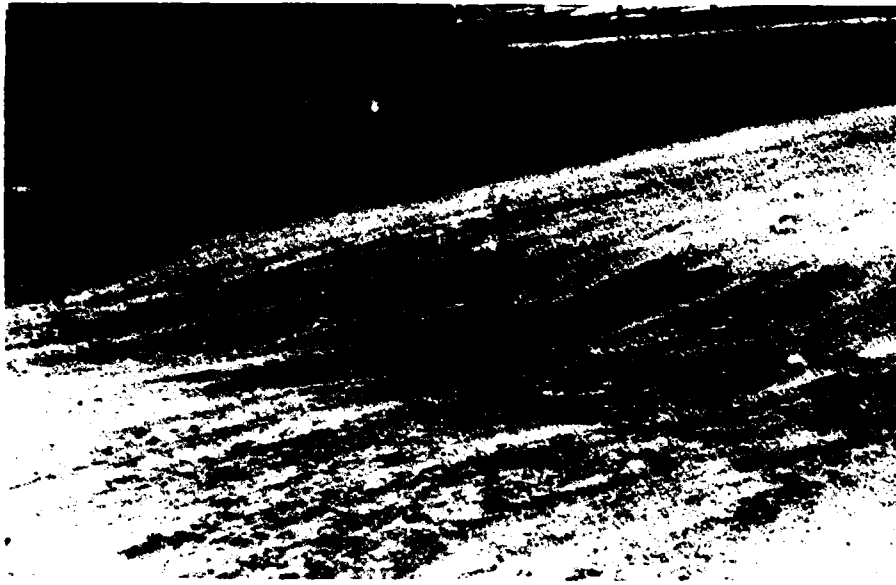


Figure 57. Depression over a culvert on frost heaving ground. Usual type on Swedish gravel roads.



Figure 58. Example of frost heaving. When this road was built, the part over frost heaving ground should have been excavated to about 60 cm and replaced by non-frost-heaving soil. By a mistake, those responsible neglected to take this precaution, and thought it would not be noticed. The photo shows the road right after the snow melted, with the sudden drop between replaced and non-replaced frost heaving sections.

upon special conditions. Under all circumstances, a magnitude of at least 10 cm must be considered from a practical standpoint as considerable. This occurs over about 15% of the distance. Certainly it would be better to draw the line somewhere near 5 cm, which then would make dangerous frost heave occur over $\frac{1}{4}$ to $\frac{1}{3}$ of the whole length.

An extract of the results from level surveys on highways is given in Tables 6 and 7. For the data of frost depths for various values of frost heave (Table 7)

occurring in Norway, the author is indebted to Chief Engineer A. Rode, Trondheim, Norway. The Swedish observations are made from data obtained by the Swedish Bureau of Roads and carried out by many Road Superintendents over various parts of the country. The various investigations are very variable, ranging from single cross-sections and longitudinal profiles to combinations of both.

From the tables (or more correctly from all the data from which the table is a condensed form) we can con-



Figure 59. Frost heaving in moraine material, noticed by the contrast to a partially uncovered boulder, which remained stationary. (In other cases boulders can be lifted with the heaving soil, leaving an open space under the boulder). Mörby, Stocksund, near Stockholm.

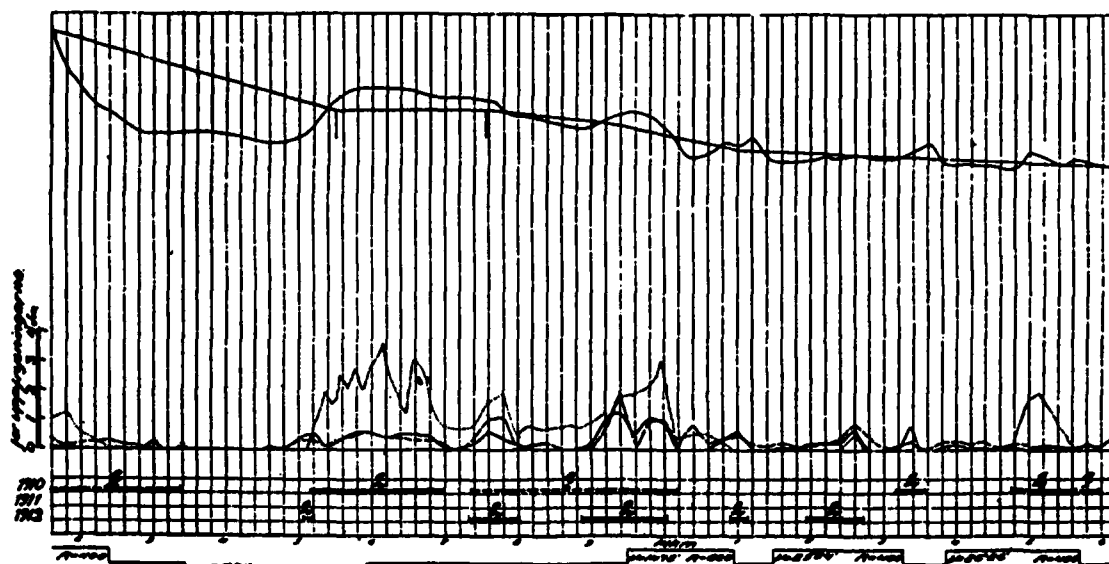


Figure 60. Profile from a section of the Bastuträsk-Skellefteå railroad, showing the ground profile (above) and the magnitudes of frost heave for different years (below); (dotted 1909-10, dashed 1910-1911, solid line 1911-12). The heavy lines underneath show the stretches and years in which sections of soil were either replaced with non-frost-heaving soil or drained. The dotted curve 1909-10, before these measures were taken, show frost heaves up to 30 cm. Soil: Normal moraine material, in a relatively saturated state. The correlation between cuts and heave is quite evident.

clude the following: frost heaves of about 20 cm are quite common, especially in northern Scandinavia. The silt and lean clay sediments show a normal frost heave of about 20 cm* not considering the special extreme cases where frost boils occur. In the latter cases, the usual heave is about 30 cm, and sometimes reaches 40 cm and even more.

On moraine soil as a rule the frost heave is considerably less, generally about one-half. However, it is quite variable on moraine material, not only different from place to place but there are local spots of excessive heave, making an uneven frost heave profile curve.

* For roads with normal embankments, the greatest height being a few decimeters above the ground surface; for greater heights of embankments the frost heave is less.

Table 5. Percentage of total length of two railways in North Sweden which is subject to frost heaving.

Frost heave (cm)	I + II			Frost heave (cm)	I + II		
	I (%)	II (%)	2 (%)		I (%)	II (%)	2 (%)
45-50	0	0.1	0.05	>40	0	0.1	0.05
40-35	0.4	0.2	0.3	>35	0.4	0.3	0.35
35-30	0.2	0.6	0.4	>30	0.6	0.9	0.75
30-25	0.7	1.0	0.85	>25	1.3	1.9	1.6
25-20	1.3	1.6	1.45	>20	2.6	3.5	3.05
20-15	2.0	4.8	3.4	>15	4.6	8.3	6.45
15-10	8.0	7.9	7.95	>10	12.6	16.2	14.4
10-5	18.7	24.8	21.75	>5	31.3	41.0	36.15

Type of soil: moraine material.

I— Line from Hällnäs to Stensele, kilometers 30-50, Winters of 1922-23, 23-24, and 24-25.

II— Line from Bstutraäsk to Skellefteå, kilometers 0-20, Winters of 1909-10, 10-11, and 11-12.

Table 6. Magnitudes of frost heave of various Swedish roads, obtained by leveling surveys.

Locality	Province	Type of dirt	Length observed*	Date	Frost Heave (cm)	depth (cm)
Vännäs, Overkalix-Korpilom- bolo, I (Overkalix)	Norrbotten	Lean clay on gravel or bedrock	s	1/2 29	16-28	—
			s	12/4/29	35-51	—
			s	28/2/30	19-27	—
			s	29/4/30	17-28	—
Vännäs, Overkalix-Korpilom-bolo, II	Norrbotten		s	28/2/30	23-46	—
Långsel, Avarfors-Vitå (Råneå)	Norrbotten	Lean clay	s	9/3/31	24-27 80	—
Renholmsvalgen, km 3 (Piteå)	Norrbotten	Varved lean clay	s	18/2/30	18-21	—
			s	5/4/30	22-28	—
Selet-Norrlångträsk (Byske, Skellefteå)	Västerbotten		40 m	14/2/30	12-30	—
Skansholm, Vilhelmina, I	Västerbotten	Moraine material	s	11/5/30	15-23	—
Malgomaj, II (Vilhelmina)		Moraine material	s	11/5/0	14-26.5	—
Lågmyren, Kusfors-Norsjö (Norsjö)	Västerbotten	Moraine material	175 m	30/12/30	2-12	—
			175 mm	3/4/31	7-22	—
Burträsk-Skellefteå, km 26 (Burträsk)	Västerbotten	Moraine material	25 m	28/12/28	5-21	—
			25 m	3/4/29	14-44	—
Lycksele-Björkberg (Lycksele)	Västerbotten	Moraine material	300 m	10/4/31	2-14	—
Brattbyåker, Umeå-Vännäs (Umeå)	Västerbotten	Lean medium Clay	2 p	27	—	—
			2 p	5/8/28	18-23	—
			250 m	8/3/29	19.5-42.5	—
			250 m	30	—	—
			250 m	9/2/31	10.5-23.5	—
			250 m	22/1/32	c/a 2-10	—
				10/3/34	10-19	—
Soärfors, Brännland-Overboda (Umeå)	Västerbotten		2 p	16/2/27	16.5-19	—
			2 p	19/3/28	16.5-18.5	—
			2 p	12/4/29	21-23	—
			2 p	30	—	—
			2 p	26/3/31	24-25	—
			2 p	27/1/32	11-13	—
				31/1/34	8-18	—
Ramvik, Härnösand-Sollefteå (Gudmundrå)	Väster-norrland	Medium Clay	40 m	19/3/27	9-15	c/a 90 105-120
Fröland, Härnösand-Sundsvall (Säbrå)	Västerbotten	Lean Clay	40 m	17/3/27	15-20	—
Alvesta-Bergunda (Kinnevalds)	Kronobergs	Morain material	3040 m	Spring '30	Frequently 8-10 cm 13-15 cm max.	20-25
			3040 m	Spring '31	Frequently 5-8 cm. 12 cm max	35-40
Växjö-Huseby (Kinnevalds)	Kronobergs	Moraine material	440 m	Spring '30	Normally about 5 cm Up to 10 cm max	—
			440 m	Spring '31	Normally about 5-10 cm 11 cm max	35-40

* "s" means one section, "2 p" two observation points from the centerline of the road surface.

These undulations or waves are very bothersome for road traffic. The cause is probably due to the unusually sudden changes in the composition of the moraines, and is sometimes due to uneven water supply. A magnitude of 15-20 cm for a moraine must be considered as large, and actually dangerous for frost boils. In exceptional cases moraine material can heave 30-40 cm.

b) *Magnitude of water-content increase.* The heave is a measure (with a small reduction, see discussion above) of the amount of water sucked up during freezing.

But what is important for roads when thawing occurs is not the total amount of water increase, but the relative thicknesses of the layers in which the increase is separated and the type of uneven distribution within the frozen soil.

If we neglect for a moment the latter factor, we can observe from the preceding tables (Tables 5-7) the data for computing the relative water-content increase. The thicknesses of the road beds are not usually given, but for ordinary gravel roads, we can use an average value of 15 cm. The frost depths are in some cases determined,

Table 7. Frost heave and maximum frost depth at certain points on the roads of Sör Trøndelag, Norway.

Locality	1929/30		1930/31	
	Frost heave (cm)	Frost depth (cm)	Frost heave (cm)	Frost depth (cm)
Trondheim-Klett (km) 2.778	12.9	—	14.9	100
Klett-Orkanger (km) 31.438	18.5	40	27	90
Klett-Orkanger (km) 31.500	9.5	40	16	c/a 100
Klett-Berkåk (km) 114 A	14	25	—	—
Klett-Berkåk (km) 114B	12	27	—	—
Klett-Svantøy (km) 33.9	—	—	13	125
Svantøy-Berkåk (km) 52.9	—	—	22.5	110
Storas-Berkåk (km) 117.35	—	—	12	80
Storas-Berkåk (km) 119.30	—	—	15	90

Table 8.

Frost heave (cm)	Corresponding amount of water sucked up (cm)	Frost depth (cm)	Orig. depth of frozen ground (Road bed assumed 15 cm deep) (cm)	% avg increase in water content (% of frozen soil)	
				by volume	by weight
10	9	50	25	36	23
100	75	12	7.5		
20	18	50	15	120	75
100	65	36	23		
150	115	16	10		
30	27	100	55	49	31
150	105	26	16		
40	36	100	45	30	50
		150	95	38	24

and from these we can conclude that for the northern roads in normal winters the depth is about 1 to 1½ meters, depending upon the geographical and terraiinal conditions of the road.

In the table above (Table 8), the values are computed for these average conditions.

One can see from the table that the average increase in water content in frost heaving soils in Norrland (northern province in Sweden) is about 20–30% by weight, or about 30–50% by volume. These results agree well with the results of numerous direct determinations (see below).

But let it be emphasized that the average water content increase in the road bed is by no means the only cause of the behavior of the roadway during thawing, or for the occurrence of frost boils. If we neglect the effect of variations in the road surfacing or loading as well as traffic loads, there are two factors that are of extreme importance, namely, the mechanical properties of the soil and the *distribution of the water excess in the road bed*. Concerning the first factor it will only be mentioned as Simon Johansson (1916) has shown, that the decrease in bearing capacity caused by a certain water excess, or the other way around, the water increase that is necessary to lower the bearing capacity a certain

amount, varies very markedly with the soil grain size composition (see Beskow 1934b, Fig. 97, p. 308). When the soil is capillary saturated to begin with, as in the case with all frost-heaving soils, it is only necessary for a small increase in water to bring a silt or fine silt to a liquid state, while on the other hand, for the same reduction in consistency (i.e., bearing capacity) for a clay a large amount of water excess is needed, and a greater amount of fatter the clay is.

The distribution of the water excess within the road bed has been shown by observations of frost structure to be very uneven. At certain levels the strata are closer together or are thicker than usual; at these points the excess is, of course, larger (Fig. 61). Sometimes single ice layers of several centimeters and even decimeters are found, which is why at such levels we can say the water content of the frozen soil is infinite.* From this

* Naturally this depends upon how large an element one uses to compute with. If we take a very small thickness and divide it into millimeters or fractions of millimeters, the water content curve will consist of short sections of 100% water content (per volume; per dry substance it is infinite) varying to sections with water content less than for the same clay in an unfrozen condition, which is due to the drying out of the clay between the ice bands, although the total water content increases.

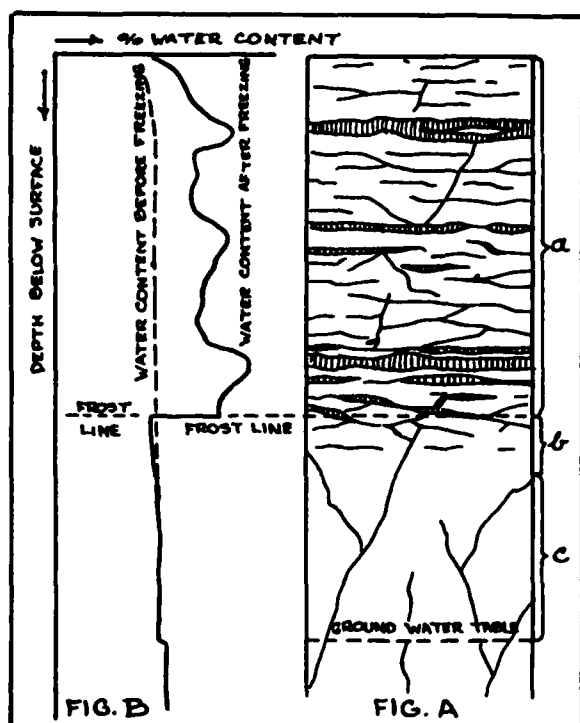


Figure 61. Diagram showing the relation between different ice layers in frozen soil (A), and the water content distribution curve (B). The soil is assumed to be a medium clay, with permanent cracks. *a* = frozen part, *b* = dried out zone below the frost line. Since the clay contains permanent cracks, the groundwater surface is real, i.e., the level where the cracks contain free water, and is therefore noticed as a kink in the water distribution curve.

extreme value all variations in water content occur down to a water content about equal to that of the unfrozen soil. Sometimes certain levels can be entirely free of ice strata.

The causes of this uneven distribution are *varying climatic factors*, and *uneven soil composition*. If the effect exists in a discontinuously frozen sediment, then the sand layer will be a level where there are no ice strata and no excess water. But less extreme changes can be noticed, as when layers of different coarseness have different amounts of heave, so that a difference in water excess occurs for various strata. Even a very small variation in the average grain size or even in the grading (and this often occurs in a silt or lean clay which might seem uniform) can cause appreciable variations. The effect of grain size variations was very markedly seen in the analyses of heave with grain size. The small variations in grading, however, cause a considerable difference in the pore volume and therefore in the water content at the beginning of freezing. In addition, if the different types of soils are separated by a sharp boundary, this boundary becomes a place where thick ice

layers occur, as has been explained before (see Fig. 17).

But more important than these differences in soil composition from a practical standpoint is the effect of the climatic factor as it affects the *rate of freezing*.

c) The effect of rate of freezing on the distribution of the excess water in the frozen soil. It has been shown above that the water sucked up to the frost line is independent of the rate of freezing.

What occurs is that the water transported upward in a given time is divided unequally in the soil volume, that is to say the water excess is different at different levels in the soil causing different density and thickness of ice layers. Numerous tests have shown that at the levels where the rate of frost penetration is very small, the ice content is large (Fig. 9), and that quantitatively the difference in the ice content at different levels is nearly proportional to the quotient of the rate of heaving and the rate of freezing.

The smaller the rate of heaving the more concentrated the ice content becomes. When the rate is so small that the rate of heat conduction balances the amount of water sucked up, the total amount of "coldness" is used up to freeze the water, so that none is left over for further penetration downward of the frost line. The frost line becomes stationary and a single ice layer grows, which can theoretically continue to an infinite thickness (under these assumptions the rate of heaving is directly proportional to the thermal energy transfer).

As mentioned before there is often found in natural ground places where there are thick ice layers corresponding to levels of relative (or absolutely) stationary frost lines (see Fig. 7 and 8). Actually what the ice distribution really registers is the surface temperature. The most important climatic factors that cause these changes in ground surface temperature and therefore the rate of freezing are, of course, the air temperature and the snow covering. For bare ground or a constant snow cover the distribution of ice layers registers all marked temperature changes which last sufficiently long. But changes in the thickness of the snow covering (or, more specifically, ground insulation) have a considerable effect added to that of the air temperature, and can often exceed the latter. On cultivated ground, or even on roads that are only snowplowed by horses, as is usual in Norrland (northern Sweden) where there is an early snowfall in the Fall, there is a marked decrease in the rate of frost penetration, and sometimes there is even melting from below. In late winter, on the other hand, when thawing occurs accompanied by very cold nights, the snow covering is reduced to ice and there is a fast penetration of frost (see Fig. 84). Nowadays that roads are generally quite efficiently plowed by motor power, the effect of the snow covering is relatively immaterial, and the biggest factor is the *air temperature*.

It is now clearly understood that the most important factor for the stability of roads during thawing is at what depths in the roadbed the larger water excesses are concentrated, and that it is more advantageous the deeper these occur. A considerable water excess deposited as ice high up in the fill, or even in the roadbed itself, will cause softening or frost boils during the thawing even if it occurs on a soil that is inclined to be very stable. The deeper the water excess occurs, the less possibility there is for the water-saturated soil to penetrate to the surface as frost boils or for large deformations to occur. Only in the most unstable soils, such as silts and fine silts, can it penetrate to the surface from depths of $1/2$ to 1 m.

Slow freezing in early winter, caused by a moderate cold spell or by snow, causes a large amount of ice in the upper layers of the road, which tends to produce difficult thawing effects. For rapid freezing in early winter, the critical upper layers become relatively barren of ice, and therefore more stable during thawing. For this reason machine snow plowing, contrary to current opinion, is not only unharmed, but is a positive advantage considering thawing (Beskow 1928).*

Lastly it should be kept in mind that, independent of other contributing factors, a certain inequality in the distribution of the water excess can occur by a change in the rate of upward flow of water due to variations in the distance to groundwater during freezing. The penetration of frost brings the frost line closer to the groundwater table, but, on the other hand, the groundwater table itself is generally lowered during the winter. Just how the distance between both boundaries will change depends on the local conditions, and only a few general values will be given here.

Variations in the height of the groundwater table are generally less the flatter the topography is. On relatively horizontal plains of clayey ground with appreciable expanse, such as the farm lands in central Sweden and certain northern coastal plains, the variations are pretty small. If the depth to groundwater when freezing starts is about $1/2$ to 1 m, it sinks to about $1 1/2$ to 3 m. Since the frost depth on the cultivated parts of this section seldom exceeds a few decimeters, there is a considerable increase in the distance to groundwater during freezing. On roads, on the other hand, one can usually assume a frost depth of around 1 m, and here both movements compensate each other, and the distance becomes approximately constant.

When it concerns a coarser soil, such as a silt sediment with somewhat greater variations in elevation and less expanse, which is the type of ground in the valleys

of northern Sweden, the variations often become much larger. Here the elevation of the groundwater is also high when the first frost comes, due to the heavy fall rains (about $1/2$ to 1 m under the surface). But the sinking of the groundwater during the winter can be so large that towards the end of the winter the groundwater depth can be as large as three, four, and even more meters. This lowering is compensated only slightly by the penetration of the frost line, so that (assuming homogeneous soil) the amount of frost heave decreases during the winter.

As will be shown later in Section IV, the theoretical importance of the groundwater surface is quite different for different soils. Concerning a fissured clayey soil, the groundwater surface is actually a place where the water content changes and is of great importance. But in non-fissured silts and fine silts, the groundwater surface is a "theoretical surface" and is not of great importance. Here the important factor for frost heaving is the distance to more permeable soil, such as gravel or sand or even fissured clay. When such is the case the distance from the frost line to the water conducting layer decreases as the frost depth increases, and the rate of frost heaving increases. But this occurs only as long as the permeable layer actually conducts water up to the freezing soil layer; if the water supply is reduced for the duration of the winter so that the groundwater flow is stopped or only sunken below this contact surface, the amount of water sucked up and therefore the heave is practically reduced to zero.

Likewise local conditions can cause an infinite number of complicated, varying, special cases which cannot be dealt with here. Regional behavior also becomes quite different in different climatic regions. If there is a continuous winter with a decrease in the groundwater, or if there is a milder climate with intermittent freezing and repeated thaws causing a renewal of the groundwater supply, this makes a great difference.

All of the factors causing uneven distribution of the water excess in frozen soil are directly shown by the ice layers found in the frozen samples. Numerous tests of the free ice volume and water content have been performed by Kokkonen (1926), and it was found that in clays the transported water in the water excess of a frozen sample does not constitute all of the free ice content, but that some of the water in the ice comes from pre-existing water in the clay. The magnitude is generally larger the stiffer (fatter) the clay is and the larger the water content is.

Figure 62 shows graphically the results of two series of tests. The shaded area between the water content curve before and after freezing shows the magnitude and distribution of the water excess.

d) Summary of the magnitude of water content increase in considerably frost-heaving ground. The fol-

* That the total frost heave increases due to this is another matter; as a rule the significance of this on roads is relatively small. The cause of this increase is, of course, that for increased frost depth, freezing occurs closer to the groundwater table.

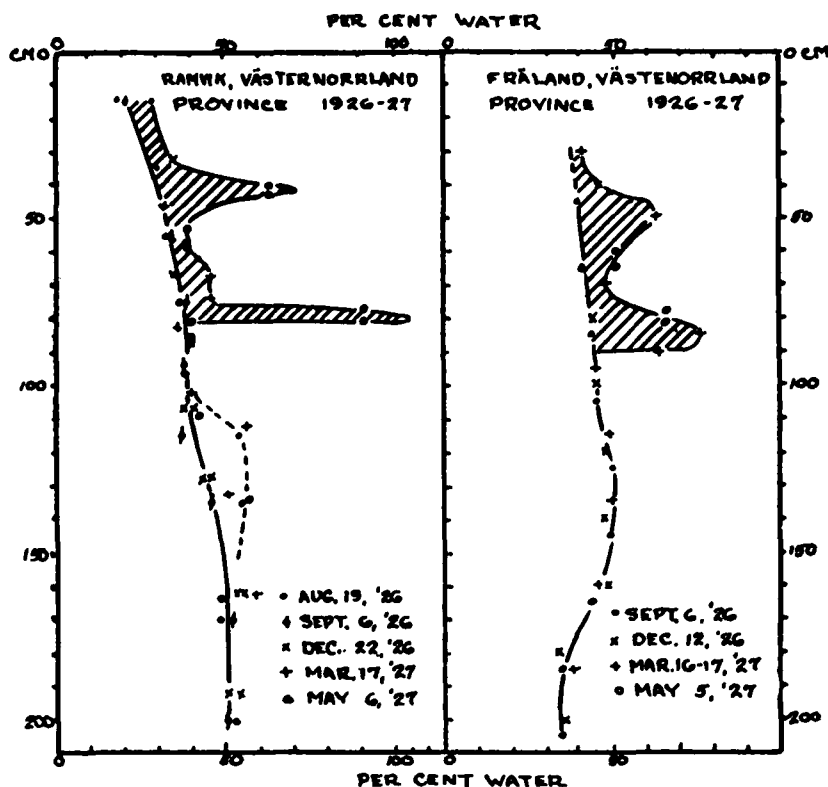


Figure 62. Example of water content increase in roadbeds during freezing. Graphical summary of moisture determinations on a series of soil samples from different depths, taken at different times during the same year. Depth under the road given in centimeters, and water content in percent of dry substance. The soils in both cases were light medium clays (at Fräland, the clay changes to a fine silt at a depth of 160 cm, which causes the inward bend of the curve at the lower part).

lowing is a summary of all the data accumulated on the water content excess occurring during freezing:

For very slow freezing—the freezing being slowed by snow insulation or by a mild winter—the water increase can be very large. Kokkonen's data of frozen cultivated ground give an ice content in percent of the total volume for clayey soils of about 25–55%, for silts 0–75%, with an average of about 50%. The frost depth is only a few decimeters due to the higher temperature. The above figures therefore correspond to a frost heave of about 15 cm which is quite what one would expect. For a silty soil with a normal water content of 35–40% by volume there is therefore an increase in water content of 100 to 150%. Similar water contents were obtained from roads in Vaxjo and Stockholm during relatively mild winters even though the frost depth was only a few decimeters. Levels taken in Vaxjo showed a heave of 10–15 cm for a frost depth of 20–40 cm (see Table 6), which gives an average water excess of about $\frac{1}{3}$ to $\frac{1}{2}$ of the total volume. Direct determinations from soil samples give similar values. The above are of course average values, but with small frost depths there are not

such large variations of the distribution of the water excess. If we have layer thicknesses of 3–5 cm, a water excess of 100–200% of the original soil volume is not unusual.

For large frost depths, the distribution is much more uneven. The rapidly frozen layers of a very strong frost heaving roadbed only increase to about 10–15% of the original volume, corresponding to an increase of the original water content of 20–30%; if we should consider very thin layers the increase can be even smaller. On the other hand, the levels containing a lot of ice, denoting a relatively stationary frost line, can have practically an infinite water content; since a single ice layer can grow to a few decimeters, we can say that the water content increase for a layer of that thickness is infinite. But neglecting exceptional cases, assuming unit layer thicknesses of 3–5 cm as before, the layers of large ice content usually have an excess water content from 100%, and sometimes up to 200% and 300% of the original volume, which means the water content is doubled and sometimes tripled and quadrupled.

Table 9. Frost-heaving rate at different pressures, for varying moraine-soils or similar types.

No.	Soil sample	Capillarity K_M (m)	Frost-heaving rate (mm/hr) at pressure (g/cm ²)				
			15	48	170	410	650
A	Miscaceous sand 1	0.95	0	—	—	—	—
B	Miscaceous sand 2	1.9	0.06	0	—	—	—
1	102 moraine	0.8	0.1	0	—	—	—
2	B1 14 rock flour	1.5	0.26	0	—	—	—
3	S0 4 gravel		(0.1)1	0.1	0	—	—
4	44 Road surface	1.6	0.24	0.19	0	—	—
5	G8 6 moraine	1.9	0.52	—	0.18	0.02	—
6	209 Road surface	2.5	[0.24]*	0.5	0.24	0	—
7	1934, 1:10 road surface		[0.4]*	0.53	0.19	0	—
8	29 Road surface		[0.26]*	0.6	0.2	0.07	—
9	4/11/33, 8 b road surface		(0.7)*	0.7	0.26	—	—
10	B1 14 Rock flour	4.4	(0.6)*	—	0.3	0.11	—
11	4/11/33, 4 road surface	4.7	[0.6]*	1.2	0.34	—	—
12	47 Road surface		[0.6]*	1.05	0.48	0.2	0.01
13	1934, 11:1 macadam		1.0	0.75	0.55	0.25	—
14	B1 5 Road surface	6.5	[0.68]*	—	0.66	0.4	—
15	201 Road surface		—	0.84	0.7	0.38	0.18

* Important! () [] the figures are wrong (too small), because of bad contact with the cold air in freezing chamber (slow freezing).

10. Limits between frost-heaving and non-frost-heaving soil

From the above considerations we can see how various factors act to cause an expansion of the soil during freezing, and how they determine whether a soil will for practical purposes heave or not heave, that is, whether soils become frost-heaving or non-frost-heaving. It is therefore practically impossible to fix a definite boundary between frost-heaving and non-frost-heaving soil.

However, when we consider the conditions which the highway engineer is concerned with, the degree of variation of these factors is so strongly marked that for practical purposes one can with fair accuracy determine certain limits or boundaries. The basic properties which can be evaluated in this respect are grain size and capillarity.

a) *Limits of grain size.* A description has already been given (Section II-B-4) of a series of freezing tests on sorted fractions, from which it was found that the limiting average grain size was about 0.1 to 0.05 mm.

Results of other tests (which were carried out with the apparatus described in Figure 36) as well as grain size analysis of various natural frost-heaving soils, are given in Table 9 and Figures 63–65. The material has been summarized into two groups: sediments and moraine material. The following is a summary of the results:

Concerning normal sediments (that is, well-sorted soils) the limiting grain size is a very definite boundary as shown in Figure 63. If we choose a suitable value consistent with these curves, we can say that this limit is at the point where 30% of the material is finer than

0.062 mm or 55% of the material is finer than 0.125 mm. All soils coarser than this are definitely non-frost-heaving.

But the soils that lie just under this limiting value—the coarsest soil that can be frost-heaving, or actually a medium coarse silt—are not under all circumstances frost-heaving, but only under certain conditions, such as a high groundwater table and a very small load pressure. Such soils are only dangerous when located in wet slopes, where the load pressure is insignificant, and when in road beds with a very high groundwater table (very poor drainage or an abnormally wet fall).

As a boundary for normal frost heaving sediments a value can be taken where 50% is finer than 0.062 mm.

For unsorted moraine soils the conditions become more complicated. The first difficulty arises from the method of analysis; the first question is at what size shall we start the analysis and neglect the coarser material. Certainly the large stones must be omitted. Since we are concerned with a continuous variation of grain sizes, such a limit must be arbitrary and conventional. For several reasons it is, however, suitable to use the limit between coarse soil and fine soil or 2 mm.

According to the Inter-Scandinavian grain size designation the diameter refers to the diameter of a round holed sieve. For road purposes it is, however, more suitable to use the nearest number in the series of sieves that the road institute together with the Swedish geological survey has agreed upon, or a 2-mm four-cornered-holed sieve. The road institute has determined that for normal soils the relationship between round and four-cornered holes is about 1:0.8 so that a 2-mm sieve

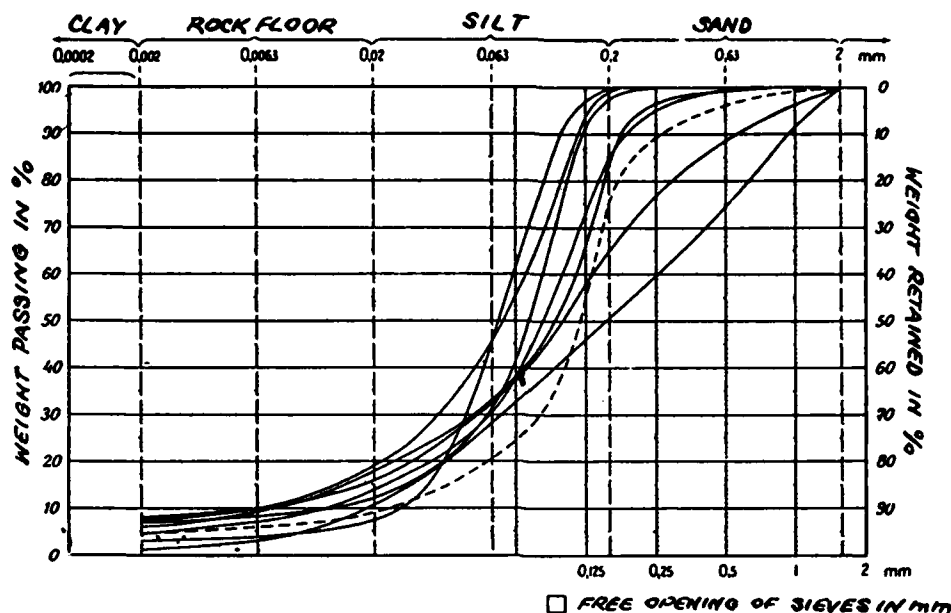


Figure 63. Grain size analysis curves for the coarsest frost-heaving and finest non-frost-heaving sediments. Dashed curve: non-frost-heaving (group 0, Table 11). Solid curve: frost-heaving group 1a and 1b.

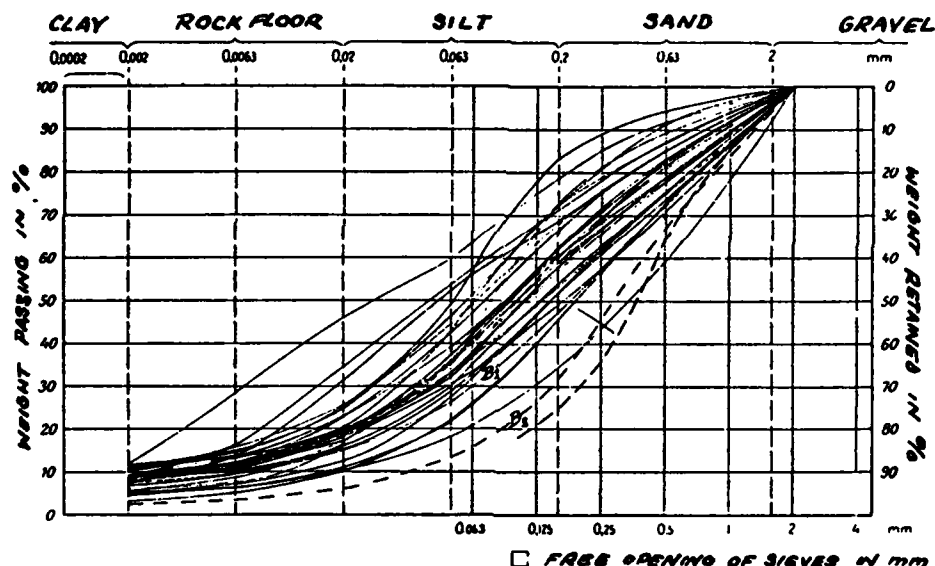


Figure 64. Grain size analysis curves for frost-heaving (solid lines) and two non-frost-heaving moraine soils (dashed curves). Curves show the composition of material passing the 2.0-mm sieve. The same analysis was made with Atterberg's method. Observe B_1 and B_2 , which are from the same stretch of road, in which B_1 is a frost-heaving and B_2 a non-frost-heaving variation of a moraine material.

with round holes corresponds to 1.6-mm-square-holed sieve. On this basis, it is possible to convert the results of the round holes to that of the square and vice versa. Diagrams 63–65 are given in a special form in which both designations are given and are related to each other by the given value of 1:0.8.

Figure 64 contains curves of fine soils which are definitely frost-heaving, and also a curve for a non-frost-heaving moraine. As shown the shape of the curves are relatively harmonic, and the limits seem to

fall fairly exact; they pass through the points where 18–20% is finer than 0.062 mm and 30% is finer than 0.125 mm. Curves B_1 and B_2 are of special interest. They consist of variations of the same moraine mantle, and were taken a few kilometers away from each other on the same roadway on which levels showed that, although they were both in about the same type of cut, the one (sample B_1) was considerably frost-heaving while the other (sample B_2) was completely non-frost-heaving. Here the limits are fixed within a very small zone.

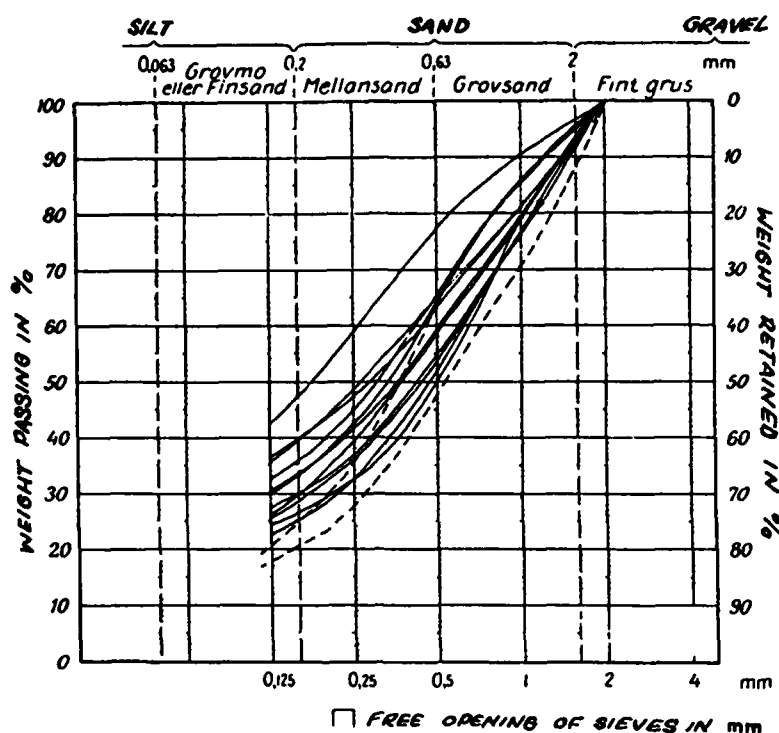


Figure 65. Sieve analysis curves of relatively coarse mixtures, on which freezing analyses were made in order to determine the limit between frost-heaving and non-frost-heaving moraine mixtures (see Table 9).

Figure 65 gives curves of the sieve analyses for a number of coarse moraine soils and moraine mixtures, on which freezing tests were performed with different pressures, and the results of which are given in Table 9.

On the basis of both of these diagrams, Figure 66 is constructed, giving on the one hand the limit of such soils that are under all circumstances non-frost-heaving from a practical standpoint, and on the other hand the frost-heaving in normal road beds in a relatively moist condition. If we state this graphical illustration in numerical values, we can state it as: non-frost-heaving moraines are those of which less than 22% of the material passes the 0.125-mm sieve and less than 15% passes the 0.062-mm sieve, computed on the basis of % weight of the material that passes the 2-mm sieve.

Note that the above refers to moraine material of approximately "normal" grading curves, and also to mixtures of materials having approximately the same type of grading curve.

For more unevenly sorted mixtures, such as disintegrated soils and certain soils found in road beds, the behavior is quite complicated and the freezing properties cannot always be determined only on a basis of sieve analysis. In such cases the composition of the finest sieve fractions must be determined either by sedimentation analysis or by hygroscopic determinations. In general it can be said that the more colloidal

material in the soil the greater is the danger of frost-heaving even though there may be very coarse soil in the mixture.

The behavior of unevenly sorted mixtures was studied by freezing tests of mixtures of sand and fat clay in different proportions.

The sand consisted of a medium sand; the clay came from Skarestå in Östergötland, with a water content of 47% on a dry weight basis. Freezing tests were carried out on these materials in different proportions in the usual apparatus.

The samples were capillary saturated, with the groundwater surface at a constant depth (about 10–5 cm). The results are given in the summary of Table 10 (also Fig. 67, curve A).

From the table we can conclude the following: By adding a small amount of clay to the sand the mixture immediately becomes frost-heaving. An addition of 5% clay (corresponding to 2½% dry substance) causes a noticeable effect, 10% considerable, and 20% a very large effect.

For increased clay content, the heave decreases, and for 40% clay it is practically zero.

The cause of this is the following: for a small amount of clay, the clay particles form a colloidal covering on the sand grains, having the physical requisites for discontinuous freezing at the same time the pores are

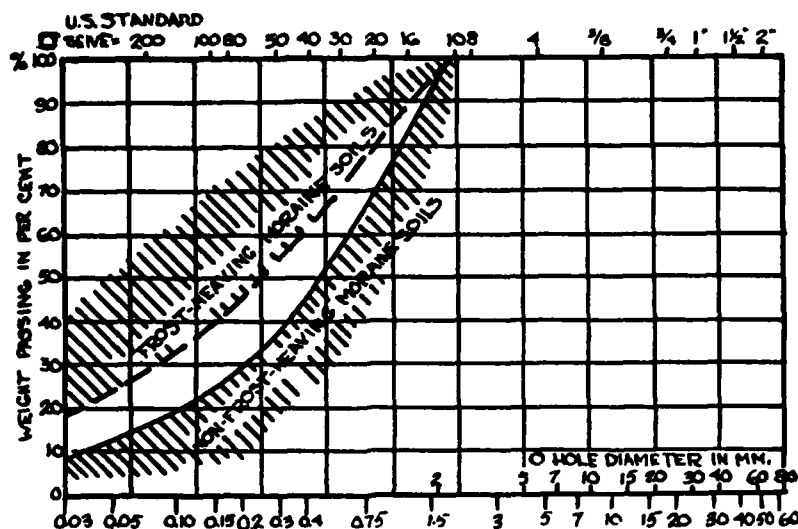


Figure 66. Limits between frost-susceptible and non-frost-susceptible mixtures of moraine materials or similar mixtures. The lower cross-hatched zone is for soils in group 2 of the same table, the middle blank zone for soils in groups 1a and 1b.

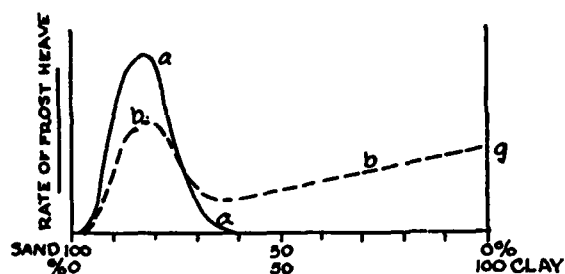


Figure 67. Schematic curves of frost-heaving rate for mixtures of sand and clay. Curve a: a very stiff practically impervious clay; constructed with the help of Table 10. Curve b: similar curve for lean, permeable clay, having a frost-heaving rate "q" in unmixed state. Full saturation and small depth to groundwater assumed.

open, and the mixture is very permeable, which allows the soil to suck up water causing a large frost heave. On the other hand, when the clay content is so large that it completely fills the sand pores, which occurs at 40% clay content, the system becomes practically impermeable, and no appreciable frost heave occurs. The optimum value is about 20% at which the colloidal covering is appreciable, while at the same time the pore system is relatively open.

Keep in mind that since sand almost half filled with clay has a very low capillarity (actually about double that of clean sand), such a mixture is very sensitive to changes in groundwater depth; although for small groundwater depths, up to a half meter for example, this mixture may be strongly frost-heaving, an appreciable

increase in the groundwater depth may exceed the capillarity of the mixture, and causes a decrease in the heave to almost nothing.

Just how the mixture will behave if on the one hand we vary the grain size of the coarse component and its consequent capillarity and on the other hand vary the fineness of the clay component and its consequent permeability, shall not be discussed here, except that we may note that if we mix a sand and a leaner clay, the relation between the heave and the clay content will be approximately as shown in Figure 67 [curve b].

b) *Limits of capillarity.* Of the direct physical properties which might possibly be used to determine the frost-heaving characteristics of a soil, capillarity has been found to be the most useful.

There are several reasons for this, principally that capillarity is an approximate measure of the grading of a soil, and is also very important in the process of upward flow in the soil. Also it is a very easy property to determine and requires only a very small sample (see Section IV).

Determinations of the capillarity of a large number of frost-heaving and non-frost-heaving soils have been made, and the results are summarized in Table 13. The table includes natural sediments, such as moraine material as well as materials of very variable grading curves. As can be seen from this table, the capillary number is a good, comparatively accurate index of the heaving properties of a soil.

We can, therefore, without much approximation fix the limit between soils which are non-frost-heaving under all circumstances and such soils which become

Table 10. Frost-heaving rate in mm/hour for mixtures of sand and fat clay (water = 47%), at different pressures. Full capillary saturation.

Percent by weight		Pressure (gr/cm ²)				
Clay (wet)	Sand (dry)	15	48	109	170	410
5	95	0.1	0	—	—	—
10	90	—	0.31	0.14	0.04	—
20	80	0.62	—	—	0.2	0.12
30	70	0.18	0.08	—	—	—
40	60	0.1	0	—	—	—

ice-stratified under a small load pressure and slow freezing as $K_F = 1.0$ m and $K_M = 1/2$ to $1 1/4$ m. [Note: K_F means the capillary height for loose standard packing and K_M means the capillary height for the densest packing.] The limit between the latter type and normal frost-heaving soils can be taken as $K_F = 1.5$ m and $K_M = 2.0$ m.

Since capillarity can be determined so easily and accurately it is the most suitable method of determining the heaving properties of a soil. The grain-size and frost properties of a so-called silty soil are difficult to determine in the field without great experience. The silty moraine soils can even give considerable difficulty to the trained technician. For approximate rapid determinations of the capillarity of the coarser soils of up to a few meters capillary height, a very simple apparatus can be used for field determinations (see Fig. 72).

c) *Hygroscopic limits.** The hygroscopic method is of little use in determining the upper grain size limit of frost-heaving soil due to the very small magnitude of hygroscopic moisture which makes the method inaccurate, and also due to the organic content. On the other hand this method is considerably more useful in distinguishing between the strongly frost-heaving soils and the comparatively harmless stiffer clays.

Figure 68 contains a number of hygroscopic values of frost-heaving soil.

It should be noted that the determinations were made on soils containing no organic material. Also, in order to fix a lower limit, the author endeavored to get a large number of the most clayey soils, which are over-represented in relation to the other types of soils from the location.

The diagram shows that the frost-heaving soils are within the hygroscopic limits of 0.4 to 4, the greatest number occurring in the interval 1 to 2. The latter corresponds to the silts and fine silts. But frost-heaving soils

continue with a large frequency to a hygroscopic value of about 4, where the frequency decreases. At this value (4) we find the dividing line between lean clay and medium clay (Ekstrom 1927). This then is essentially the maximum limit for soils which can form frost boils.

Naturally, this limit is not as sharp as the upper limit of the coarser soils. For increasing clay content, there is a gradual change of the soil properties that reaches over a wide region. The most important of these properties is the reduction in the permeability which decreases the amount of water sucked up and decreases the tendency to become fluid which requires a greater water excess for a given reduction in rigidity [author means that the fatter clays do not become fluid and form frost boils as readily] (see Simon Johansson 1916). Of course both of these factors act together so that for increased clay content the soils become more favorable from the standpoint of frost damage. It is important to note that the magnitude of the water excess which is released during thawing and also the time which this excess takes to be absorbed is of significance. The fatter the clay, the longer time it takes and the more force is required for the tight clay layers between the melting ice layers to take up this melting water excess and to form a fluid mass. The medium clays can then be taken as relatively but by no means completely resistant. The leaner clays with a hygroscopic value of 4 to 5 must therefore be classified as always being dangerous, although to a lesser degree. In other words, these clays are only dangerous when there is a very thin surface and very unfavorable hydrological conditions.

Therefore, the extreme limit for soils susceptible for frost boils can be put at the hygroscopic value of $W_h = 5$. However, this is by no means the limit for frost heave; this limit lies much further on the clay side. For increasing clay content the amount of frost heave is diminished, but theoretically never reaches zero. But from tests analogous to those shown in Table 3, the frost heave for a very fat clay (hygroscopic value $W_h = 16$) was imperceptible. From a technical standpoint just where the boundary between frost-heaving and non-frost-heaving can be drawn is not at all definite; however, it would be useful to define the limit as that heave which is of no

* Translator's footnote: The author does not define the term "hygroscopicity"—it is the amount of water (in percent of dry soil) adsorbed on the surface of the soil particles from an atmosphere of water vapor of known relative humidity. Since the method used to determine hygroscopicity is not given, the values in the following discussion should be considered merely as relative values.

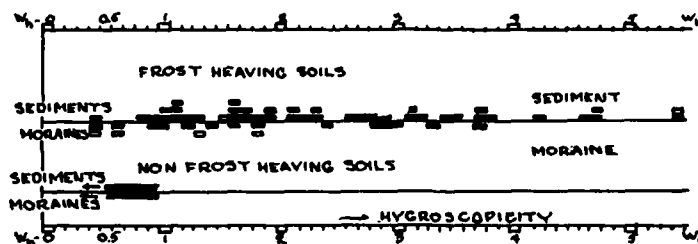


Figure 68. Hygroscopicity (W_h) of frost-heaving and non-frost-heaving humus-free mineral soils. Upper part: soils from frost boil regions (solid bars) and frost-heaving regions (unfilled rectangles).

significance on railroads. Just what numerical value can be assumed for this is difficult to say, but about 4 to 5 cm total heave would be about right. The hygroscopic value of clay corresponding to this value has not been ascertained, but certainly the entire group of medium clays (up to $W_h = 7$) must be considered as frost-heaving. This specific technical railroad problem requires further study.

d) *Summary.* To summarize, we can set down the following facts:

1. Soils with a capillarity of K_F less than 1 m (coarse silts, sand, and gravels) are under no circumstances frost-heaving. For sediments this is defined as material of which less than 30% passes the 0.062-mm sieve and less than 55% passes the 0.125-mm sieve. For moraine, it is the material of which less than 15% passes the 0.062 and less than 22% passes the 0.125 sieve, all computed in percent of the material that passes the 2-mm sieve.

2. For small loads (and high groundwater), soils with a capillarity of $K_F = 1-2\frac{1}{2}$ m and $K_M = 1\frac{1}{4}$ to 4 m may be dangerous (silt sediments: 30–50% less than 0.062 mm). Such soils may cause bank slides even if they don't have any heave in roadways. For an extremely high groundwater and slow freezing they may even be dangerous in the roadbed.

3. Soils with a capillarity of K_F greater than 2 m and K_M greater than 3 m (fine silts and finer sediments of which more than 50% is less than 0.062 mm) are under all circumstances frost-heaving. These soils usually have hygroscopic value of W_h greater than 1.

These values are for the upper limit of grain size which are critical. For the lower grain size limit the following data may be given:

1. *Sediments.* The soils which are essentially frost heaving and cause frost boils have a hygroscopic value up to $W_h = 4$, which is the division between lean clay and medium clay. However, even the leaner of the medium clays ($W_h = 4-5$) may become dangerous under very variable hydrographic conditions and under a very small load pressure. The extreme limit may then be put at $W_h = 5$ for soils which may form frost boils. But stiffer clays may still be frost-heaving, and from a practical

standpoint the entire range of medium clays may be considered frost-heaving. Therefore, the ultimate limit for any danger at all must be put at $W_h = 10$.

2. *Moraines.* The limit is here quite difficult to fix definitely. Only the silt and fine silt sediments are really dangerous to form frost boils. For a considerable clay content, and especially when there is a very even distribution of grain size causing a small pore volume, the permeability and therefore the possibility of frost-heave become very small.

The above summary is organized in Table 11.

IV. THE HYDRODYNAMIC CONSIDERATIONS OF FROST-HEAVING*

The transportation of water to the freezing layers occurs by capillary suction or flow. The factors that control the rate of flow are the following:

1. Capillary pressure (k) at the frost line, the maximum value of which is the capillarity of the soil (K).

2. The permeability of the soil (P).

3. The distance to the groundwater table (the surface where the relative capillary tension is zero) = ℓ .

For such capillary flow the following formula may be given:

$$Q = \frac{k - \ell}{\ell \cdot m}; \quad k \leq K$$

where Q is the rate of flow of water (quantity of water in unit time) and m is the specific resistance of soil, which is equal to the reciprocal of the permeability (P):

* The literature on capillarity, permeability, and flow of water in the soil is voluminous, so that even a brief summary of the important findings of different investigators would be unduly long. Due to the large amount of work done, there seems to be a divergence of opinion on much of the phenomena of water in soil, and there is also a confusion on definitions and terminology. In this treatise, therefore, where we are concerned only with that part of the subject that has to do with frost heaving, no historic discussion is given, and only the most recent and basically sound ideas are considered. For the part on capillarity, reference is made to Beskow (1930c); for the other parts, the bibliography contains a few recent articles on this subject.

Table 11.

Soil group		Average diameter (mm)	Amount passing sieve		Capillarity K_F (m)	Hygro- scopicity W_h
			(0.062 mm)	(0.125 mm)		
0	Non-frost-heaving under any circumstances	Sediment Moraine	0.1 —	<30% <15%	<55% <22%	<1 —
1a	Causing frost-heave only at surface and for very high groundwater	Sediment Moraine	0.1–0.07 —	— 30–50%	— —	— 1–1 ³ / ₄
1b	Same, except affects whole road base for very high groundwater	Sediment Moraine	0.08–0.05 —	— 15–25%	— 22–36%	— —
						1 ¹ / ₄ –2 ¹ / ₂
2.	Normally frost-heaving and liable to frost boils for groundwater depths 1 ¹ / ₂ m (moraine 1 m)	Sediment Moraine	<0.05 —	>50% —	— 2–20	— 1–4
				>25%	>36%	—
3.	Frost-heaving clays but not liable to boils	(Sediment)	—	—	—	20–?
						5–(10?)
4.	Non-frost-heaving stiff clays	(Sediment)	—	—	—	? (>10?)

$$m = \frac{1}{p}.$$

If the amount of water is so large that the entire capillary pressure is realized, the formula is:

$$Q = \frac{K - \ell}{\ell \cdot m}.$$

The magnitude of the factors entering into these formulas are of course of importance to know. The maximum capillary pressure (K) and the specific resistance (m) are functions of the grain size composition and distribution of the soil (also density of packing) or the specific surface and the pore volume.

A. Capillarity

For rapid determinations of the capillarity of small samples the author's capillarity meter has been used (Beskow 1930c). The apparatus is shown in Figures 69–71. Figure 72 shows a simplified form, for use in field determinations to determine the limits between frost-heaving and non-frost-heaving soils.* The relation between capillarity and grain size has been empirically determined by the author as well as Atterberg (1903), Beskow (see 1930c) and Engelhardt (1929). The author's

determination has been carried out on the basis of Atterberg's original material, on pure fractions. Capillarity determinations agreed quite well with those of Atterberg (Beskow 1930c, Table I, p. 20) but were carried out to considerably larger values, up to 70 m. These determinations verify the following empirical formula for the capillarity (K) as a function of the particle diameter (d):

$$K = c_k \frac{1}{d}$$

where c_k is an exponent, composed of the capillary constant and degree of packing. For constant temperature and degree of packing, c_k is a constant; for these tests, the packing was relatively dense and the temperature was the room temperature (about 20°C). The value of the exponent c_k was determined as 0.060 when K is given in meters and d in millimeters. For purely spherical-shaped particles, the exponent (as determined by tests on shot, Beskow, 1930c, p. 22) is 0.053.

For natural soils the exponent has a different value depending on the gradation of the soil. Generally we can say that the less well-sorted the soil is, the higher its capillarity becomes in relation to its average particle diameter. For the relatively well sorted sediments, the exponent is between 0.060 and 0.085 (see Beskow 1930c, p. 26). The average particle diameter is in this case computed by using the logarithm of each fraction [i.e., logarithmic mean].

If, however, the average grain size is computed by

* This type is essentially similar to Engelhardt's capillarity meter (Engelhardt 1929, Fig. 1 and 2), although both methods were worked out simultaneously and independently. Another type of capillarity meter was constructed by engineer A. Eriksen, and published by Fleischer (1934).

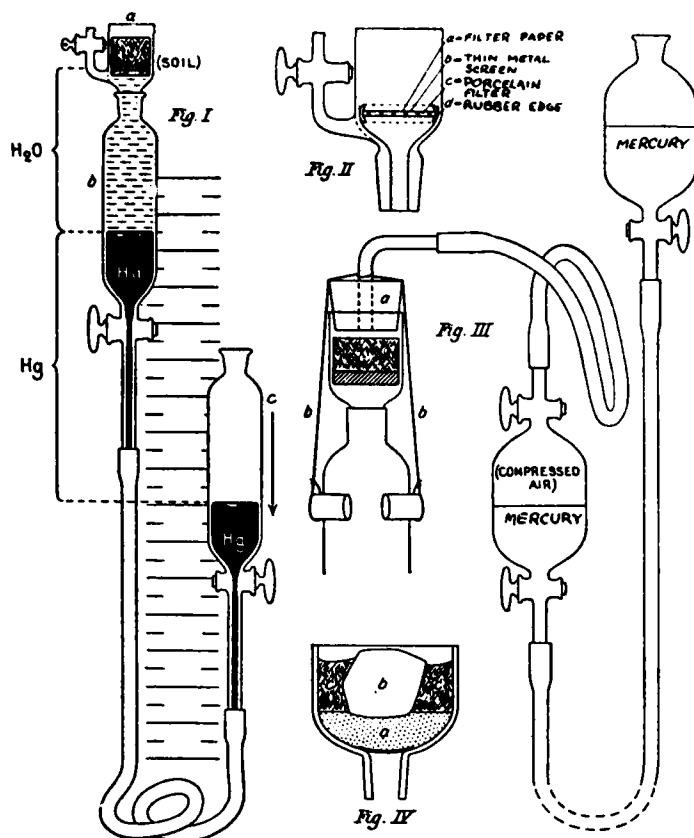


Figure 69. Diagram of author's capillarimeter. Figure I is the normal apparatus, Figure II a detail of the specimen container, where a = filter paper, b = metal screen, c = porcelain filter, d = rubber edge. In a later model, c and d were replaced by a perforated glass bottom welded to the container. Figure III shows the arrangement for the determination of the capillarity for a piece of undisturbed natural soil (a = sand bed, b = soil specimen, c = stiff clay). Figure III shows the arrangement with air pressure for determining capillarity greater than 9.5 m.

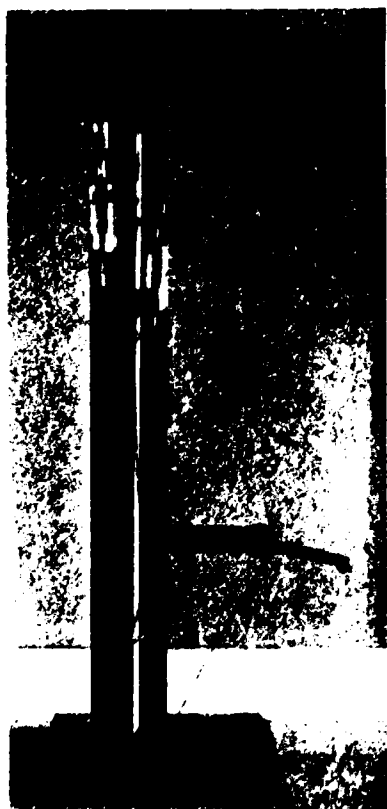


Figure 70. Author's capillarimeter, 1933 model.



Figure 71. Detail of Figure 70, showing the setup right after a test, when the air sucked in during the test is being driven out.

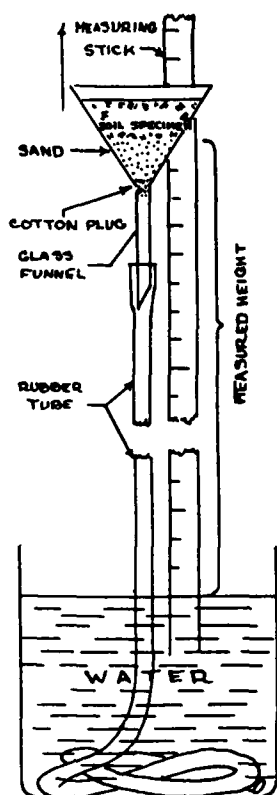


Figure 72. Simplified type of capillimeter intended for field use for the determination of the capillarity of sand and coarse silts, and thus distinguished between frost heaving and non-frost-heaving sediments (from Beskow 1933a, 1934b). Mercury is not used in this apparatus; instead the whole system is filled with water. After the soil sample is placed at a consistency at about the liquid limit (during which the tube is kept closed by a clamp), the excess water is sucked out by holding the funnel a few decimeters above the water surface. The funnel is then raised slowly, and the level at which the air breaks through is observed. By using a table, ladder, etc., capillaries up to 3 m can be determined, which is a large enough range to determine the degree of frost-heaving danger.

using the inverted value of the diameter for each fraction, the exponent c_k seems to have a constant value of about 0.06 and independent of the grading of the soil (see Beskow, 1930c, p. 27). The capillarity is also approximately inversely proportional to the specific surface. Zunker (1930, p. 98) gives the following general formula for the capillarity as a function of specific surface, pore volume and degree of packing;

$$K = 0.3 \cdot a^2 \cdot \frac{1-v}{v_{ol}} \cdot U \text{ [cm]}$$

where K = capillarity
 a^2 = capillarity constant
 v = apparent pore volume
 v_{ol} = "pressure and air-free" pore volume
 U = specific surface.

For coarse soils, or where the accuracy required is not great, the difference between v and v_{ol} can be neglected, and also if temperature variations are neglected, the formula becomes:

$$K = 4.5 \cdot \frac{1-v}{v} \cdot U \text{ [cm]}.$$

The specific surface U is obtained from the reciprocal of the effective particle diameter d in millimeters.

$$U = \frac{1}{d}.$$

For Atterberg's pure fractions the pore volume is very nearly 40%; if the value is substituted in formula (12) we get

$$K = 4.5 \cdot \frac{1-0.4}{0.4} \cdot \frac{1}{d} = 6.0 \cdot \frac{1}{d} \text{ [cm]} = 0.060 \cdot \frac{1}{d} \text{ [m]}.$$

The value of the exponent in the formula $K = c_k (1/d)$ ($c_k = 0.060$, when K is given in meters and d in millimeters), is exactly the same as obtained empirically by the author.

The largest difficulty in determining the capillarity is in the packing. The effect of the degree of packing is so large that for sediments of a normal grading, the capillarity for the densest packing is about twice that for the loosest packing (this is much larger than the error which might arise due to segregation of sizes). The divergence is larger, the less well-sorted the soil is. For the usual well-sorted sediments fairly accurate values can be obtained by having the soil at a liquid consistency, and shaking it slightly, and removing the excess water by applying a very small suction (a few millimeters of mercury).

It cannot be claimed that this method is entirely satisfactory either. To say "a liquid consistency" is not being very definite and leaves a wide margin. The variation becomes greater for less well-sorted soils.

Because of this, a definite value of consistency has been chosen, and this is the cone method of the Geotechnical Commission. This is the consistency at which a cone of 60° containing 60 gr of materials sinks 5 mm.* According to Ekstrom (1927, p. 121) this value is very nearly the same as Atterberg's liquid limit. Thus for this degree of packing we can denote the corresponding capillarity by K_F [the F stands for liquid limit]. To show the small variations in values obtained the following results of F. Rengmark are given:

Soil type		Capillarity values
Silt U296	230, 230, 260, 230, 245;	$K_F = 240 \text{ cm}$
Silt M102	(<335), 315, 315, 315, 325, 355;	$K_F = 320 \text{ cm}$
Silt GB20	325, 325, 325, 325, 315;	$K_F = 325 \text{ cm}$
Fine Silt BW19	(<670), 790, 625, 625, 600, 585;	$K_F = 625 \text{ cm}$
Clayey Silt H28	<960, 940, 950, 965;	$K_F = 950 \text{ cm}$
Lean Clay GB21	12.2, 12.5, 12.5, 12.1, 12.1;	$K_F = 12.3 \text{ m}$

Another type of packing is the "densest possible packing," by which is meant the densest packing that

* This does not include slumping that occurs after some time as in clays (plastic flow), but means only the instantaneous slump.

can be obtained without changing the homogeneity (i.e., without stratifying) or crushing.

The "densest possible packing" is principally applicable as a standard of packing, and is also of particular use. In these tests this was obtained in the following manner: the soil is put in the receptacle in a fairly liquid consistency, and either shaken or tamped with a glass rod. Then the surplus water is removed by a vacuum of a few centimeters of mercury, using greater pressure for stiffer soils, so that the rate of flow of water is small so that the lower part does not silt up due to the finer particles being washed through (see Beskow 1930c, p. 11). Then the sample is shaken and tamped again, this time a little harder, and the surplus water sucked out again. The procedure is repeated, each time tamping harder and using more suction (a vacuum up to $\frac{1}{2}$ to $\frac{2}{3}$ of the permeability) until the sample cannot be packed any further, which is the densest state to which the soil can be packed. If successive capillary determinations are made and the soil is tamped after each determination, the values of the capillarity become asymptotic and become constant when the optimum packing is reached. K_M is used as the symbol for this densest possible packing or the optimum capillarity.

The practical disadvantages of this method are, however, significant. The most important one is that even though the greatest precaution may be taken, the bottom part becomes silted up with finer material, and therefore causes too high values to be obtained. To be more certain, several determinations should be made of the same sample, from which the abnormally high values may be separated and the average of the rest used.

The degree of packing usually found in nature is *between* these two types, and although varying, usually lies nearer one than the other. For the condition we are concerned with—suction to the frost line—the natural condition certainly lies close to the looser degree of

packing (K_F). Of course, the bottom moraine materials are originally packed densely, but in the frost zone they are usually loosened by previous freezing. K_F then becomes closest to being the actual capillarity, and is the minimum value. K_M then becomes a maximum value, and is approached by natural soils which are densely packed.

K_F is the most suitable value of capillarity as a *characteristic soil property*, at least as far as natural sediments are concerned.

For unsorted soils, moraines, disintegrated soils, road bed mixtures, and others, the case is a lot more complicated. For the first it is best to determine the capillarity on material free of gravel, or the material that passes the 2-mm sieve. For such soils, in which the fine soils fill the pores of the coarser particles when packed, the values obtained are only slightly higher than those for the soil in a natural condition, due to the fact that the larger particles have hardly any effect on the capillarity (see Beskow 1930c). In other cases it is useless to try to obtain definite values of capillarity.

It is also true that for moraine soils it is practically impossible to use the cone method to determine the consistency, and that if one wants to find K_p , he has to be satisfied to try to bring the soil to an approximate consistency. The value of K_F then becomes relatively uncertain. K_M is also very difficult to get for unsorted materials because it is so easy for the bottom of the sample to silt up with the finer material. Determinations of the capillarity of moraine materials are therefore only approximate.

Values of capillarity for the accepted grain size classification and, for the soil groups classified according to average grain size, are given in Table 12.

A number of values of capillarity of principally frost-heaving soils are given in Table 13.

Table 12.

Pure fractions			Natural sediments		
Grain size classification	Grain size (mm)	Capillarity	Soil type	Capillarity	
				Loose K_F	Dense K_M
Coarse sand	0.2-0.6	3-10 cm	Coarse sand	3-12 cm	4-15 cm
Medium sand	0.6-0.2	10-30 cm	Medium sand	10-35 cm	12-50 cm
Coarse silt	0.2-0.06	30-100 cm	Coarse silt	30-200 cm	40-350 cm
Silt	0.06-0.02	1-3 cm	Silt	1.5-5 m	2.5-8 m
Fine silt	0.02-0.006	3-10 cm	Fine silt	4-10 m	6-12 m
Very fine silt	0.006-0.002	10-30 m	Lean clay	8-15 m	10-18 m
Coarse clay	0.002-0.0002	30-300 m	Medium clay	14-? m	15-? m
Fine clay	0.0002	300 m			

Table 13. Capillarity of various sediments and moraine soils.

a. Sediments						b. Moraine soils†					
Soil number and type			Frost heaving index*	Capillarity (m)		Soil type	Frost heaving index	Capillarity K _M (m)	Soil type	Frost heaving index	Capillarity K _M (m)
				K _F	K _M						
Gl	1	Fine sand	0	—	0.95	102	0	0.8	B 12,000	2	3.4
Bw	10	Mica sand	1 a	0.70	1.35	B 9,640	0	1.35	Bw 15	2	4.6
No	5	Fine sand	0	1.15	1.9	Gä 6	1a	1.9	No 4	2	4.9
Gl	2	Fine sand, micaceous	0	—	1.9	Kc 7	1a	2.85	V: 7	2	7.8
Bw	25	Coarse silt	1 b	1.6	2.15	V: 4	1b	2.9	Bw 6	2	10
VI:	7	Coarse silt	1 b	—	2.5	Bw 4	2	3.1	Bw 16	2	11
GB	5	Coarse silt	1 b	1.8	—	V: 2	2	3.3	Bw 18	2	11.5
No	8	Coarse silt	1 b	—	2.9						
Bw	1	Coarse silt	1 b	1.85	3.0						
Bw	23	Coarse silt	1 b	1.9	3.8						
Bw	2	Coarse silt	1 b	2.0	—						
U	296	Silt	2	2.4	6.1						
Bw	14	Clean silt	2	2.7	4.8						
GB	9	Clayey silt	2	3.0	7.8						
M	102	Clean silt	2	3.15	5.0						
GB	20	Silt	2	3.25	6.0						
GB	15	Fine silt	2	—	6.0						
No	7	Fine silt	2	—	7.7						
Bw	13	Unsorted clayey silt	2	4.6	9.5						
GB	18	Fine silt	2	4.7	7.2						
GB	16	Fine silt	2	—	7.8						
A	84	Fine silt	2	4.9	9.3						
Bw	26	Clayey fine silt	2	5.3	9.0						
Bw	21	Clean fine silt	2	6.1	9.7						
Bw	19	Fine silt	2	6.4	9.6						
Bw	22	Fine silt	2	6.6	10.1						
Bw	7	Clayey fine silt	2	7.1	—						
45990		Lean clay	2	9.0	c/a 15						
Bw	24	Clayey fine silt	2	9.3	—						
H	28	Lean clay	2	9.6	—						
GB	21	Lean clay	2	12.3	—						
GB	2	Medium clay	2	14.0	—						
GB	17	Medium clay	2	15	—						

* See group classification Table 11.

† Determined on material passing 2 mm sieve, densely packed in the soil container of the capillarity meter.

K_F = capillarity determined on soil specimens placed at the liquid limit, and then the water surplus removed by snow suction, at a vacuum of only 10–50 mm Hg.

K_M = capillarity determined at maximum density.

B. Permeability

The resistance to flow of water (non-turbulent) through a capillary tube is inversely proportional to the fourth power of the diameter, and directly proportional to the permeability. In a complex capillary system such as a soil, assuming the packing and grain shape to be the same, the pore space is proportional to the second power of the grain size, and can be taken as:

$$P = c_p \cdot d^2$$

where d = average particle diameter and c_p is an exponent, which takes into account the degree of packing and the changes in viscosity of the water due to temperature

changes. For a constant temperature and degree of packing the value of c_p is constant.

The validity of this formula has been proven empirically by permeability tests on soils of different coarseness. The author has used Atterberg's pure fractions for the purpose, as well as soils with very different gradation curves and grain shape.

The soils in these tests were all brought to the same degree of packing, the densest possible packing (see above). This was obtained by mixing the soil with distilled water and shaking it until it had a fluid consistency, then placing it in a capillarity meter and subjecting it to a slight vacuum. This was repeated until the soil could be compressed no more.

Table 14. Permeability of Atterberg fractions.

Grain size (mm)	Permeability P^* (at) +20°C	Capillarity K (m)	Real particle size d from capillarity $d = 0.06 \times 1/K$	$P/d^2 = c_p$	Relative permeability $P \cdot K^2$
0.2-0.1	37.5	0.41	0.147	1.73×10^3	6.3
0.1-0.05	5.3	0.99	0.066	1.21×10^3	5.2
0.05-0.02	1.74	1.9	0.0322	1.67×10^3	6.0
0.02-0.01	0.46	4.4	0.0136	2.48×10^3	8.9
0.01-0.005	0.11	8.2	0.0073	2.07×10^3	7.4
0.005-0.002	0.020	17	0.0035	1.62×10^3	5.8
0.002-0.001	0.0012	70	0.00086	1.64×10^3	5.9

* (cm³/hour) (cm/cm) (cm²) = cm/hour.

The permeability was determined by the quantity of water collected in a burette in a certain time interval under a constant hydrostatic pressure. Due to the very large effect of even small variations in packing of the soil on the accuracy of the determinations, it was not thought necessary to keep the temperature absolutely constant; the temperature T was on the average about +20.5°C, with a maximum variation of $\pm 3^\circ\text{C}$ which seldom occurred; as a rule the temperature was +20.5°C.

The results of the permeability tests are summarized in Table 14.

The permeability P is given in centimeters per hour and signifies the thickness in centimeters of a layer of water that passes through a soil of a given thickness of the soil sample. It is also interpreted as cubic centimeters per hour (square centimeters per centimeter) which means it is the quantity of water in square centimeters that passes through a soil of one square centimeter in cross-sectional area when the thickness of the sample is equal to the head acting.

As the last column in the table shows, these tests prove the validity of the formula. The separate values of the product $P \cdot K^2$ of course vary considerably, but these variations are due to the variations in the tests, the principal ones being the difference in the degree of packing. The distributions of each of the variations are completely unsystematic and show no tendency for any one value, even though the tests cover a wide range, the diameter of the particles of the coarsest fraction being 170 times the smallest.

Permeability is therefore directly proportional to the square of the particle diameter, assuming other factors are constant; for Atterberg's pure fractions, for close packing, the exponent c_p in the formula

$$P = c_p \cdot d^2$$

has the value of about $1.8 \times 10^3 (= 1800)$ when d is given

in millimeters and P in cubic centimeters of water per hour and the cross-section in square centimeters when the head is equal to thickness of the sample, and the temperature is 20°C.

Permeability is therefore, like capillarity, a single function of grain size; there is then a definite relation between permeability and capillarity, which is given by the following:

$$K = c_k \cdot \frac{1}{d}$$

$$P = c_p \cdot d^2 = c_p \cdot \frac{c_k^2}{K^2}$$

The formula can be written more conveniently as:

$$P = C_{pk} \cdot \frac{1}{K^2}$$

where

$$C_{pk} = c_p \cdot c_k$$

If we assume only pure fractions, where $c_p = 1.6 \times 10^3$ and $c_k = 0.060$, then $C_{pk} = 5.8$.

The above formula, however, only holds for the same grading characteristics, and the given value of the exponent and the viscosity constant vary, and therefore the values of the exponents also vary, as shown by Table 15.

For varying grading characteristics, the capillarity and the permeability vary, the former much less than the latter.

This can be understood by qualitative reasoning. If we take a fine grained fraction, a fine silt for example, and mix with it an equal amount of coarse material such as small stones, the capillarity of the mixture is unchanged (same as for fine material). The permeability, on the other hand, which depends on the number of pores as well as the size of the pores, must diminish due

Table 15.

Temp (°C)	Capillary constant a^2 (mm ²)	Coefficient of viscosity (η)	Relative values of c_k , c_p and C_{pk} at various temperatures (+20°C = 1)			C_{pk} for unsorted fractions (absolute value)
			c_k	c_p	C_{pk}	
±0	15,406	0.0179	1.040	0.564	0.610	3.5
+5	15,251	0.0152	1.029	0.665	0.706	4.1
+10	15,105	0.0130	1.020	0.785	0.817	4.7
+15	14,959	0.0114	1.010	0.885	0.903	5.2
+20	14,821	0.0101	1.000	1.000	1.000	5.8
+25	14,686	0.00894	0.991	1.130	1.109	6.4
+30	14,556	0.00800	0.982	1.262	1.208	7.0
+35	14,424	0.00723	0.973	1.399	1.322	7.6

Table 16. Permeability and capillarity of mixtures of Atterberg fractions.

Component fractions (mm)		Proportion A:B	Capillarity K			Permeability P			Relative permeability P K ² A+B
A	B		Of components A	Of components B	Of mixture A+B	Of components A	Of components B	Of mixture A+B	
0.01-0.005	—	—	8.2	—	8.2	0.11	—	0.11	7.4
0.01-0.005	0.02-0.01	1:1	8.2	4.4	6.35	0.11	0.46	0.14	5.6
0.01-0.005	0.05-0.02	1:1	8.2	1.9	7.3	0.11	1.74	0.10	5.3
0.01-0.005	0.2-0.1	1:1	8.2	0.41	7.3	0.11	37.5	0.082	4.4
0.01-0.005	0.2-0.1	1:2	8.2	0.41	6.5	0.11	37.5	0.74	3.1

Table 17. Permeability and capillarity of some natural soils.

Type of soil		Permeability P cm/hour (at) +20°C	Capillarity K (m)	Relative permeability P K ²
Medium sand		80.5	0.20	3.2
Fine sand		1.20	0.54	0.35
2.	Moraine silt	0.140	1.3	0.24
12.	Coarse silt	0.29-0.25	2.72	2.1-1.8
8.	Coarse silt	0.113-0.073	2.9	0.95-0.63
BW 1.	Coarse silt	0.36	3.0	3.2
GB 20.	Silt	0.017	3.65	0.23
1.	Silt	0.090-0.061	4.6	1.9-1.3
M 102.	Silt	0.032	4.9	0.77
7.	Fine silt	0.0088	7.7	0.52
A 84.	Fine silt	0.0013	9.4	0.11
10.	Lean clay (very fine silt)	0.0013	>9.75 ^m	

to the decrease in cross-sectional area (40%) and also to the fact that the path is made longer since the water has to travel around the large included particles.

It is obvious that the decrease in permeability depends upon the grain size of the mixed material, or rather on the difference between the grain size of both fractions. If to a fine grained soil fraction a fraction very nearly the same size is added, the effect is only an increase of the pore size (also a very small decrease in the pore volume) and the permeability increases; if the difference in size of the fractions increases, a point is soon reached where the permeability begins to decrease

rapidly, and continues until it reaches an asymptotic value which is somewhat less than the original permeability.

This phenomenon has been experimentally investigated by tests on different mixtures of Atterberg's pure fractions, the results of which are given in Table 16. The permeabilities of the separate fractions are also determined.

The practical significance of this is that the relation between capillarity and permeability changes as the fractions in the mixture are changed, and that the change is quite large. Experiments have not been performed on

more complex mixtures, wherein other factors besides grain sizes are changed, but of course they have been carried out on natural mixtures, i.e., on natural soils.

In Table 17, a number of such determinations are given, from which we can see that the product of the permeability and the square of the capillarity—here called *relative permeability*—which for unsorted fractions is constant, varies greatly for natural soils. The values for natural sediment are lower than for pure fractions, and for moraine material are considerably lower, being only one hundredth as great.

The product of the permeability and capillarity is a measure of the grading. The more unsorted a soil is, the smaller is the relative permeability. This explains why *very poorly sorted soils, especially normal moraines, are so little frost-heaving*, in spite of their high capillarity.

In the above the relations between permeability and the various mechanical properties of the soil are determined only for a number of special cases; for example for similar degrees of sorting and grain shape, for similar packing, etc. A formula containing all variables has been established by Zunker (1930). The grading and packing act together on the *pore volume*, the effect of which is given by an experimental factor easily obtained. His formula for permeability (Zunker 1930, p. 157) is,

$$P = \frac{\mu}{\eta} \left(\frac{v_{ol}}{1 - v} \right) \cdot \frac{1}{U^2} \text{ [cm/sec]}$$

where v = "apparent pore volume"

v_{ol} = "actual pore volume"

U = specific surface

η = coefficient of viscosity (see Table 15)

μ = a number that characterizes the shape, surface properties and degree of mixture (not grading) of the soil grains.

If instead of U , the grain diameter d is introduced, from the relation $U = 1/d$, and the value for the pore volume for Atterberg's pure fractions, i.e., 40% is inserted, the equation becomes:

$$P = \frac{\mu}{\eta} \left(\frac{0.4}{1 - 0.4} \right) \cdot d^2.$$

The value of μ for ordinary natural soils lies between 0.015 and 0.010; if the value for η at 20°C is inserted the permeability becomes:

$$P = \frac{0.01}{0.01} \frac{0.4}{0.6} \cdot d^2$$

$$P = \frac{2}{3} \cdot d^2 \text{ [cm/sec]}.$$

Converted to cm/hour the exponent c_p in the formula for permeability $P = c_p \cdot d^2$ becomes

$$P = \frac{2}{3} \cdot 3600 = 2.4 \times 10^3$$

which agrees well with the empirical value for the determined exponent (Table 14), which lies between 1.21×10^3 and 2.48×10^3 .

By the use of the formulas for capillarity and permeability Zunker derives the following formula as a relation between the two:

$$P = \frac{\mu\lambda}{K^2} \text{ [cm/sec]}$$

(Zunker 1930, the formula 190a, p. 172). λ is a coefficient of viscosity which is constant for a constant temperature (at 20°C it is 1957); μ is the shape constant.

The significance of this formula is that permeability is for practical purposes determined by the capillarity (the value of μ varies only between 0.018 and 0.004 for soils, and is as a rule quite constant, being about 0.01); Zunker himself expresses this fact as [translated from German]: "In all tests in which the value of the capillarity is determined this value can be used for the solution of K (= permeability P)."

This is not always the case; it has been shown above that the permeability is by no means necessarily determined by the capillarity, and that the relation between the two, $P: 1/K^2$, varies considerably depending upon the grading. We can say in general that Zunker's formula for the relation of permeability and capillarity is incorrect. This is obvious since both formulas from which this relation was obtained contained the specific surface and the pore volume as factors and this one does not.

Table 18 contains the values of permeability for different grain size groups and natural sediments.

It should be pointed out that the computations of the permeability assume that the soil is in its densest state, that it is homogenous, that the pores are entirely filled with water, and that no cracks or fissures exist. In nature, however, these assumptions are seldom fulfilled, and certain disturbances occur, which complicates matters.

Air pockets often occur in the coarser soils, which has the effect of considerably decreasing the permeability. In filter sands and in pumping from wells, small air bubbles from the water can accumulate so rapidly that the permeability is rapidly decreased. This can also happen in laboratory determinations of permeability and have a serious effect.

The effect of cracks depends partly on their arrangement, and partly on just how much they are filled with

Table 18. Permeability (at 20°C) and capillarity of various coarse materials (well sorted sediments). At temperatures near 0°C permeability is reduced to 60% of the values given.

Type	Pure fractions			Natural soils				
	Grain size (mm)	Capillarity K	Permeability P (at +20°C) (cm/hr)	Soil type	Grain size (mm)	Maximum capillarity K _M	Permeability P (cm/hr)	
							Normal values	Limiting values
Coarse gravel	20-6	< 1cm	640×10 ⁻³ -58×10 ³	Coarse gravel*	20-6	—	—	—
Fine gravel	6-2	1-3cm	58×10 ³ -6.4×10 ³	Fine gravel*	6-2	1-5 cm	15,000-1,000	25,000-150
Coarse sand	2-0.6	3-10 cm	6.4×10 ³ -0.58×10 ³	Coarse sand*	2-0.6	4-15 cm	1,500-70	2,500-15
Medium sand	0.6-0.2	10-30 cm	580-64	Medium sand*	0.6-0.2	12-50 cm	125-5	250-1.5
Coarse silt	0.2-0.06	30-100 cm	64-5.8	Coarse silt*	0.2-0.06	40-350 cm	10-0.3	25-0.1
Silt	0.06-0.02	1-3m	5.8-0.64	Silt†	0.06-0.02	2.5-8 m	1-0.02	2-0.005
Fine silt	0.02-0.006	3-10 m	0.64-0.058	Fine silt†	0.02-	6-	0.1-0.002	0.2-0.0005
Very fine silt	0.006-0.002	10-30 m	58×10 ⁻³ -6.4×10 ⁻³	Lean clay†	c/a 0.006	c/a 12 m	0.005-0.0005	0.01-0.0002
Coarse clay	0.002-0.0002	30-300 m	6.4×10 ⁻³ -0.064×10 ⁻³	Medium clay**	—	c/a 18 m	—	—
Fine Clay	<0.002	>300 m	64×10 ⁻⁶ -0.64×10 ⁻⁶	Stiff clay	—	c/a 15-?m	0.001-0.00005	0.002-0.00002
				Very stiff clay	—	—	—	—
								4-7
								7-10
								<10

* Non-frost-heaving.

† Frost-heaving; Frost boils.

** Frost-heaving.

water and air, the latter depending largely upon the way the water seeps through. Water-filled fissures allow seepage, and increase the permeability, the greater the larger and more numerous they are. In clay layers the cracks in the top layers allow considerable seepage through the otherwise impermeable material. But when it concerns the capillary suction of water above the groundwater table where the fissures are not filled with water, the fissures have a tendency to hinder the process.

The same reasoning applies for laboratory determinations. Water filled fissures increase the permeability; especially in the case of tests on clays has it been noticed that the very fine fissures can increase the permeability many times the actual value. Similarly, it has been noticed that air filled cracks decrease the permeability.

Strict attention should be given these facts, both in the laboratory tests, as well as in the practical application of the values of permeability to hydrodynamic determinations.

C. The process of water suction

In this section, only capillary flow in capillary saturated soils will be dealt with, since only this type of flow is important for frost heaving. Water flow in non-saturated soils, although of great theoretical and practical interest, will therefore not be discussed here.

1. Assuming a homogeneous soil column

a) *Maximum capillary suction.* The following formula has been given for the capillary suction in a homogeneous soil column:

$$Q = \frac{k - \ell}{\ell \cdot m}$$

where Q = the quantity of water sucked up in unit time ($\text{cm}^3/\text{cm}^2 \text{ hr} = \text{cm/hr}$)

k = capillary pressure

ℓ = the distance to the groundwater surface

m = the specific resistance.

k has a maximum value which cannot be exceeded, which is called the capillarity or capillary height, K . When $k = K$ the maximum possible capillary suction is then

$$Q = \frac{K - \ell}{\ell \cdot m}$$

For homogeneous soils, the specific resistance m is equal to the reciprocal of the permeability:

$$m = 1/P$$

and the formula can be written:

$$Q = \frac{(K - \ell) \cdot P}{\ell}$$

However, it is more practical and more correct to use the first formula, with the resistance instead of the permeability, because it applies when the soil is not completely filled with water and not homogeneous, and therefore the actual specific resistance is greater than $1/P$.

It should be kept in mind that in the above formulas, as in all such formulas, the values of the permeability and resistance (P and m) must be reduced to the actual occurring temperature, due to the change in viscosity. For the coefficients for this reduction see Table 15, exponent c_p .

The reason why the distance to groundwater is such an important factor is that in the formula for frost heave it enters in the denominator and therefore when it increases the heave decreases, but the quantity Q also decreases, therefore the effect of ℓ is a double effect.

The limit, where $Q = 0$ occurs when $\ell = K$; that is the distance to the groundwater surface is equal to the capillary height; above this level, no capillary suction can occur.*

When ℓ is large compared to K , and especially when ℓ is very nearly equal to K , the effect of the depth to groundwater on the rate of flow is very great. This is the case for the coarser frost-heaving soils, having a capillarity of a few meters. Take for example a silt with a capillarity of 2 m; for a groundwater depth of 1.5 m its rate of suction is

$$Q_1 = \frac{2 - 1.5}{m \cdot (1.5)} = \frac{1}{m} \cdot \frac{1}{3}$$

For a groundwater depth of 1.75 m, Q becomes

$$Q_2 = \frac{2 - 1.75}{m \cdot (1.75)} = \frac{1}{m} \cdot \frac{1}{7}$$

An increase in the groundwater depth from 1.5 to 1.75 m, which is $1/6$ of the original value, causes a decrease to less than $1/2$ of the original rate of suction; for a further increase of 0.25 m the rate reduces to zero. Therefore the coarser the soils are, the more sensitive to groundwater changes they are.

Clays, on the other hand have such a large capillarity, that the effect of groundwater changes is small. When ℓ can be neglected compared to K , the formula becomes approximately

* However, water transportation can occur in a gas phase as well as by capillarity. This type is only of significance for the very coarse soils (gravels and sands) and for the greatly fissured clays. For the ordinary tight soils, capillary flow is the most important.

$$Q = \frac{K}{\ell \cdot m}$$

Variations in the rate of suction then becomes inversely proportional to the change in distance to groundwater; the increase from 1.5 to 1.75 in the previous example causes here a small decrease to $\frac{6}{7}$ of the original.

If we insert K and m in the formula as functions of the grain size, we get

$$Q = \frac{c_k \cdot c_p \cdot d^2}{\ell \cdot d}$$

$$Q = c_k \cdot c_p \cdot \frac{d}{\ell}$$

For a given constant value of ℓ (assuming that temperature and grading is constant) for clays, the rate of suction is approximately directly proportional to the particle diameter.

If for a certain groundwater depth, for example a meter, we allow the grain size to decrease, it is obvious that until the capillarity can exceed 1 m, no water flow can occur; then each decrease in grain size (increase in capillarity) causes a very large relative increase in $(K - \ell)$, and therefore a large increase in the rate of flow. For clays on the other hand, a decrease in grain size causes a proportional decrease in the rate of flow, it is then obvious that the curve for the rate of upward flow or suction for constant groundwater depth and varying grain size must reach a maximum for a certain particle diameter. A number of such curves, for different groundwater depths, are given in Figure 73.

From Figure 73 we can see that the grain size for maximum capillarity falls in the region of fine silt, i.e., the soils which in practice show the most frost heaving.

b) *Capillary suction in natural ground.* In experimental tests, capillary suction usually occurs in the form of a capillary rise in dry soil, that is to say that the

boundary between the water-saturated soil and the air-filled soil moves upward. In natural ground this process is insignificant; the important process here is the water suction to a relatively stationary surface, from which the water is taken away, usually through evaporation.

The rate of capillary rise in a dry soil sample, even if it is fully homogeneous and densely packed, is, as a rule, considerably less than the value computed from the capillarity and permeability. The reason for this is the following: In order that a rapidly penetrating water surface and the maximum capillary pressure shall occur, it is necessary that in front of the advancing water surface a very rapid wetting of the soil particles occurs, so that the adsorption films can cover the particle surfaces before the meniscii develop in the voids; otherwise the maximum curvature does not occur and consequently the maximum capillary pressure does not occur. When the rate of penetration is fast, however, wetting cannot take place fast enough so that (especially if the soil particles are the least bit coated with certain substances such as grease, resin, etc.) the maximum capillary pressure does not occur, and the rate of capillary rise is less than the maximum.

Experimentally, the maximum rate of upward flow can only be obtained when the rate of evaporation from the capillary surface is as large as the greatest possible amount of capillary suction; a condition of dynamic equilibrium is then reached, a condition of continuity in which the stationary meniscii in the pores are fully formed, and continuously suck up the same amount of water that is removed by evaporation. In this manner high values of capillarity can be obtained experimentally (see Beskow 1930c, p. 48-53; Fig. 9 and 10).

This type of capillary flow, having a relatively stationary capillary surface, is the most common type of capillary suction in nature. As soon as the possible evaporation from the ground surface exceeds the maximum rate of capillary flow in the ground, the capillary

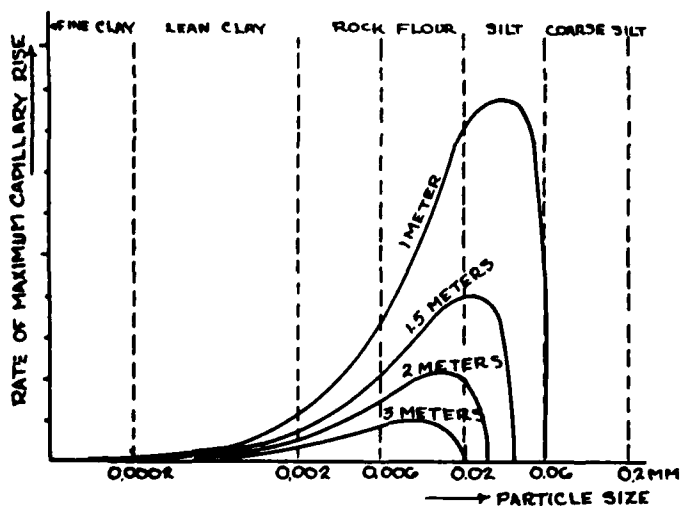


Figure 73. Curves of maximum capillary rise of sorted soils as a function of the average particle diameter, for a number of different distances to the ground water table.

surface no longer remains stationary, but moves downward; in this region a "dry zone" is formed, in which water movement can only occur in the adsorption films surrounding the particles. The thickness of this dry zone changes, i.e., the capillary surface moves downward or upward, depending upon the rate of evaporation and thus the depth to groundwater changes.

A similar condition of continuity as discussed above occurs in the upward capillary flow of water to a growing ice layer at the frost line; the only principal difference is that the water is removed in a solid state rather than a gas state.

Water flow to the frost line is then, from the above reasoning, seldom of maximum magnitude; this corresponds to the case of upward flow to a slowly evaporating surface, where then the menisci are not fully developed, and therefore the maximum capillary pressure cannot exist. A comparison of the computed maximum rate of upward water flow in a frost heaving soil and the actual amount of flow to the frost line, would show that the latter is much less, and for the coarser soils is only a small fraction of the maximum.

The difference between the maximum rate of flow and the actual flow to the frost line, or *the reduction factor required for the formula of maximum capillary suction*, in order to be valid for actual cases of soil freezing, is therefore larger the coarser the soil is. The reduction factor is, however, not constant for any one type of soil, but depends also on the *load* and the *distance to the groundwater table*. These phenomena have been dealt with in a previous section.

2. Water suction in heterogeneous soil

a) *The effect of fissures on the rate of flow.* We have previously assumed that the soil between the groundwater surface and the frost line is a fissure-free, homogeneous mass. For the coarser frost-heaving sediments containing no clay this assumption is valid. However, in the clay soils the direct capillary flow is interfered with or broken by a more or less well-developed fissure network, which is formed in the so-called dried upper crust, which as a rule extends to 2- or 3-m depth. Moreover, in some sediments varying, and other alternations of layers of different coarseness can occur and complicates the process of water suction. The significance of these factors will be discussed briefly below.

Fissures only prevent capillary suction when they cross the prevailing direction of capillary flow. This flow departs very little from the vertical, and therefore vertical cracks are of little importance. The deeper fissures are usually vertical or depart only a few degrees from the vertical and hardly diminish the flow at all, and in fact, as we shall show later actually aids the upward flow. Fissures lying closer to the horizontal occur prin-

cipally nearer the surface, lying approximately in the upper meter. This is partly due to the fact that close to the surface there is much more drying out and that the pressure is much less, which encourages the formation of cracks, but is also due to (and this is just as important) the freezing of the soil during previous years. Due to ice stratification, horizontal fissures are formed which are difficult to "close up" in clays. By careful digging during the summer or fall, long after the frost has disappeared, one can observe in clays thin discontinuities or fissures which have the same arrangement and structure as the ice stratification occurring in the same soil in a frozen condition.

The degree to which fissures lying cross-wise to the direction of capillary flow prevent flow, depends largely on the widths of the fissures. Water transportation across the fissures can be considered as occurring in two ways: in a gas phase (diffusion), and in a liquid phase, by capillary flow at points where the two surfaces touch each other. The rate of diffusion is inversely proportional to the distance (width of crack), and the number of points of contact are larger the smaller the distances between the two surfaces are.

If we imagine the fissure to widen gradually, we get to a width where there are no contact points, and the surfaces are completely separated, and over which the entire water transportation must occur in a vapor phase, by *diffusion*.

Diffusion. Capillary flow is caused by a capillary pressure difference. Flow by diffusion is caused in a similar manner by a pressure difference in the direction of flow, but the difference is in the vapor pressure. In order for diffusion of water vapor from the lower side to the upper side to occur, the relative vapor pressure must be larger on the lower side than the upper. Such a pressure difference can occur in two ways. One possibility is a *temperature difference*, in which the upper surface would be cooler. Such a condition certainly occurs during freezing from above. The average temperature gradient during the winter at depths from 0.2 to 1 m is about 0.02–0.04°C/cm. Since air has a much smaller conductivity, the temperature change in a crack is much larger; the effect of this difference is to cause water transportation in a gaseous or vapor state, cooling the evaporating surface, and warming the surface of condensation. If we take the ratio between the coefficient of conductivity of air and soil to be about 1:50 (see Tables 20 and 21), the temperature changes across a fissure would be 1–2°C/cm. For temperature close to 0°C the change in partial pressure of water vapor is about 0.35 mm Hg/°C. This temperature then would correspond to a vapor pressure difference of 0.35–0.7 mm Hg, or 0.477–0.954 gr/cm² cm.

For the rate of diffusion of one gas through another

gas the following formula applies:

$$v = -kst \cdot \frac{dp_1}{dx}$$

where k = constant (constant of diffusion),

s = area of a cross section taken at right angles to the pressure drop

t = time

dp_1/dx = the pressure change across the section

v = volume of flowing gas, reduced to the pressure at which it occurs.

If centimeters and seconds are used as units, the value of k for water vapor in air at a temperature around 0°C is $0.20 \text{ cm}^3/\text{sec}$.

For the pressure drops of 0.447 and 0.954 gr/cm^2 obtained above, v becomes in the first case $0.2 \times 0.477 = 0.0954$, and the latter $v = 0.2 \times 0.954 = 0.191 \text{ gr/cm}^2$, computed in cubic centimeters of vapor at a pressure of 1 gr/cm^2 (at 0°C), per second and square centimeters of cross-sectional area. The unit weight of vapor at that pressure is $0.775 \times 10^{-6} \text{ gr/cm}^3$. The amount of water per day per square centimeter in the first case is then $0.0954 \times 0.775 \times 10^{-6} \times 86,400 = 0.00639 \text{ gr}$, and in the latter $= 0.0128 \text{ gr}$, corresponding to a water layer of 6 to 13 hundredths of a millimeter in thickness.

If we compute how much cooling of the lower surface that such evaporation involves, we find however, that it is $2/3$ of the total heat conducted upwards by flow (the latent heat of vapor just above $0^\circ = 595\text{--}590 \text{ cal}$, the heat coefficient of soil $= 0.003$, the temperature gradient -0.02 to 0.04°C/cm , the rate of evaporation $= 0.74 \times 10^{-6}$ to $0.148 \times 10^{-6} \text{ gr/cm}^2 \text{ sec}$). Consequently, the above values of 0.06 to 0.13 mm/day are actually reduced to about a half; these computations only show the order of magnitude. It is not necessary to give the relatively complex exact computations here.

We can say in other words: the diffusion due to temperature difference is so small, that for water flow in frost-heaving soils (being only a few millimeters per day), it is of no importance.

The other possibility for a vapor pressure difference is the difference in capillary tension between the upper and lower surface. The effect of capillary tension on the vapor pressure is to change the vapor pressure as the curvature of the meniscus is changed. For a *convex* meniscus the vapor pressure above is *greater*. Above a *concave* meniscus (i.e., a capillary meniscus) the vapor pressure is *less*.

The higher the negative capillary pressure is, the larger is the curvature of the meniscus, and the lower is the relative vapor pressure. The adsorption films around the particles in the voids are of course convex, but here the vapor pressure is reduced by adsorption from the

particle walls (hygroscopic action). Probably the thicknesses of the adsorption films so accommodate themselves to an equilibrium condition, that their vapor pressure is the same as that of the meniscii (or else there would be a continuous circulation of vapor between the meniscii and the particle surfaces).

If we now go to the case we are considering, a cross-wise fissure without any points of contact of the upper and lower surfaces, we find, therefore, that in order to have a diffusion across the fissure, a capillary pressure difference between the upper and lower surfaces is necessary. But we also find that the pressure difference must be quite large in order to have any considerable diffusion, although the effect of capillary tension on the vapor pressure is very small.

The following computation gives an idea of the quantitative values involved.

If we assume the meniscus in a capillary to be a hemispherical shell, we have the following relation between the capillary pressure k and the radius of the meniscus r :

$$k = a^2 \cdot \frac{1}{r}$$

where a^2 is the capillary constant, which for water between zero and room temperature is about 15. Then k becomes

$$k = 15 \cdot \frac{1}{r}$$

where k and r are in millimeters.

The change in relative vapor pressure corresponding to a certain curvature is given by:

$$p_w = p \pm \frac{2\alpha \cdot s_a}{s_v \cdot r}$$

where p_w = vapor pressure over the curved surface

p = vapor pressure over a plane surface

α = surface tension

s_a = vapor density

s_v = water density

r = radius of curvature.

The sign for the last term is plus for convex, minus for concave surfaces. In the latter case, for meniscii, we can then write:

$$p - p_w = \frac{2\alpha \cdot s_a}{s_v \cdot r}$$

For water the values are given for different temperatures in Table 19 (top).

By inserting these values in the formula we get $p - p_w$ as a function of r . In Table 19 (bottom), $p - p_w$ is com-

Table 19.

	0°C	5°C	10°C	15°C	20°C	25°C
p (gr/cm ²)	6.24	8.90	12.5	17.4	23.8	32.4
s _a	4.84×10 ⁻⁶	6.81×10 ⁻⁶	9.41×10 ⁻⁶	12.8×10 ⁻⁶	17.3×10 ⁻⁶	23.1×10 ⁻⁶
s _v	0.999	1.000	0.9997	—	0.9991	0.9971
α (dynes/cm)	75.49	74.75	74.01	73.26	72.53	71.78
α (gr/cm)	0.0770	0.0762	0.0755	0.0747	0.0740	0.0732

Capillary pressure k (m)	Meniscus radius r (cm)	0°C	5°C	10°C	15°C	20°C	25°C
Difference of vapor pressure p-p _w in gr/cm ²							
≈ 0.0015 l/r	(cm)	0.745×10 ⁻⁶ l/r	0.928×10 ⁻⁶ l/r	1.42×10 ⁻⁶ l/r	1.91×10 ⁻⁶ l/r	2.65×10 ⁻⁶ l/r	3.38×10 ⁻⁶ l/r
0.1	0.015	0.0000496	0.0000618	0.0000946	0.000127	0.000174	0.000229
1.0	0.0015	0.000496	0.000618	0.000946	0.00127	0.00174	0.00229
5.0	0.0003	0.00248	0.00309	0.00473	0.00636	0.00853	0.0114
10.0	0.00015	0.00496	0.00618	0.00946	0.0127	0.0174	0.0229

puted for a number of different values of r and their corresponding capillarity.

From the table we see that the vapor pressure differences are very small. For low temperatures, close to zero, which we are interested in, we must have a pressure difference of 100 m per mm of fissure width in order to have a vapor pressure difference of the same magnitude as for the illustrated example where a diffusion of a layer of water 0.06 mm per day occurred! For 0°C the rate of diffusion v is

$$v = \frac{0.00067k}{d} \text{ mm of water/day}$$

where k = the capillary pressure difference in meters and d = the width of the fissure in millimeters. No appreciable amount of water can therefore be transported in such a manner, unless the capillarity can exceed a few hundred meters. For frost-heaving soils, this type of diffusion is insignificant.

We can therefore conclude that fissures having no contact between the upper and lower surfaces are practically perfect insulators for water flow. The significance of fissures is then clear: water movement across a fissure can only occur over points of direct contact.

Capillary conduction at the contact surfaces. The exact quantitative computation in this case is entirely dependent on the number of contact points, and their arrangement, which naturally varies a great deal. In general we can say that for a soil column which is not completely detached by an air discontinuity, and in which there are certain areas in which there is contact and over which capillary flow can occur, the amount of water that can flow in unit time through a contact area is the same as that which would flow through the entire cross section Y .

The rate of flow is therefore larger at the contact surfaces than at other points in the soil column by the ratio $Y:a$ and the pressure drop is also greater. If we assume the soil to be completely homogeneous, and therefore the permeability to be the same at all points, the rate of flow across a surface a , is

$$Q_a = \frac{Y \cdot Q_y}{a} \text{ ([cm}^3\text{/cm}^2 \text{ sec] or [cm/sec])}$$

where Q_y = the rate of flow across the entire cross section, also, the capillary pressure gradient across a , is

$$\frac{dk_y}{dl} = \frac{Y}{a} \cdot \frac{dk_y}{dl}$$

where dk_y/dl is the pressure gradient for the entire cross section (at some distance away from the fissure).

The flow of water occurs in such a manner that when it approaches a fissure, the flow is concentrated at the contact surfaces, that is to say the stream lines are concentrated through the contact surfaces, and after passing spread out to the same shape they had before. This is shown diagrammatically in Figure 74. Along with the convergency of the stream lines, there is a corresponding concentration of the potential lines, denoting a steeper pressure gradient.

It is of course obvious that there is a very large potential drop at the contact surfaces. But the potential drop is proportional to the resistance; for practical reasons it is more convenient to consider the effect of a crack as an increase in the resistance to flow.

The following is a quantitative computation of the effect of a fissure:

If we assume the contact areas to be divided at a number of places with an equal spacing and length, and equally divided over the cross-section, the entire system

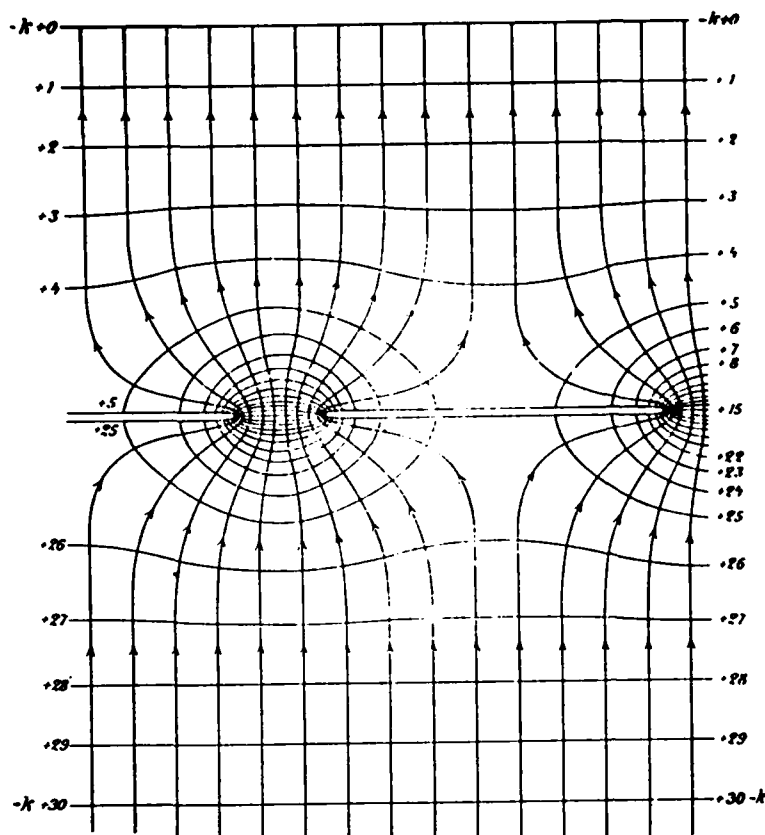


Figure 74. Schematic diagram showing the interference to upward flow of water due to lateral fissures. The diagram shows the equipotential lines (horizontal lines of equal water pressure), and the stream lines which are perpendicular to the equipotential lines. The water pressure for each equipotential line is given by the number to the right (relative values) which starts as a negative value ($-k$) near the surface and increase ($+1, +2$, etc.).

can be divided up into separate systems, each having one contact plane. In such a single system the converging flow net has the shape of a truncated pyramid (or cone), the bottom of which is equal to the cross section of the entire column, Y , and the top of which is equal to the area of the contact surface y . The height depends, besides these areas, on the steepness of the pyramid, that is, the top angle. If we express a in terms of the square root of the base (\sqrt{Y}) and the height of the untruncated pyramid x , we get

$$a = \frac{\sqrt{Y}}{x}.$$

We then get the following formula for the resistance of the truncated pyramid, M_1 , when the specific resistance of the soil is $1/P = m$ [hr/cm]:

$$M_1 = \frac{m}{a} \cdot \left(\frac{1}{\sqrt{y}} - \frac{1}{\sqrt{Y}} \right) \text{ [hr/cm}^2\text{]}.$$

This formula applies of course for the single system. The resistance per unit area (M_A) is inversely proportional to the number of single systems per unit area, and M_A is therefore

$$M_A = \frac{A}{n} \cdot \frac{m}{a} \cdot \left(\frac{1}{\sqrt{y}} - \frac{1}{\sqrt{Y}} \right)$$

where A = the area of the single contact (cm^2) and n is the number of contacts (i.e., number of systems). But $A/n = Y$; therefore

$$M_A = Y \cdot \frac{A}{n} \cdot \left(\frac{1}{\sqrt{y}} - \frac{1}{\sqrt{Y}} \right) \text{ [hr]} (= \text{hr cm}^2 \text{ cm/cm}^3).$$

The increase in resistance for each fissure is $M_A - M_{AO}$, where M_{AO} is the resistance per unit area for normal flow through a layer of the same thickness as double the height of the pyramid (= a single layer on each side of the fissure), that is, the resistance that would be in the disturbed zone if the fissure did not exist. M_{AO} is

$$M_{AO} = Y \cdot \frac{1}{Y} \cdot \frac{2m}{a} \cdot (\sqrt{Y} - \sqrt{y}) = \frac{2m}{a} \cdot (\sqrt{Y} - \sqrt{y}).$$

The increase in resistance $M_A - M_{AO}$ is therefore

$$M_A - M_{AO} = \frac{2m}{a} \cdot \left(\frac{Y}{\sqrt{y}} - 2\sqrt{Y} + \sqrt{y} \right).$$

Then factoring out \sqrt{Y} :

$$M_A - M_{AO} = \sqrt{Y} \cdot \frac{2m}{a} \cdot \left(\sqrt{\frac{Y}{y}} - 2\sqrt{\frac{y}{Y}} \right).$$

From this latter formula, we see that for constant conditions ($Y/y = \text{constant}$) the increase in resistance is directly proportional to the square root of the base surface Y (or inversely proportional to the square root of the number of contact surfaces per unit of area), that is to say, is directly proportional to the average distance between the centers of the contact planes. For a constant percentage of contact surfaces, the resistance depends on at how many points these surfaces are divided, and the resistance is larger the fewer, and smaller the more numerous these points are. For example, if four contact planes be put together to form one (four times as large) the resistance is therefore increased four times.

On the other hand, if n/A and therefore Y is constant, while the distance between the surface varies, we get, putting $r = \sqrt{Y/y}$

$$M_A - M_{AO} = \left(r - 2 + \frac{1}{r}\right) \cdot C: \text{ where } C = \sqrt{Y} \cdot \frac{2m}{a}.$$

For large values of $r = \sqrt{Y/y}$ and therefore for small relative contact surfaces, the increase in resistance is approximately proportional to Y/y .

The numerical value of the form constant a lies between 1 and 2 and probably closer to 2. The following computation illustrates the numerical magnitude of the increase in resistance.

Assume $Y/y = 100$, and that one contact surface occurs on each square decimeter, therefore $Y = 100 \text{ cm}^2$. If $a = 2$ we get

$$\begin{aligned} M_A - M_{AO} &= 10 \cdot \frac{2m}{2} (10 - 2 + 0.1) = \\ &= 10 \times 8.1 m = 8 \times 1 m \text{ hr.} \end{aligned}$$

It is probably difficult to see how the hour time unit enters: however giving the units as hours $\times \text{cm/cm}$ (or hour $\times \text{cm}^2 \times \text{cm/cm}^3$), we have the time in hours required for a water layer 1 cm thick to pass for a pressure difference of 1 cm. The specific resistance m is simply the resistance under these conditions of a layer 1 cm thick and therefore the value of $M_A - M_{AO} = 81 \times m$ simply means that the resistance caused by the fissure corresponds to the resistance to ordinary flow of a layer 81 cm thick.

Then in order to compute the capillary suction in a soil column containing fissures, all we need to do is to substitute the equivalent height of soil column which would have the same resistance as the column with the fissures. Thus for the formula for capillary suction

$$Q = P \cdot \frac{k - \ell}{\ell} = \frac{k - \ell}{\ell \cdot m}$$

(and other similar formulas) all that is necessary is to

multiply m by ℓ and we get

$$Q = \frac{k - \ell}{(\ell + \delta)} \cdot m$$

where δ is the sum of the increase in resistance for each fissure or

$$\delta = \delta_1 + \delta_2 + \delta_3 + \dots + \delta_n$$

where each separate value is determined by the formula

$$\delta_1 = \sqrt{Y} \cdot \frac{2}{a} \cdot \left(\sqrt{\frac{Y}{y}} - 2 + \sqrt{\frac{y}{Y}} \right)$$

or if $a = 2$

$$\delta_1 = \sqrt{Y} - \left(\sqrt{\frac{Y}{y}} - 2 + \sqrt{\frac{y}{Y}} \right).$$

We can tell from the various probable values of Y and Y/y that the effect of the fissures in increasing the resistance is very large. The above example showed that for 1 cm^2 of contact surface on each square decimeter, the effect of this single fissure of the area is to increase the resistance 80 times; and five such fissures gives a resistance of 4 m , etc. Thus it is obvious that *numerous fissures can lower the capacity of soil to suck up water to a small fraction of the normal flow.*

b) Effect of stratification and other heterogeneous properties of soil on water flow and frost heave. The computation of the capillary flow in a soil consisting of different layers of varying grain size is relatively simple, even though it may become quantitatively cumbersome in reality.

The fundamental basis is that from the groundwater table to the frost line, the negative capillary pressure can nowhere exceed the capillarity of a layer in which it exists. It is obvious that the rate of flow is the same at all levels, and therefore the gradient of the capillary pressure difference* in each layer is inversely proportional to the permeability of the layer. If we then start from the bottom and sum up the resistances of the different layers, we have then at any level the resistance of the total column below that level. If we know the capillarity of any layer, we can one by one compute the rate of upward suction. But the layer which from this gives the least rate of flow is the one which limits the entire flow; this capillarity cannot be exceeded, since air would be sucked up and the capillarity would be broken.

* This is the active capillary pressure gradient, i.e., the capillary pressure difference in centimeters of water minus the height above the groundwater table in centimeters. If the absolute pressure difference $dk/d\ell$ is n centimeters of water per centimeter of height (ℓ), we have $n - 1 = c \times 1/P$, where P = the permeability and c = constant.

Let us set up an exact analysis: Let us assume a series of layers numbered from below a, b, c, d, etc. Their respective capillaritys are K_a, K_b, K_c, \dots , and their specific resistance m_a, m_b, m_c, \dots . If the sediments have a similar grading we have

$$\frac{m_a}{K_a^2} = \frac{m_b}{K_b^2} = \frac{m_c}{K_c^2} \dots = C = \text{constant}$$

since for similar grading characteristics, the resistance is proportional to the square of the capillarity ($m = c \times K^2$). If we call the thickness of the layers a, b, c ..., the absolute resistance of each layer M_a, M_b, M_c , is then:

$$M_a = \ell_a \cdot m_a; M_b = \ell_b \cdot m_b \dots \text{etc.}$$

If we compute successively using the upper surface of each layer as the surface to which the flow is going, we have the following formulas for the maximum upward flow to the surface of layer a, b, c... etc.:

$$Q_a = \frac{K_a - \ell_a}{m_a \cdot \ell_a}$$

$$Q_b = \frac{K_b - (\ell_a + \ell_b)}{m_a \cdot \ell_a + m_b \cdot \ell_b}$$

$$Q_c = \frac{K_c - (\ell_a + \ell_b + \ell_c)}{m_a \cdot \ell_a + m_b \cdot \ell_b + m_c \cdot \ell_c} \text{ etc.}$$

If in this manner we add layer on layer, we find, as stated above, that in reality the largest rate of flow Q can never be larger than any of the computed values of Q . So for example, the maximum flow to the layer e, which we can call Q_{Ae} must be

$$Q_{Ae} \leq Q_a, Q_b, Q_c, Q_d, Q_e$$

The layer having the smallest rate of flow determines the possible rate of flow for all the overlying layers. The cause of this is, as has been said, that at any layer the capillary pressure cannot exceed the capillarity of the soil. This can be more clearly seen by graphical presentation.

Figure 75 shows schematically a sediment series a-f. The capillarity of each layer is laid out to scale on the left side. On this diagram, the capillary pressure curves are constructed (1-4) for flow to various heights in the sediment. These are in fact the gradient, i.e., the pressure change per unit length, $dk/d\ell = g$ which is inversely proportional to the specific resistance for each corre-

sponding layer, which in turn is proportional to the square of the capillarity.*

Thus we see here how the capillary pressure curve cannot exceed the capillarity of any of the layers below; for flow to layer c, layer b is the deciding factor, for flow to layer e or f, layer d is the deciding one.

The effect on the rate of flow can also be shown graphically, by having the gradient angle for each layer as a relative measure of the rate of flow. Thus in the given case the relation between Q_1 and Q_2 , etc., is:

$$\frac{Q_1}{Q_2} = \frac{t_b v_1 - t_g \delta}{t_g v_2 - t_g \delta}; \frac{Q_1}{Q_a} = \frac{t_g v_1 - t_g \delta}{t_g v_3 - t_g \delta} \dots \text{etc.}$$

where $t_g \delta = \text{unity}$.

The similar case of varved sediments is schematically illustrated in Figure 76. Here the soil is assumed to consist of layers of equal thickness and equal properties (varves) each varve consisting of a layer of fine material, a, with a capillarity K_a (here taken = 100) and a coarser layer b, with a capillarity K_b (here taken = 30).

We find here that the more varved the material is, the closer the active capillary pressure k_a is to K_b . When the varving is large, we can for practical cases take $k_a = K_b$, i.e., the largest possible capillary pressure is equal to the capillarity of the coarsest soil in the varved material. This also applies if the varved material consists of many different components.

Naturally, the rate of flow in this case is less than the maximum rate of flow that would occur if the soil consisted entirely of material a, where the pressure can equal the capillarity K_a (the angle for the dashed line from K_a is larger than for line 5 in layer a). Now in the case of flow to the frost line, if we assume that an ice layer is growing on the upper surface of one of the fine-grained layers a (in Fig. 76 at the surface of layer a_5),

* As an example of how this is computed we will assume flow to the surface of layer d. Then the average gradient $G_d = K_d/L_d$. The total resistance, M_d is then

$$M_d = \ell_a \cdot m_a + \ell_b \cdot m_b + \ell_c \cdot m_c + \ell_d \cdot m_d$$

where

$$\ell_a + \ell_b + \ell_c + \ell_d = L_d.$$

The average specific resistance is then M_d/L_d . If we call the gradient $dk/d\ell$ in layer a, g_a , in layer b, g_b , etc., we get

$$\frac{g_a - 1}{G_d - 1} = \frac{m_a}{M_d/L_d} = \frac{m_a \cdot L_d}{M_d}; g_a = \frac{m_a \cdot L_d \cdot (G_d - 1)}{M_d} + 1;$$

$$\frac{g_b - 1}{G_d - 1} = \frac{m_b \cdot L_d}{M_d}; g_b = \frac{m_b \cdot L_d \cdot (G_d - 1)}{M_d} + 1 \text{ etc}$$

Thus for each layer, the gradient can be computed, and the capillary pressure curve constructed, as shown in Figure 75.

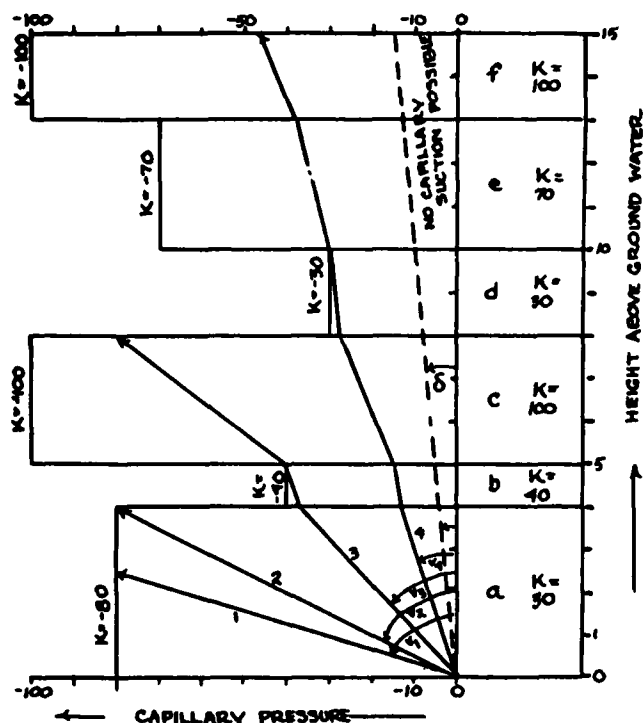


Figure 75. Schematic diagram, showing the effect of different layers on the capillary rise. Right side: the soil column, consisting of layers a to f, having capillarities shown. The capillarities are also shown graphically at the left as the maximum possible capillary pressure in the soil pores (negative pressure). The diagonal lines with arrow heads 1, 2, 3, 4 represent the pore water pressures at different levels, giving the maximum possible rate of capillary rise for water flowing to the various levels shown by the corresponding arrow heads. The slope of these lines within each layer ($v_1 + v_2$ for the bottom layer) is a measure of the rate of capillary flow if the slope angle δ is subtracted from it (the dashed line at a slope δ is the pressure condition at which there is no flow).

then the possible active capillary pressure at the frost line is considerably less than the capillarity of the soil. If the capillary pressure that would exist at the frost line if the soil consisted entirely of the fine-grained material (a) is less than the capillarity of the coarser soil, K_b , the effect of the coarser soil is to increase the rate of flow. This is shown graphically in Figure 76 where from point k_{a5} a dashed-line is drawn to the origin, this line being the capillary pressure line if the soil consisted entirely of material a. As shown, the angle for this line is considerably less than for the solid line 5, i.e., the rate of flow is less.

The following gives the exact computation: the total resistance, M_{ab} of the soil column is

$$M_{ab} = n \cdot L \cdot m_a + (1 - n)L \cdot m_b$$

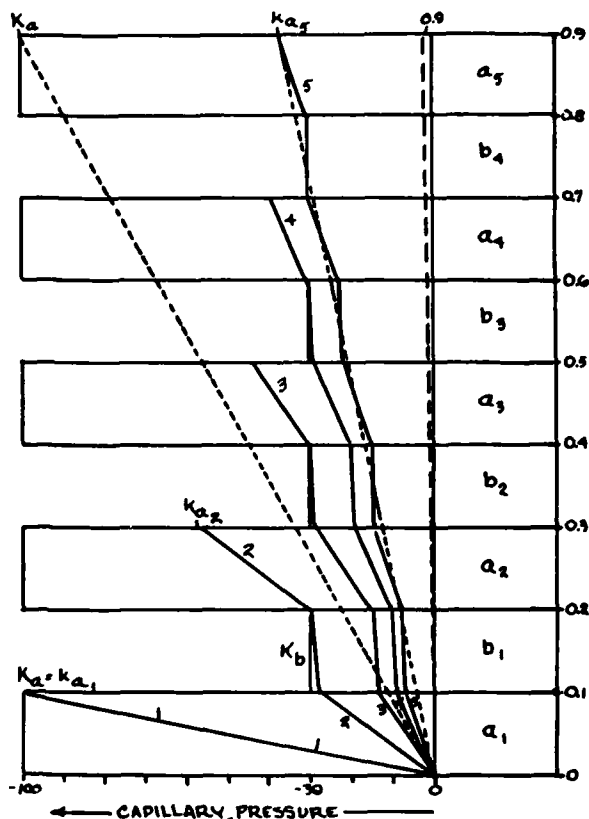


Figure 76. Schematic diagram similar to Figure 75, showing the capillary conditions in a regular stratified (varved) sediment, made up of alternating coarse and fine grained layers. Lines 1, 2, 3 etc., represent the pore water pressures at different heights, if the maximum capillary flow occurs to the surface of layers a_1, a_2 , etc. The capillarity thus occurring at the surface are k_{a1}, k_{a2} , etc.

where n is a number less than 1, and is the ratio of the material a to the total length of the column L , and $m_a = c \cdot K_a^2$, $m_b = c \cdot K_b^2$. The driving capillary pressure is K_b ; then the rate of flow Q is

$$Q_{ab} = \frac{K_b - 1}{Lc[n \cdot m_a + (1 - n)m_b]}.$$

The rate of upward flow (for the same capillary pressure) for a soil column consisting entirely of material a is

$$Q_a = \frac{K_b - 1}{L \cdot c \cdot K_a^2}.$$

We then get

$$\frac{Q_{ab}}{Q_a} = \frac{(K_b - 1) \cdot L \cdot c \cdot K_a^2}{Lc [n \cdot K_a^2 + (1 - n) \cdot K_b^2] \cdot (K_b - 1)} =$$

$$= \frac{K_a^2}{n \cdot K_a^2 + (1 - n) \cdot K_b^2}.$$

From this we find that since K_b is less than K_a , Q_{ab}/Q_a is greater than 1, i.e., Q_{ab} is always larger than Q_a .

We find, furthermore, that the difference becomes greater the smaller n becomes, i.e. the thinner the layer a becomes. When n approaches 1, the value of Q_{ab}/Q_a approaches 1, the value of Q_{ab}/Q_a approaches $K_a^2/K_a = 1$, i.e., the difference vanishes. But when n approaches zero, the value of Q_{ab}/Q_a approaches K_a^2/K_b , which is the greatest possible difference.

Let us take a numerical example to illustrate. Assume $K_a = 10$ and $K_b = 2$. Then we get from the formula

$$\text{for } n = 1 \quad Q_{ab}/Q_a = 100/(1 \times 100 + 0.4) = 100/100 = 1$$

$$\text{for } n = 1/2 \quad Q_{ab}/Q_a = 100/(1/2 \times 100 + 1/2 \times 4) = 100/(50 + 2) = 1.922$$

$$\text{for } n = 1/4 \quad Q_{ab}/Q_a = 100/(1/4 \times 100 + 3/4 \times 4) = 100/(25 + 3) = 3.57$$

$$\text{for } n = 1/10 \quad Q_{ab}/Q_a = 100/(1/10 \times 100 + 9/10 \times 4) = 100/(10 + 3.6) = 7.35$$

$$\text{for } n = 1/100 \quad Q_{ab}/Q_a = 100/(1/100 \times 100 + 99/100 \times 4) = 100/(1 + 3.96) = 20.2$$

$$\text{for } n = 0 \quad Q_{ab}/Q_a = 100/(0.100 + 1 \times 4) = 100/4 = 25.$$

If the capillarity is expressed in meters, this corresponds to a varved lean clay, with the fine varve of clayey fine silt ($K_a = 10$ m) and the coarse varve of silt ($K_b = 2$ m). Now if the depth to the groundwater is not too great (considerably less than K_b), 1 m for example, it is quite likely that the capillary pressure at the frost line, if the entire layer consists of material a , would be about 2 m, and perhaps less. *In such a case the effect of the varving is to cause a very large increase in the rate of upward flow.*

The actual increase, however, is not proportional to the increase computed by assuming the capillary pressure at the frost line to be unchanged, but is considerably less. In order to have a condition of continuity, the increase in capillary suction must be accompanied by an equal increase in the rate of flow through the adsorption films against the ice contact, but this can only be possible if the pressure is diminished.

Thus looking at it quantitatively: Assuming first a homogeneous soil column and a capillary pressure k_a , the rate of flow Q_a is

$$Q_a = \frac{k_a - \ell}{M_a}$$

where the total resistance is $M_a = \ell \cdot m_a = \ell(1/C_{pk}) \cdot K_a^2$. The compressive pressure p_a at the frost line is $p_a = k_a + a$ where a is the load pressure. But we also have for Q

$$Q_a = \frac{1}{\left(\frac{p}{b} + c\right)^2}.$$

If a is assumed to be a clay, in which c is for all practical purposes 0, and ℓ/k_a is so small, that ℓ can be neglected, then both formulas become:

$$Q_a = \frac{k_a}{M_a}$$

$$Q_a = b^2 \cdot \frac{1}{p_a^2}, \text{ where } p_a = k_a$$

In the actual case, where the soil column consists of materials a and b , the total resistance M_{ab} and the capillary pressure at the frost line k_{ab} , Q_{ab} becomes for the same approximations as above:

$$Q_{ab} = \frac{k_{ab}}{M_{ab}}$$

$$Q_{ab} = b^2 \cdot \frac{1}{p_{ab}^2}, \text{ where } p_{ab} = k_{ab}.$$

From which we get

$$\frac{Q_a}{Q_{ab}} = \frac{p_a \cdot M_{ab}}{p_{ab} \cdot M_a} = \frac{b^2 \cdot p_{ab}^2}{b^2 \cdot p_a^2} = \left(\frac{p_{ab}}{p_a}\right)^2$$

$$\left(\frac{p_{ab}}{p_a}\right)^3 = \frac{M_{ab}}{M_a}; \quad \frac{p_b}{p_a} = \sqrt[3]{\frac{M_{ab}}{M_a}}$$

since

$$\frac{Q_{ab}}{Q_a} = \left(\frac{p_a}{p_{ab}}\right)^2$$

we get then the required relation:

$$\frac{Q_{ab}}{Q_a} = \sqrt[3]{\left(\frac{M_a}{M_{ab}}\right)^2}.$$

From formulas given above, $M_{ab}/M_a = u$ can easily be computed.

Assuming three different values of u , 0.5, 0.1 and 0.05, the value of Q_{ab}/Q_a becomes = 1.59, 4.64, and 7.38

or in other words the effect of the varves is to cause an increase in the rate of heave of 59, 364, and 638% respectively.

This shows, therefore, that the increase in frost heave due to the varving is very large, and varved soils can be many times more frost heaving than the corresponding homogeneous soil. This is especially true for the finer grained sediments, for the increase is greater the thinner the clay varves are. It should also be noted that for coarse varves, or only one single varve of coarse material, the soil is very sensitive to a sinking of the groundwater level, and deep drainage has therefore a very great effect.

Lastly there is one more case, which is certainly of theoretical interest, and which not too seldom can be realized practically, is the case where the grain size of a sediment changes in a vertical direction, either becoming coarser with depth or vice versa. The first is more important, since here the rate of upward flow can be very large. The negative capillary pressure grows from zero at the groundwater surface to k at the surface at which the water is being delivered. If now the soil at the groundwater table is very coarse, and gradually becomes just fine enough at each elevation going upward to retain the water column, we get for the same capillary pressure at the surface, a soil with the least possible resistance, and therefore the rate of flow is the largest possible.

This can be explained quantitatively as follows:

If we call the capillary pressure x , and the height above the groundwater surface y , the capillary pressure at the top surface K , and this height above the groundwater L , the curve of capillary pressure vs height will pass through the origin and the point $x=K, y=L$. In order to fulfill the conditions of the problem, this curve of capillary pressure must also be the curve of capillarity vs height. Both the capillary pressure and the capillarity are then x . But the resistance is proportional to the square of the capillarity (x^2) and the pressure gradient dx/dy for the same rate of flow at all levels is proportional to the resistance. The curve satisfying these conditions is

$$-x = \frac{1}{c} \cdot \tan(c \cdot y)$$

where c is a constant, obtained from the value of the capillarity at the top surface and the distance to groundwater, i.e., by putting $x = K$ and $y = L$. We then get

$$\tan(c \cdot L) = -c \cdot K.$$

This can be solved either graphically, or by the use of tables by finding the angle (in radians) whose tangent is $-cK$. If for example $K = 10L$, L can be taken as unity and one gets

$$-\tan c = 10c.$$

Since the trigonometric tables give functions in angular measure and not in radian measure we can change this to

$$-\tan\left(c \cdot \frac{180}{\pi}\right)^0 = 10 \cdot c.$$

Then one can find the angle in degrees whose tangent is ten times the angle (in radians); this is the value of c .

For above K/L values one gets $cL = 1.504$, or $c = 1.504$, where $L = 1$. A few other values are given in the following table:

$K = 2L$	$cL = 1.166$	$= 66^\circ 48'$
$K = 5L$	$cL = 1.432$	$= 82^\circ 3'$
$K = 10L$	$cL = 1.504$	$= 86^\circ 12'$
$K = 100L$	$cL = 1.564$	$= 89^\circ 21'$
$K = \infty L$	$cL = 1.5708 (= \pi/2)$	$= 90^\circ$

For the large values of K/L , c can be obtained by the following approximate formula:

$$c = \frac{\pi}{2 \cdot L} - \frac{2}{K \cdot \pi}; \quad cL = \frac{\pi}{2} - \frac{2}{(K/L) \cdot \pi}.$$

If the value of π is inserted we get:

$$cL = 1.5708 - 0.31831 \cdot \frac{L}{K} \text{ (approx.)}.$$

This can be used when the value of K/L is about 100 or larger. Under this value the error becomes too great.

The limiting value which c approaches when K/L approaches infinity, is $\pi/2$, corresponding to 90° .

For a given soil, the rate of flow is proportional to the pressure drop per unit length, minus the head due to hydrostatic pressure drop which is 1:1. The pressure drop is in this case

$$\frac{dx}{dy} = \frac{1}{\cos^2(cy)}$$

if the values for the top surface are inserted, we get the pressure difference at that surface and the rate of flow Q becomes then:

$$Q = c_1 \left(\frac{1}{\cos^2(cL)} - 1 \right).$$

But the rate of flow Q_a that would occur if the soil column had the same soil throughout as it has at the top surface, is

$$Q_a = c_1 \cdot (K/L + 1).$$

The ratio between Q and Q_a is then

$$\frac{Q}{Q_a} = \frac{\frac{1}{\cos^2(cL)} - 1}{\frac{K}{L} - 1}$$

The following are the values of Q/Q_a for the four K/L values obtained above:

$$K/L = 2 \quad Q/Q_a = 5.64/1 = 5.64$$

$$K/L = 5 \quad Q/Q_a = 1.4/4 = 12.8$$

$$K/L = 10 \quad Q/Q_a = 227.7/9 = 25.3$$

$$K/L = 100 \quad Q/Q_a = 7770/99 = 78.5$$

Thus the increase in rate of flow is very large. The rate of flow can, for such a properly graded soil composition, be increased several tens of times that of a uniformly distributed soil composition. It should be noted that this only applies to the maximum rate of flow; the relative increase for the flow occurring at the frost line is considerably less. And further note that this ideal system is so constituted, that even the slightest drop in the groundwater surface can completely cut off further flow, since the capillarity of the coarse bottom-most layer is exceeded.

But on the other hand, if the groundwater level rises, there is a very small increase in the rate of flow if K/L is not very small. If $K/L = 10$, and the groundwater raised reducing L to $1/2$ (i.e., cutting the groundwater distance in half) the resistance is reduced by the loss of resistance of the lower half, but which is only a small fraction of the resistance of the upper half, so that the maximum driving net pressure $K - \ell$ is increased by a small fraction (from 0.9 to 0.95K). Now if we start with this high groundwater and lower the groundwater surface, the rate of flow decreases very little until we have doubled the distance ($=L$) where the flow suddenly decreases to zero. Such an occurrence in nature can seem quite perplexing.

Another case of regular gradation of particle size in the vertical direction, is one in which the capillarity decreases downward proportionately with the depth—the resistance then can be visualized as a cone with its apex downward. If the groundwater is at the apex of the cone, the ratio $Q/Q_a = 3$ (the reciprocal of the ratio between the volume of a cone and a cylinder having the same base and height); if the level is higher, the cone is truncated, and for Q/Q_a we have the relation

$$\frac{Q}{Q_a} = \frac{(K-L) \cdot M_a}{(K-L) \cdot M_b} = \frac{3 \cdot K_a^2}{K_a^2 + K_b^2 + \sqrt{K_a^2 \cdot K_b^2}}$$

The limiting value is 3, which occurs when $K_b = 0$. If K_b is small in comparison to K_a , the increase is very nearly three times.

Practically, such extreme cases as mentioned above never occur. But gradation with depth is not unusual, and can more or less approach these ideal cases. Thus these examples give a limiting quantitative value for practical cases.

c) *Summary of the effect of heterogeneous soil distribution on the water suction phenomenon and frost-heaving.* To summarize, the effect of fissures, varves, or changes in the coarseness of the layers on the flow to the frost line and the consequent frost heave is the following:

1) If the soil between the frost line and the groundwater surface contains transverse (approximately horizontal) fissures, these fissures usually cause a considerable decrease in the rate of water suction and frost heave. The effect depends on the distribution and magnitude of the points or planes of contact; to a lesser degree the area of the crack is important, and also the closer the contact surfaces are spaced, the larger is the effect of the fissures in reducing frost heave.

Vertical fissures, on the other hand, do not hinder the flow, and often have a decidedly opposite effect.

2) If a coarse layer occurs high up (right under the frost line) in frost heaving ground, it can very greatly diminish and may even prevent frost heave. This layer is completely effective in preventing heave if its capillarity is less than the distance from the groundwater surface to this layer. The effectiveness of "sand insulation" depends on this fact, that is to say it "breaks" the capillarity to the road bed. And even if the capillarity of the coarse layer is larger than the groundwater depth, it of course won't prevent heave entirely, but will diminish it appreciably. Thus a thin layer of coarse silt lying above a strongly frost heaving clay is very effective. It should be noted, however, that the effect of diminishing the frost heave is greater, the thinner this silt layer is!

3) Varved soils, i.e., soils with regular alternations of coarser and finer layers, most often cause a much larger heave than if the entire soil mass consisted entirely of the finer material. The requirement for such a condition is that the capillarity of the different soils be large in comparison to the groundwater depth. The reason why the heave is increased, is that the driving capillary pressure at the frost line does not exceed the coarsest material (silt) and this material therefore does not diminish the driving force. But also, on the other hand, the coarsest layer has a larger permeability, and therefore reduces the total resistance of the soil column. This reduction in resistance becomes larger, the larger part of the total thickness that the coarser material occupies, i.e., the smaller the relative thickness of the fine clay layers. This agrees with practical experience with varved clays, consisting of thin clay varves in silt or fine silt, in which there is extremely large frost heaving. In such material

the ice layer grows on the upper surface of the clay varves.

4) If the soil distribution is such that the grain size changes considerably with depth, this has a large effect on the water sucked up. If the grain size decreases upwards, so that the soil closest to the groundwater is coarse, and gradually becomes finer, the possible amount of flow to the frost line becomes greater.

On the other hand if the grain size varies in the opposite direction, so that the finest soil is underneath, the amount of water sucked up and the frost heave becomes much less.

For these cases, very characteristic and irregular changes occur due to changes in groundwater depth. In the former case (fine grained soil on top), a lowering of the groundwater causes hardly any decrease in heave, until a certain level is reached, where suddenly the heave decreases to practically zero. In the later case (soil finest at the bottom), lowering the groundwater only slightly has a very large effect.

D. Groundwater

The most useful, and important, but least theoretically understood part of the fundamental hydro-geological science is, without doubt, the phenomenon of groundwater. The phenomenon of groundwater occurrence is extremely important for all types of vegetation and therefore the usefulness of the ground in agriculture and forestry. Lately, groundwater has been found to play a very important role in the process of soil freezing, and its consequent effect on highways and railroads. Without making the pretense of going into the subject thoroughly, a brief analysis of the principles of the science of groundwater will be given, with special regard to the effect on the latter named phenomenon.

The term groundwater has formerly been understood to mean the water existing in the ground that can be obtained (by digging, drilling, or from natural springs) in a free state. According to the science of groundwater, this definition does not include all the water in the ground, but only the "free" water; therefore not the hygroscopic or capillary water (the fact that no sharp dividing line between the latter and "free" water exists, has already been mentioned). There is of course no point in trying to change the generally accepted meaning of this term, except to define more exactly and describe more completely the physical significance of it.

Tied together with the science of groundwater is the groundwater table, which is the level or surface to which the water reaches, or with respect to the ground surface, the level where the water begins.

1. Different types of soil water

The different names for the water existing in a soil

refer to different degrees of freedom of the water, i.e., in what manner and with what force the water is tied to the soil substance.

In general, the strength with which the water is bound is larger the closer the water molecules are to the mineral particle surfaces. The strongest bound water is "chemically bound," which enters into the mineral particles themselves, as a part of the solid substance; generally this water can only be separated by heating to high temperatures. Next there is the water that is bound to the surface of the mineral particles, due to the forces of the molecular charges (adsorption forces); this water is called the "adsorbed water" or the "hygroscopic water." The latter name is used in relation to air, and refers to the action of the adsorption forces in lowering the vapor pressure of the adsorbed water, so that in a soil only part of the water can evaporate to the surrounding air in the pores, or if the soil is dry it can absorb a certain amount of moisture from the air.

If we now allow the water content in an air dried soil (i.e., a soil having air filled pores and thin films of hygroscopic moisture around the particles) to increase, first the hygroscopic water films begin to increase in thickness until the force of adsorption on the outermost layer or film of water is extremely small. Then an entirely different force begins to act, the "surface tension" between water and air. As we already know, the general mechanical effect of surface tension on a water surface is a force which tends to level out the surface or make the curvature of the surface as small as possible. Now the soil contains irregularly shaped pores, with the moisture contained in corners and other points of contact. The surface tension thus has the effect of trying to surround the air in the pores, tending to form spheres, which, however, is not possible. This condition, in which the adsorption force is very small and the surface tension tends to form equilibrium with the entrapped air is called "funicular water" (in German "Porenwinkelwasser," see Zunker 1930). This funicular water is important for sandy soils, for it is the state in which it exists between a dry condition and a fully saturated one. For such a coarse soil there is practically no hygroscopic moisture, so that with the addition of water it enters right into the funicular state.

What happens now when the pores become completely filled with water, i.e., the soil becomes capillary saturated? We can give this state the usual name of "capillary water," although the term is far from being ideal. In this state, the adsorption forces act all over and hold the water molecules in the form of a meniscus, the force decreasing with distance from the particle walls. This effect of the force of adsorption is important when the soil freezes, for it lowers the freezing temperature of the water (Fig. 23); also in the fine grained soil it has a large

mechanical effect, acting as tight rubber membranes squeezing the soil together and affecting its swelling and shrinkage properties. There is really no sharp distinction between adsorbed water and capillary water. By capillary water we mean the water in the pores, the "freest" or least bound water away from the particle walls; as the water approaches the particle walls it becomes tighter bound or adsorbed and is called adsorbed water. Thus the essential meaning of capillary water is the "freest" water in the soil.

2. The groundwater table

The capillary water in the pores of a soil can exist under various hydrostatic pressures. The pressure can be positive or negative, or larger or smaller than the atmospheric pressure. If the pressure in the pores is greater than atmospheric, the water must flow out of the pore space (in a bore hole or ditch for example) and "run out as free water." **Groundwater is therefore the water in a soil whose hydrostatic pressure is greater than atmospheric.** The groundwater surface is therefore a pressure surface, but just what the value of this surface

is and where it lies is another matter. This is a complicated matter and varies greatly depending upon the soil properties.

One of the important properties having an effect is the grain size, or more correctly the pore size and shape which depend on the grain size, and upon which the capillarity depends, being inversely proportional to the pore size (see Sect. IV). If a dry column of sandy soil is placed with its bottom end in water, the water will rise to a definite height above the water surface, the height being the capillary rise. If the soil is extremely coarse, the capillarity will be zero, and the system is not able to lift the water above the free water level; thus the capillary level and the groundwater level coincide; but as soon as the soil has any capillarity at all, the capillary level rises above the groundwater level.

Let us assume a number of tubes filled with sand inserted into a container of water, and that the water has risen in the columns to its full height and that the system is in a state of hydrostatic equilibrium (Fig. 77a). The free water in the container of course represents the groundwater level. In the sand column at this level, the

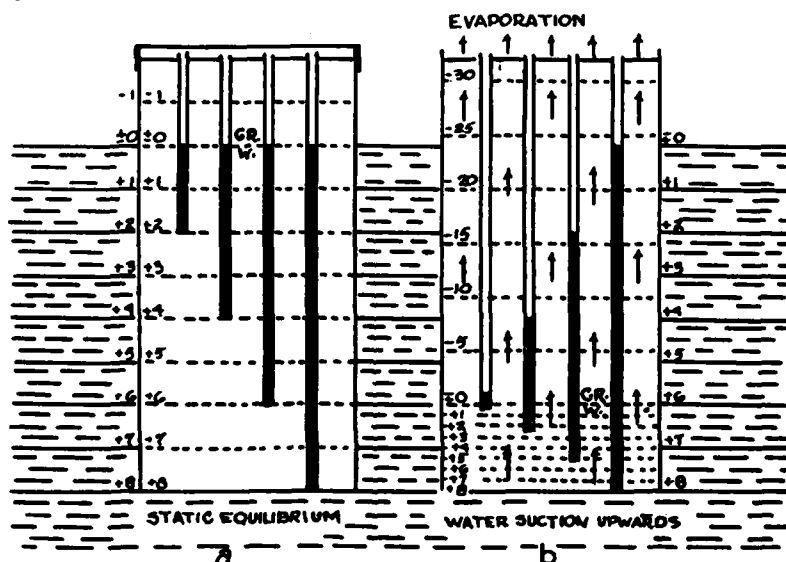


Figure 77. The difference between groundwater conditions for static equilibrium and for capillary flow in soil. The diagram shows two similar large soil columns, *a* being of any type soil, and *b* being a non-fissured fine silt, packed in a cylinder and lowered into the water. In *a*, evaporation is prevented by means of a cover, and static equilibrium is therefore reached in the system. In *b*, constant evaporation, and therefore a constant upward flow of water occurs (dynamic equilibrium). Each horizontal numbered line represents a given water pressure. Imagine standpipes inserted at different depths. In *a*, then, the groundwater will rise to the same height as the surrounding free groundwater. In *b*, however, the evaporation and the upward flow will cause a steeper pressure gradient. The groundwater level can in this case be quite low but cannot be practically determined by standpipes. The deeper the hole is under the groundwater table, the higher will be the final level of the water in the pipe. (Note that the rise takes long.) The diagram shows the final equilibrium conditions. The groundwater table is in such soils a pressure surface without any practical significance. The critical factor is the distance to very permeable layers.

pressure in the pores is the same as atmospheric; above this level there is a linearly increasing negative pressure, which is largest just under the meniscii at the capillary boundary, and which is equal to the total capillary pressure. Thus the capillary boundary is the boundary between the large and the small amount of water; the groundwater surface, on the other hand, is not such a boundary but a pressure boundary. In a homogeneous soil then, the groundwater table is only a pressure surface; but when the first discontinuity occurs such as a pit or groove, below this level, it will fill up with water. A clay containing cracks or fissures will have these cracks or fissures filled with water below the groundwater table, whereas the cracks above this level will not be filled. Thus for a clay the groundwater table is in addition to a pressure boundary, also a boundary where the water does not rise any further in a crack or a boundary between different water contents. If we bore a hole through a sand having a dry surface we first reach a zone (after a gradually increasing zone of moisture, funicular water) where the sand is capillary saturated, and then at some depth under this (in sand 1–5 cm) the level where free water flows, or the groundwater table.

The zone between the groundwater surface and the capillary boundary can be called the capillary water zone. It is larger the finer grained the soil is, and, assuming that static equilibrium has been reached, it is the same length as the capillary rise (4–15 cm for coarse sand, 12–50 cm for medium sand). But the finer the soil is, the more rarely is it in static equilibrium, and this is the reason for the very complicated phenomena of the groundwater in the finer grained soils.

Let us look at the case where a sand column is immersed in water, *before* the condition of equilibrium is reached, i.e., during capillary flow. As the meniscus surface moves upward the meniscii have their full curvature, and thus the pressure in the water just under the meniscii is the full capillary pressure.* At the bottom surface of the sand column the free water pressure exists (proportional to head or depth of immersion of the column). The pressure difference between these two surfaces is distributed linearly, as a constant pressure drop upward. The zero pressure surface, or the groundwater surface in this case, lies in such a position that the distance to both surfaces is proportional to the capillary pressure (negative) and the pressure (positive) at the bottom of the sand column. For example, if a fine sand having a capillarity of 1 m is immersed to a depth of 10 cm and the capillary surface stands just even with the

exterior free water surface, the “groundwater surface” in the sand is only 1 cm from the lower surface, i.e., 9 cm underneath the exterior water surface.

Likewise, when water flows downward—such as if for example a water filled column of soil is lifted high enough above the water surface to exceed the capillary height—the groundwater surface is also displaced but in the opposite direction; the system of isobars in the flow net becomes, in this case, expanded, while in the former case it was compressed.

This occurrence is very important because most of the year there is a change in the groundwater level going on and water movement occurring.

Let us assume a homogeneous fine silt layer, lying on top of a very permeable gravel. Assume the thickness of the layer to be 3 m, and the water pressure at the top surface of the gravel surface to be plus 2 m. This means that, if a hole is drilled down to the gravel layer, the water will rise 2 m above the layer, i.e., 1 m under the ground surface; thus we are inclined to say that the groundwater depth is 1 m. This is correct, if we assume that hydrostatic equilibrium has been reached. This is approximately true during the period after the fall rains have stopped and before the ground surface has begun to freeze. The groundwater surface would be 1 m under the ground surface. If we drill a hole that reaches just slightly below this surface, the hole ought to fill up with water to this level. And it will fill up, provided we wait long enough, or if the conditions remain constant long enough for it to rise. The flow of water can only occur as a “water excess” at the bottom of the hole, and due to the small permeability of the material and the very small pressure difference causing this flow, it can take a very long, almost infinite time to occur. But let us not consider the difficulties of empirically determining the groundwater level and go further into the problem.

Now what happens when freezing, and the consequent flow of water to the frost line, occurs? There is a negative pressure at the frost line, a capillary suction, which causes the water to flow upwards. A condition of *hydrodynamic equilibrium* is then reached, and the flow of water past any cross section in unit time is the same at all levels, i.e., the pressure difference between the frost line and the lower surface is equally divided in the fine silt.

The position of the groundwater surface (surface of zero pressure) can then be determined from the following formula:

$$\frac{\ell}{a - \ell} = \frac{k}{h} ; \quad \ell = \frac{ka}{h + K}$$

where ℓ = the depth of the groundwater below the frost line, at which the negative capillary pressure is $= k$, and a is the total distance from the frost line to the bottom of

* We must assume that either the particle surfaces are covered with adsorption films (i.e., the sand is damp) or the wetting occurs very rapidly. For a rapid rise in a dry soil the capillary pressure is less than the full pressure, and the rate of rise becomes less than that computed on this assumption.

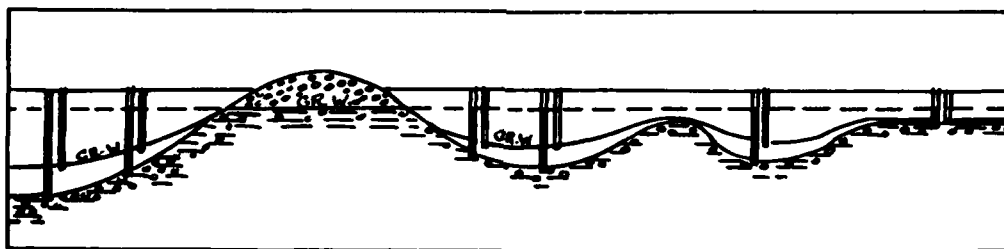


Figure 78. Schematic diagram showing the analogy to Figure 77b in nature. The cover material is fine silt resting on saturated fine gravel. If a hole is bored down to the gravel, the water will rise in the hole to the dashed line, but the groundwater surface in the fine silt lies further down. Practically, the important factor which largely determines the rate of capillary rise is the depth to the top of the gravel containing the water supply. The water pressure there [at the gravel surface], measured by the height to which the water rises in the hole, is of lesser importance. Note that it is assumed that the soil has no fissures and has low permeability (very fine silt). In fissured clay the conditions are quite different, in that the dashed line represents the groundwater surface, under which the fissures (and of course all other cracks or holes) are filled with water.

the gravel layer, where the water pressure (positive) is $= h$.

If we go to the numerical example that has been mentioned, and for simplicity assume that the frost line is at the ground surface and that the capillary stress $k = 5$ m, we get

$$\frac{\ell}{3 - \ell} = \frac{5}{2}; \quad \ell = \frac{15}{7} = 2.14 \text{ m}$$

i.e., the groundwater depth has increased from 1 to 2.14 m, and this occurs at a rate which is practically instantaneous, when considering a fine silt without any clay, having very little shrinkage, and a large permeability compared with clay.

In general the flow through a homogeneous soil can be found by the formula:

$$Q = P \cdot \frac{\delta_1 - \delta_2 - \ell}{a}$$

where a = distance

ℓ = difference in height between two pressure-surfaces δ_1 and δ_2

P = permeability.

The given formula for capillary suction is only a special case of this general formula where the pressure surface is the atmospheric or zero pressure surface; the pressure difference then is equal to the capillary pressure k . But in the case treated above it is, however, more practical to figure the rate of upward flow from the difference between the capillary pressure at the frost line (k) and the pressure at the bottom of the fine silt layer (h), and the distance between them (a). The result is exactly the same as if the groundwater surface is used; but in this case the groundwater surface is practically impossible to determine, and can only be computed from k , h and a , so that computing using the groundwater surface is a

needlessly long way of doing it. Thus, similarly, the formula for the capillary suction becomes

$$Q = P \cdot \frac{-(-k) + h - a}{a} = P \cdot \frac{k + h - a}{a}$$

(for practical cases $h \geq$ the capillarity of the coarser bottom layer; i.e., in this case, where the bottom layer is gravel (zero capillarity $h \geq 0$).

From the formula we can say that essentially, the rate of upward suction depends on a . If k is constant (which is the case when there is complete evaporation, where $k = K$) and if $h - a$ is constant, which means that the water level obtained by boring a hole to the gravel layer is at a constant distance from the surface independent of the thickness of the layer of fine silt, then the maximum rate of suction is inversely proportional to a .*

Now let us consider the case where the surface of the gravel layer is no longer plane but varies as shown in Figure 78. Even if hydrostatic equilibrium exists in the gravel, so that the water will rise to the same level in bore holes made anywhere, and thus the groundwater has essentially a constant depth, the zero pressure surface (= theoretical groundwater surface) is quite undulated, being lower where the finer material is thickest and vice versa, as shown in the figure. Now on such a terrain, if we want to find, by a boring, the depth to groundwater, it is not possible to obtain the existing zero pressure surface in the fine silt, since the flow of water is so extremely small. When the bore hole first reaches the gravel layer, the water starts to run, and rises rapidly to a certain level and stops. We are now led to think that this level is the groundwater level, and since the water is just as deep from the ground surface all over, we would conclude that the danger from frost heave is the

* For soil freezing this proportionality is true for clays, but not exactly true for coarser soils, such as fine silts, which we are here considering. However, the variation is in the same direction and the proportionality is not too badly off.

same all over. This is of course all wrong. If the results of such borings are interpreted correctly, they are of extreme value, especially since they determine the thickness of the fine silt layer at different points, or the depth to the water conducting bottom layer.

How does this result agree with the foregoing valid premise that the groundwater depth is of extreme importance in the phenomenon of water suction? There is of course no contradiction at all, and instead the validity of the statement still remains: the rate of upward water suction is dependent upon the depth to groundwater, but this depth is in certain cases essentially a function of the thickness of the frost heaving soil layer, or of the depth to the coarse permeable layer.

To summarize we can say that as soon as water movement begins, the static equilibrium is disturbed, and the groundwater table is lowered. The deeper a hole is bored, the higher the water level (the apparent groundwater surface) rises. However, some soils are so permeable (capillarity low) that the upward flow is very small and the groundwater is practically the same as for static conditions.

3. Classification of various soil types considering groundwater

Soils for this purpose can be divided into three principal types, and the soils found in nature can be said to approach any one of these types. These are:

1. *Gravels and sands.* Very permeable. The groundwater surface can easily be found by measuring the depth to the water surface in a bore hole.

2. *Fine silts containing no clay.* Very small permeability. Very small pores, and an absence of fissures. The groundwater level cannot be found practically, and has no significance for frost heaving properties.

3. *Clay.* Quite permeable due to fissures. The groundwater surface can easily be determined from borings, and the groundwater depth is of extreme importance for frost heaving properties.

In the first group, there are only the very coarsest frost-heaving soils, such as the silts. But nevertheless soils coarser than silts are important too, because in stratified soils, layers of coarse material may occur together with fine layers.

In the case of type 3, the groundwater surface can usually very easily be determined empirically, and it is a very critical quantity. From a hydrodynamic standpoint, the groundwater surface occurs in a fissured clay much the same as the gravels—there is free water movement independent of capillarity and only dependent on the slope, since the pore space (cracks and fissure in clay) is so large that there is no capillarity. In both cases the groundwater surface is also a boundary of water content, and is not merely a theoretical surface.

The quantitative differences are, however, quite large: while the percentage of voids in gravel is about 30–35 and up to 40% by volume, the percentage of voids due to fissures and cracks in clays is only a few and at most 5%. But the greatest difference is due to the fact that while the gravel particles consist of massive stone, the large secondary “particles” in clay, or the clay lumps separated by the cracks, consist of a fine porous soil mass, which not only holds a large amount of “solid” or immobile water, but whose water content can change considerably, and can by the help of the immense capillary forces that are possible in clay, move very rapidly into the clay mass itself.

In a fissured clay, the water rises quite rapidly in a newly bored hole; in fact in a fairly stiff, greatly fissured clay it occurs almost instantaneously.* The amount of water sucked up is a function of this very definite measurable groundwater depth, and the *rate of frost heave is for practical purposes inversely proportional to this depth.*

The fissure system in a clay exists, however, only in the upper part, in the so-called dry crust. In general the number of fissures decreases rapidly with depth, and disappears in this locality usually at a depth of around 3 m. In this comparatively sparse fissured zone, there is consequently very little mobility of the groundwater; the more the groundwater is lowered, the less mobile the groundwater becomes, and where the fissure disappears, there is no mobility at all. Naturally a surface of zero pressure always exists, but where the groundwater has been lowered below the fissured zone, this surface has an entirely different practical physical significance: a theoretical pressure surface without much hydrological meaning.

The same phenomenon occurs when the type of soil varies sideways in a horizontal direction and there is a continuous change from a clay to a fine silt. In such a case, determining the groundwater profile is extremely difficult.

In case 2—fine silt without any fissures—as has been pointed out, the groundwater cannot be obtained from a boring, and the groundwater surface has no meaning. This does not mean that a boring in such a material determines the depth to groundwater, *but determines the depth to the permeable water bearing layer.*

If this water bearing layer is very permeable and the water pressure is considerable, the water breaks through suddenly and rises immediately to its final height in the bore hole. This height of rise—a direct measure of the excess pressure at the bottom of the layer of fine material—does not have any meaning. The significance of

* The smaller the volume of the fissures are, the longer it takes them to fill up. In transition materials such as lean clays, it usually takes a few hours for the water to rise to its full height in a bore hole.

this rise as far as frost heaving is concerned, is greater the coarser the soil is. For example, in a silt having a relatively low capillarity, the water pressure at the bottom of this layer is very important. For a very fine silt, on the other hand, with a high capillarity, it is quite unimportant.

Let it be pointed out still once more that the water pressure at the top of a gravel layer just below a finer material is very important, in that it determines whether there is access of water to the upper layer or not. If the water level is balanced just around this contact surface between the two layers, a very small change in water pressure can cause the flow and the frost heave to suddenly decrease to almost nothing.

What is important to observe then, is at what depth the coarse material occurs, and to what height the water rises in it. It is of course necessary that the coarser layer be water bearing, in order to measure the height in the bore hole. In most cases, the coarse layers are always water bearing after heavy rains, especially after the fall rain. With a little experience, it is a simple matter to determine if a layer will be water bearing or not. In the summer, when the conditions are not the same as when freezing occurs, soil profiles can be used to predict whether the coarse soils layers will be water bearing under normal fall and winter conditions, if it takes considerable rain to become water bearing, etc.

Along the coastal regions of Norrland [northern Sweden], you generally find in the valleys a fairly thin covering of the fine silt-lean clay resting on a gravelly moraine material or on a thin gravel layer covering bed rock. If the sediment cover is a fissureless or almost fissureless fine silt, the depth to the gravelly moraine material or to bed rock is extremely important, especially since the most severe frost heaving occurs when this cover is very thin or when gravel or bed rock lies only a short distance below the ground surface.

To summarize then, we can say the following: for every investigation of the frost heaving properties of any ground, borings are necessary in order to determine the distribution of the soil types and the groundwater conditions. Concerning groundwater, it is expedient to distinguish between two principal cases.

I. The case where during boring of test holes, free water rises in the holes without any essential change in the soil composition. The kinds of soils in which this occurs are sand, coarse silts, certain clays (those which are fissured), and all such varved sediments in which the coarse varves are considerably permeable (coarse silt or coarser).

On such material, one can measure the level of the groundwater in the bore hole; this level is the *ground-*

water surface, and is essential in order to know frost heaving properties.

It should be kept in mind that it is the groundwater depth *during the time of freezing* that is important; if the determination of the depth to groundwater is made at some other time, groundwater variations must be considered, for changes can even occur during the beginning of freezing.

II. The case where the sediment cover consists of so-called loam, i.e., fine silt with practically no clay. Free groundwater comes out of a bore hole when it first breaks through to another coarser layer (gravel, sand, gravelly moraine, material, or rock bottom) and rises to a considerable height. Here the most important fact is where the water broke through, and next the height to which it rose; but *this level does not represent the groundwater surface*.

The actual groundwater surface is in this case impossible to determine practically, and hasn't any significance for frost heaving.

Silt lies essentially somewhere in between both of these cases. If we are dealing with a silt, and the water breaks through a sandy or gravelly layer and rises to a considerable height in the bore hole, we have to bore a new hole a few meters away from the first one, which does not go down to the permeable layer, or stops between the bottom layer and the level of the water in the first hole. It is good practice to first drill only to $\frac{1}{3}$ of the distance between the water surface and the bottom layer, then wait about a $\frac{1}{2}$ hour to see if any water comes in it, then drill to $\frac{1}{2}$, and then to $\frac{3}{4}$ of the depth. If the water begins to trickle fast enough for at least some free water to enter the hole in say a $\frac{1}{2}$ hour, then it is possible to determine the groundwater surface. If the water gradually rises in the hole to nearly the same height as in the other hole (if the difference in level after a few days is not more than a few decimeters), we can consider with tolerable approximation the level in the first hole to be the level of the groundwater surface. To determine the groundwater level more accurately, more exact bore holes are needed, which do not go very far below the water surface, since the error becomes greater the deeper the hole is.

Since such a long time is required for observation—often several days—final level can be computed from a curve of the rate of rise (by extending it until it becomes asymptotic). A useful method is to take two holes of equal depth about 1 m apart, and in one fill it with water to a level above the groundwater surface; the common level which is being approached in both holes in opposite directions is in this way much easier to determine.

V. TEMPERATURES IN FREEZING GROUND

A lot of investigation has been done on the temperatures at various depths in various soils during different times of the year and much of the information is obtainable in many handbooks [for example, in *Blanck*, Vol. 6, 1930 (Schubert 1940) *Einführung in die Geophysik*, Vol. II (Keranen 1929)]. We shall not discuss these volumes in detail, but shall only give some of the data which illustrate the most important temperature conditions existing in soil (magnitude of temperature gradient, conductivity, temperature at the frost line, etc.) under which the natural freezing process occurs.

As we have seen, the temperature gradient dt/dx (temperature change per unit length) is a measure partly of the rate of flow of heat, and partly of the conductivity of the medium. If the rate of heat flow, i.e., the amount of heat flowing past a surface in unit time, is called Q_v , we have the following formula:

$$Q_v = \lambda \cdot \frac{dt}{dx} \text{ cal}$$

where λ is taken as the coefficient of heat conductivity. For a constant value of this coefficient, the rate of heat flow is therefore proportional to the temperature gradient, and for a constant gradient it is proportional to the heat conductivity coefficient.

Graphically, from the diagram of temperature vs depth, the slope of the temperature curve is a measure of the gradient, which is numerically equal to the tangent of the angle at any point on the curve. Now if we assume the conductivity constant to be a constant,* we have the following rule: When the temperature curve is a straight line (constant gradient), no temperature change occurs, but there is only heat flow, in which just as much heat is conducted away from a point as to a point. Where the temperature curve is curved, there is a temperature change, and thus when the gradient in the direction of the temperature drop increases, cooling occurs, and when it decreases, warming occurs. Or looking at the curves from the side of the positive temperature, when the curve is convex cooling occurs, when concave warming occurs.

Generally in the earth's crust heat flows upward from underneath due to higher temperatures within the ground. The gradient for this heat flow is for usual soils about 1°C per 33 m, or $0.03^\circ\text{C}/\text{m}$. An average value for the coefficient of heat conductivity for the upper part of the earth's crust is $\lambda = 0.006 - 0.008 \text{ cal/cm sec } ^\circ\text{C}$ (see Table 20).

* This is approximately true for any one medium, especially for rather small temperature differences, since the effect of temperature on the conductivity constant is very small.

For capillary saturated mineral soils the value is about $\lambda = 0.003 - 0.004$. The gradient is about $0.06^\circ\text{C}/\text{m}$ and this can be taken as the average value for the loose top mantle for the conditions under which frost-heaving occurs.

The exact determination of the different heat coefficients is considered briefly below, and a number of basic values of the coefficient for different substances are given.

A. Heat and temperature conductivity coefficients

The flow of heat in a medium, as we know, always depends on the existence of a temperature difference. The quantity of heat Q_v in calories flowing through a surface $U \text{ cm}^2$ during S seconds is

$$Q_v = \lambda \cdot \frac{dt}{dx} \cdot U \cdot S$$

where dt/dx is the temperature gradient, or the temperature difference per unit length in the x direction which is the direction of heat flow.

From this we get

$$\begin{aligned} \lambda &= \frac{Q_v}{U \cdot \frac{dt}{dx} \cdot S} \text{ cal/cm}^2 \cdot \text{degrees/cm sec} = \\ &= \text{cal/cm degree sec} \end{aligned}$$

If U , dt/dx and S are taken as unity, then $\lambda = Q_v \text{ cal/cm degree sec}$, i.e., the coefficient of heat conductivity is the quantity of heat in calories that flows past a surface of unit area in unit time and acted on by a unit gradient.

Temperature changes at any point can only occur when the quantity of heat flowing is different at different places, i.e., at any given cross section, the heat absorbed and the heat conducted away is different.

The temperature change that occurs at any point due to heat conduction, derived directly from the heat conductivity coefficient, is expressed in terms of the temperature conductivity coefficient. [Also known as "diffusivity"; however, the literal translation of the author's term—"temperature conductivity coefficient" will be used hereafter]. This is denoted by a^2 and

$$a^2 = \frac{\lambda}{s \cdot c} = \frac{\lambda}{C} \text{ cm}^2/\text{sec}$$

where s = density (gr/cm^3) and c = specific heat ($\text{cal}/\text{gr } ^\circ\text{C}$) and $s \cdot c = C$ is the heat capacity per unit volume ($\text{cal}/\text{cm}^3 ^\circ\text{C}$). For any given value of the coefficient of heat conductivity, the temperature conductivity—the rate at which a body heats up—is inversely proportional to the heat capacity.

The units for this coefficient are $^\circ\text{C}$ per $(^\circ\text{C}/\text{cm-cm}) \cdot \text{sec}$, and are therefore, the temperature change in de-

Table 20. Heat conductivity coefficient λ (cal/cm² sec °C) of various materials.

	Coefficient		Mean value
	Range of variation		
	Extreme limits	Range of concentration	
Volcanic rocks			
Granite (9 det.)	0.003–0.009	0.004–0.007	0.0055
Gneiss (6 det.)	0.0035–0.009	—	0.006
Porphyry (1 det.)	0.008	—	0.008
Basalt (4 det.)	0.003–0.007	—	0.005
Serpentine (1 det.)	0.007	—	0.007
Average of volcanic rocks (21 det.)	0.003–0.009	—	0.006–7
Sedimentary rocks			
Limestone and dolomite (10 det.)	0.002–0.006	—	0.0045
Marble (12 det.)	0.0055–0.009	0.006–0.008	0.007
Sandstone (2 det.)	0.0037–0.0072	—	0.005
Quartzite (8 det.)	0.0069–0.0116	—	0.009
Schist (2 det.)	0.0022–0.0036	—	0.003
Rock salt (2 det.)	0.009–0.0167	—	0.012
Loose soils			
Rubble (3–8 cm) (density 1.85)	0.000889	—	0.0009
Sand, dry (5 det.)	0.0006–0.0010	—	0.0009
Sand, moist (6.9% water)	0.0027	—	0.0027
Clay, dry (5 det.)	0.0015–0.0029	0.0021–0.0026	0.0024
Minerals			
Quartz, to axis	0.025	—	0.025
Quartz, \perp to axis	0.015	—	0.015
Quartz, amorphous (2 det.)	0.00330–0.00332	—	0.0033
Calcite, axis (2 det.)	0.0129–0.0132	—	0.013
Calcite, \perp axis (2 det.)	0.0108–0.0111	—	0.011
Ice–50°	0.00665	—	—
Ice–25°	0.00595	—	—
Ice–10°	0.0055	—	—
Ice \pm 0°	0.00535	—	0.0054
Water (0° to room temp.)	0.0013–0.0015	—	0.0014
Brick (4 det.)	0.00097–0.001750	—	0.0013
Pine wood (\perp fiber) (3 det.)	0.00025–0.0029	—	0.00027
Pine wood (fiber) (1 det.)	0.00053	—	0.00053
Sawdust (density 0.19)	0.00014	—	0.00014
Cork (3 det.)	0.000093–0.00012	—	0.0001
Peat, hard-pressed (density = 0.320) 37°	0.00018	—	0.00018
Air 0° 0.000057	—	—	0.000057

grees that occurs in one second, when the change in the temperature gradient is one degree per centimeter per centimeter distance. This of course refers to an infinitesimally thin layer for, as a rule, the change in the temperature gradient is not constant and the heat flow is variable. For the rate of heating to be uniform (i.e., equal temperature increases) the temperature gradient must be proportional to the distance from the surface being heated and thus $a^2 = \lambda/C$ is a constant.

Table 20 gives a summary of the values of heat conductivity coefficients for a number of different substances. Usually tables of values of the heat conductivity coefficient contain old, erroneous or uncertain values; this table has only the values of newer determinations and those older ones that have been later confirmed. All of these figures are taken from Landolt-Bornstein (Vol. II, page 1925, Supplementary Vol. I, p. 711, and II b, p. 1295).

The effect of water content of the soil on the value of the heat conductivity coefficient is very great. A sand saturated with water conducts heat much better than a sand with air-filled pore space. And even if the sand pores are air-filled, there is still some moisture content in the soil in the form of "porewater" [funicular water] which gives it a greater conductivity than completely dry sand.

For the coarse grained soils, gravels and sand down to the silts, we can say then, that the *heat conductivity is least in a completely dry state*, and increases with increased water content, and is greatest for complete capillary saturation.

In fine-grained soils, where the thickness of the adsorption water films is of the order of magnitude of the particle diameter itself, the phenomenon is much more complicated. Of the three possible components in a mineral soil—mineral particles, water, and air—the con-

Table 21. Temperature conductivity coefficients (a^2), heat capacity (C), and conductivity coefficients (λ) for various soils.

Type of soil		$a^2 = \lambda/C$	C	$\lambda = C \cdot a^2$
Rubble, dry, 3–8 cm density = 1.85		—	—	0.0009
Sand, dry (5 determinations)		—	—	0.0006–0.0010
Sand, moist (6.9% water)		—	—	0.0027
Clay, dry (5 determinations)		0.00343–0.0072	—	0.0015–0.0029
Damp sand from 60–90 cm deep	a	0.00616	0.274	0.00168
	b	0.00627	0.258	0.00162
Sandy lean clay (15–60 cm deep)	a	0.0052–535	0.346	0.00180–0.00185
	b	0.0058–685	0.373	0.00218–0.00250
Same, very wet (60–90 cm deep)	a	0.00600	0.449	0.00268
	b	0.0105	0.378	0.00397
Clayey sand, saturated, 60–90 cm deep. Fritzen	a	0.00774	0.342	0.00265
	b	0.00795	0.549	0.00435

ductivity of the particles is much greater than the water, and that of the water, much greater than air. For the coarser soils it is only the volume or the ratio between water and air in the pores that can vary; the greatest conductivity therefore occurs at greatest water content. This applies for the fine grained soils too but only for conditions under capillary saturation. But due to the very large adsorption forces in clays, the water content can be considerably greater than capillary saturation, since the particles are surrounded by expansive elastic adsorption films. A wet plastic clay, in which the particles are some distance from one another, therefore has a smaller conductivity than clay at capillary saturation, where the particles are as close to each other as they can possibly be.

*For clays then, the coefficient of heat conductivity is a maximum at the saturation point; and from this point it decreases with increasing water content. As the water content decreases under the saturation point, the decrease in the conductivity is much less than for sands, since in dry clay, the flaky, scaly particles are more in contact.**

A number of determinations of the heat conductivity coefficients as well as heat capacity and temperature conductivity coefficients for different soils are given in Table 21. The values are taken partly from Landolt-Bornstein, and partly from Schubert (1930, p. 371).

Schubert's values are based on average annual values from temperature readings at different depths in natural soils.

From the table we see that the heat conductivity coefficient for the coarse soils (sand) in a dry condition can be taken to be about 0.001; for fine grained soils (clay)

it increases to 0.0025–0.003. Also, only a small increase in water content for sand increases the constant greatly, to about 0.002 or 0.003. For capillary saturated sands, silts, and fine silts, the value of the coefficient lies at about 0.004 and 0.003, which comes near the value for clay near the saturation point. For a soft plastic clay, the coefficient is lower, about 0.003–0.0025. (The limit to which the value can go for increasing water content is the value for pure water or about 0.0014.)

Inclusion of humus material, and loosening due to roots, of course diminishes the conductivity. The considerably lower values of conductivity for soils in wooded areas are due to these causes.

Infrozen soil, the conductivity is usually much greater than in the same unfrozen soil, since the conductivity of ice (0.0054) is about 3 or 4 times as great as water (0.0014). The increase is of course variable, depending upon how the soil is frozen. An important effect is that the increase in conductivity due to the formation of ice is partly counteracted by a loosening due to the ice-banding (decrease contacts of particles). Due to this reason, the conductivity of moist (non-saturated) soils is changed only very slightly. In capillary saturated non-clayey soils, the increase is about 0.001 in magnitude or, for example, from 0.004 to 0.005. The conductivity of the ice strata is 0.0054; if the soil is a clay, the soil between the strata becomes compressed, and the conductivity increases somewhat. However, in clays, the ice layers often become quite porous, and therefore the conductivity of the ice decreases greatly. In general, then, the conductivity of frozen clay is not greater than unfrozen clay, but in coarser soils, the coefficient of conductivity increases for frozen soil about 0.0005 to 0.001 over unfrozen soil. In percent, this increase is about 10–15 and at the most 20–30%.

If, therefore, the change in the heat conductivity is nothing or relatively small, the effect on the temperature conductivity coefficient is greater, due to the large

* This assumes that the dry clay has not been pulverized. If the clay dries in a natural state, the capillary forces are so large that the pore volume is reduced to a rather moderate value. If the clay is ground up or pulverized after being dried, the soil becomes very loose, and the conductivity is much lower.

decrease in heat capacity when water freezes to ice (from 1.00 to 0.45 cal/cm³). This decrease in heat capacity occurs simultaneously with the eventual increase in conductivity; thus for the temperature conductivity coefficient (a^2) [diffusivity] we have,

$$a^2 = \frac{\lambda}{C}$$

where C = the heat capacity of the medium.

This effect is easily computed.

In a soil consisting of $a\%$ by volume of soil substance, and $b\%$ water (where a plus b = 100%) we have

$$C = \frac{a}{100} \cdot C_a + \frac{b}{100} \cdot C_b$$

where C_a = the heat capacity of the soil and C_b = heat capacity of the water. Now assume that the soil freezes, and that a part of the water content changes to ice. If the volumetric percent of soil and water and ice is respectively α , β , and γ , the heat capacity is

$$C = \frac{\alpha}{100} \cdot C_a + \frac{\beta}{100} \cdot C_b + \frac{\gamma}{100} \cdot C_c$$

$$(\alpha + \beta + \gamma = 100; \beta + 0.9 \cdot \gamma = b)$$

where C_a , C_b , C_c are the heat capacities of the mineral substance, water and ice respectively. As an average value of the heat capacity for the mineral substance we can take $C_a = 0.55$; $C_b = 1.00$ and $C_c = 0.45$.

A soil with 30% water has before freezing a heat capacity, $C = 0.7 \times 0.55 + 0.3 \times 1 = 0.685$; if all the water (without an additional flow to the freezing zone) freezes, the heat capacity is $C = 0.678 \times 0.55 + 0.322 \times 0.45 = 0.375 + 0.145 = 0.518$.

The ratio of C unfrozen is then $= 0.685/0.518 = 1.32$.

For a water content a) of 35% and b) of 40%, similarly the ratio of C unfrozen to C frozen is a) $-0.7075/0.508 = 1.39$; b) $= 7.30/0.507 = 1.44$.

If we assume a water content of 30% before freezing, and that after freezing there is 20% ice and 20% of unfrozen water, we have:

$$C \text{ unfrozen} = 0.7 \times 0.55 + 0.3 \times 1 = 0.685$$

$$C \text{ frozen} = 0.6 \times 0.55 + 0.2 \times 1 + 0.2 \times 0.45 = 0.62$$

$$\text{Ratio of } C \text{ unfrozen to } C \text{ frozen} = 1.103.$$

If we change the values of ice content to $a = 40$ and $b = 60\%$ while the ratio between soil and unfrozen water remains constant, the values become

$$\text{a) } C \text{ unfrozen} = 0.45 \times 0.55 + 0.15 \times 1 + 0.4 \times 0.45 = 0.5775$$

$$\text{b) } C \text{ frozen} = 0.3 \times 0.55 + 0.1 \times 1 + 0.6 \times 0.45 = 0.535$$

Ratio of C unfrozen to C frozen =

$$\text{a) } 0.685/0.5775 = 1.185$$

$$\text{b) } 0.685/0.535 = 1.280.$$

The theoretical maximum decrease in temperature capacity occurs when the ice increase in a freezing soil (with maximum pore volume) becomes infinite, i.e., the frozen soil consists of a single ice layer. We can get:

$$C \text{ unfrozen} = 0.6 \times 0.55 + 0.4 \times 1 = 0.73$$

$$C \text{ frozen} = 1 \times 0.45;$$

$$\text{ratio of } C \text{ unfrozen to } C \text{ frozen} = 0.73/0.45 = 1.62.$$

To summarize, then, we find that the heat capacity is decreased during freezing, which causes an inversely proportional increase in the temperature conductivity. This increase in frost-heaving soils has a magnitude of about 10–30% and can sometimes (for very massive ice formation) approach 50%.

The total increase in the coefficient of temperature conductivity in frozen soil is (for capillary saturated soil) about the following:

Ratio of a^2 frozen to a^2 unfrozen is

$$= 1.0 \times 1.1 \rightarrow 1.3 \times 1.4 = 1.1 \rightarrow 1.8$$

therefore an increase of 10 to 80%. The usual values are between 20 and 50%, or between $1/5$ and $1/2$.

The increase in this coefficient for frozen soils is in some cases important to know, especially for the computation of the rate of penetration of the temperature change from temperature measurements at different depths at different times. Also the temperature conductivity coefficient is generally of much less importance than the heat conductivity coefficient, in that the latter is the fundamental constant for all computations of heat conditions in the ground, therefore for soil freezing and soil melting. Thus the heat flow is computed from the temperature gradient and the heat conductivity, the heat content from the temperature (= temperature difference from equilibrium temperature) and the heat capacity, both of which control the freezing and thawing of soil.

B. Temperature variations at the ground surface and their penetration downward

If we have a conductor, which as far as heat is concerned, is in equilibrium—either a continuity condition, in which an equal amount of heat flows without temperature changes, or the special case where there is no heat flow, i.e., the temperature change occurs somewhere on this conductor, a change in equilibrium conditions occurs, and the temperature changes over the body towards a new condition of equilibrium. If a permanent increase in temperature occurs, the temperature change is conducted gradually through the whole conductor—

although in most cases with decreasing magnitude, for, as example, if heating occurs on the end of a rod on the side of a wall, and the other end the temperature is kept constant, in which case the final equilibrium condition is one in which the temperature difference is divided linearly. If the body is subjected to a temperature change all around, for example, if it is immersed in a warm liquid, the equilibrium condition would be one in which an equal increase in temperature over the entire body has occurred.

If now the ground surface temperature changes, and then the change remains constant, it will be continuously penetrated downward to a certain constant depth.

The temperature variations occurring on the ground surface, are, however, not constant but vary continuously, and can over long periods occur in regular intervals (cyclical variation) of equal length, the most pronounced being the daily and yearly intervals. For this type of temperature change, which consists of a rhythmic oscillation of the surface temperature over a known period, the following schematic analysis can be used.

Assume that the temperature at the ground surface is rising. Due to heat conduction downward, the temperature successively increases downward, at a rate depending on the temperature conductivity coefficient. But the surface temperature eventually reaches a maximum and begins to decrease. Then the temperature at the surface is lower than the temperature at some distance beneath the surface, i.e., the gradient at the surface has an opposite direction, and the heat begins to flow to the surface from underneath. If we consider the temperature curve, the heating has caused an increase in this curve, which is rapidly leveled off downward. If then again, the surface temperature drops, the drop is projected downward, until the minimum temperature is reached at the surface and the surface temperature begins to rise again; we thus get a "maximum cold" which penetrates downward. In Figure 79 the temperature curves for eight phases of complete cycles are constructed schematically. Figure 80 gives empirical curves for daily variations (from Vujevic in Keränen 1929).*

The exact mathematical treatment of temperature variations with depth for regular cyclical surface temperature variations, is given in Schubert's work (1930, p. 364) and Keränen's (1929, p. 169).

Schubert makes the assumption that the surface temperature variation is a sine function of time. He gives the following variation:

If the amplitude of the surface variation is $2p$, then the amplitude of variation at a depth x under the surface is

$$2p_x = 2p \cdot e^{-vx}$$

or, if the ratio $2p_x/2p$ is put = s ,

$$s = e^{-vx}.$$

The phase length d is then

$$\phi = vx$$

where $v = \sqrt{\pi/a^2 T}$ and T = time cycle.

The value of the temperature conductivity coefficient [diffusivity] $a^2 = \lambda/C$ is as has been mentioned (Table 21) quite variable. It is largest for dry soils with small pore volume, and lowest for very wet soils, since the water has a very low temperature conductivity coefficient (0.0014). The maximum value of a^2 is about 0.01 (for granite $a^2 = 0.01$ to 0.015); the minimum value occurring in very wet plastic clays approaches 0.0025. Schubert (1930, p. 368) gives as an average value from 16 German test stations of $a^2 = 0.0065$.

At any instant, the temperature curve has the shape of a damped wave. The depth (ℓ), to which a single change reaches is

$$\ell = 2\sqrt{\pi \cdot a^2 T}$$

where T is the time cycle. This depth is of interest, since it is the boundary to which appreciable temperature changes reach; the amplitude of variation is at this boundary only 0.2% of that occurring at the surface.

From the formula we also find that the depth is proportional to the square root of time. Yearly variations therefore reach to a depth of $19.1(\sqrt{365})$ times the depth of daily variations.

Table 22 gives a number of depths of penetrations for daily and yearly variations for a number of different temperature conductivity coefficients.

We see from the table that in clay soils, the depth of penetration is about $1\frac{1}{2}$ m per day or 10 m per year, for wet ground, and 0.7 to 0.8 m and 15 m for dryer ground; the maximum depth for loose soils is 1 m per day or 20 m per year. In bedrock, the depth is generally much greater than in loose soil. For unmetamorphosed sedimentary rocks (sandstones and shales, etc.), the depth is about the same as the high values for loose soils; for crystalline rock, (granite, marble, quartzite, etc.) the depth is about $1\frac{1}{2}$ m per day and 20–25 m per year.

The mixture of organic material in soil decreases the depth of penetration considerably. The lowest values occur for clean dry soils, about 30–40 cm per day, 6–8 m per year. The effect is relatively independent of water content for the temperature conductivity coefficient of water ($a^2 = 0.0014$) lies just at the same value. The effect

* The axis is in this case sloping, caused by the seasonal changes during the year (in this case warming up during the summer).

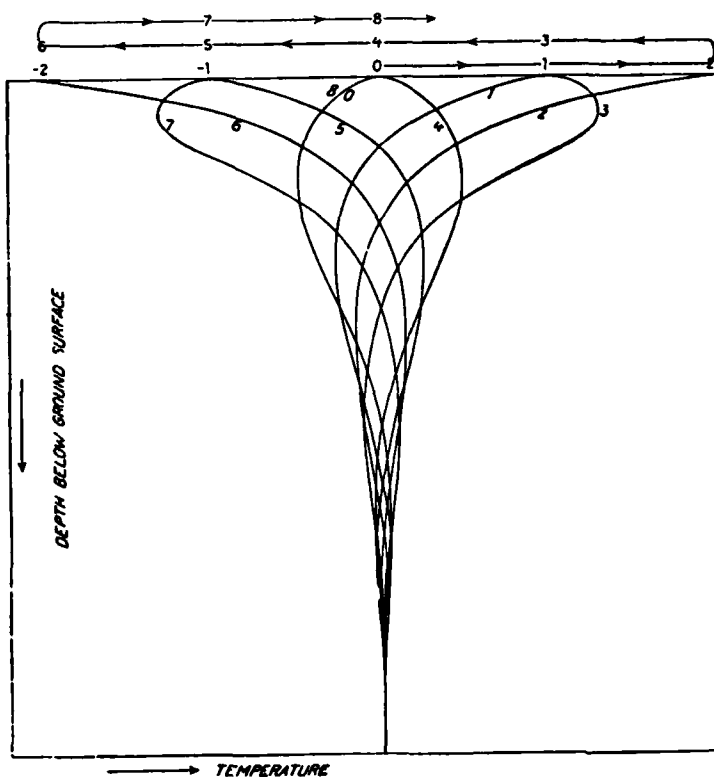


Figure 79. Schematic diagrams of depth-temperature curves at different periods assuming the surface temperature to vary between a maximum and minimum value (such as the change between day and night, or summer and winter). The diagram shows the curves for eight different stages of a temperature cycle. Compare with Figure 80.

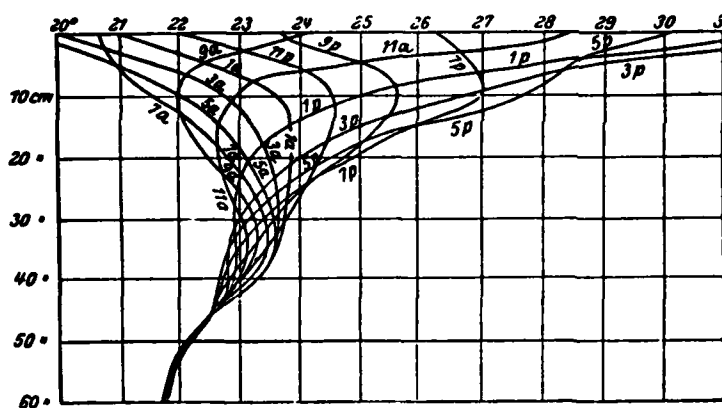


Figure 80. Depth-temperature curves at various periods of a 24-hour day. These are the means for the month of June at Belgrade (from Vujevic in Keränen 1929, Fig. 6). Notice the inclination of the temperature axis, due to warmer summer temperatures.

of mixing humus in mineral soil is about proportional to the amount added. For humus, loose forest soils, as well as in humus clays, a value of $\frac{1}{2}$ m per day is a good average value. In Figure 80, the depth (where the vortex stops) is 45 cm, which agrees with the value for a loose decomposed soil.

C. Temperature at the frost line

It is often assumed for the computation of the position of the frost line from temperature measurements, that the frost line coincides with the zero isotherm, i.e., that the temperature at the frost line is 0°C . This is fallacious, at least during the duration of freezing, for we know that in all soils the freezing temperature of water is more or less lowered.

From the data on freezing temperature depressions in different soils (Fig. 23) the maximum values of frost line temperatures (or the minimum value of freezing point depression at the frost line) can be obtained. This minimum value is of course the temperature at which freezing begins at a given water content (for frost-heaving soils, always at capillary saturation). However, the table actually shows that the temperatures at the frost line are always lower than this value, i.e., the depression is greater. The cause is, of course, that in frost-heaving soils an actual undercooling occurs, which adds to the definite freezing point depression of a soil. But it is significant that this only occurs while freezing is occurring, and that for any given soil the temperature depression is greater the quicker freezing occurs.

Table 22.

	Temperature conductivity coefficient a^2	Depth of penetrations of temperature oscillations whose periods are:	
		24 hours (cm)	years (m)
Soils			
Peat	0.001	32.9	6.3
	0.002	46.5	8.9
Plastic clay	0.003	57.0	10.9
	0.004	65.9	12.6
	0.005	73.7	14.1
Average, soil	0.0065	84.0	16.0
	0.008	93.1	17.7
	0.010	104.0	19.9
Rock	0.0115	112.0	21.4
Ice	0.013	119.0	22.8
	0.015	137.0	26.2
	0.020	148.0	28.2

For melting of frozen soil, on the other hand, at least for ice-stratified soil, the phenomenon is quite the opposite. When such a frozen soil heats up, the ice melts successively and follows the stability curve for freezing, but in the opposite direction. The clean ice layer, however—with the exception of the actual contact surface—has a different freezing and melting point than the soil. If melting occurs slowly, there is a change in equilibrium at the contact surface, due to expansion of the nearby soil and the sucking up of the melting nearby water; but the excess water which cannot be taken up by the soil forms an ice layer between the thawed water saturated soil, which does not have a melting point depression, but freezes at exactly 0°C . Thus the boundary for the melting of soil is the place where this ice layer disappears, and the *receding frost line* is the zero temperature line.

To summarize, then, we can say that for coarse non-frost-heaving soils the temperature for freezing and thawing is practically the same, and is very slightly less than 0°C . For frost-heaving soils there is quite a difference between the frost line temperature during freezing and thawing: the receding frost line temperature is 0°C or very slightly less, while the frost line temperature during freezing is considerably lowered, being greater the finer the soil is and the faster it freezes.

Below is given the results of a few exact determinations of the temperature during frost penetration. The determinations were made partly with a thermometer, and partly by a thermoelement at different depths during freezing, and depths were measured (Fig. 81). There are no exact data on the rate of freezing; all determinations are, however, either average values for longer periods of freezing, or single values for a definite, nearly average rate of freezing. They can, therefore, be taken as good

normal values of the frost line temperatures occurring for normal rates of freezing.

Types of soil	Frost line temp ($^{\circ}\text{C}$)	
	Limits	Avg.
Medium sand (only damp—6% moisture)	0.015–0.02	—
Silty moraine, Stockholm	0.02–0.08	0.05
Silt, Norrbotten	0.15–0.2	—
Lean clay, Västernorrland	0.4–1.0	0.5

D. The temperature gradient in freezing soil

It is, of course, obvious that the magnitude and the changes in the temperature gradient are largest at the surface and decrease with depth. For rapid heating (for example, by direct sun rays) or cooling the temperature gradient at the surface can be as high as a few degrees per centimeter; but as a rule even the gradient at the surface is much more damped, and at a few centimeters depth exceeds this value.

In freezing soil the temperature gradient is always quite damped, especially at and under the frost line. The reason is, of course, that due to the change in form of the water, the large amount of heat is absorbed or extracted during freezing or thawing delays temperature changes from one side to the other side of the freezing point. In frozen coarse soil which has practically a constant freezing temperature throughout, a large gradient can occur; but as soon as it concerns clays, with lowering freezing temperatures, there is a large energy transformation in changing water to ice which dampens the temperature variations considerably.

But even without considering the soil properties, there is in all soils considerably damped temperature variations below the frost line, and the gradient is very small. Thus the frost line represents an isotherm that has a very large resistance against being moved. Thus when freezing begins, there is quite a temperature variation at the surface, which is transmitted down towards the frost line which absorbs these variations and moves downward very slowly. Thus in a general way, heat flow in frozen soil occurs by a smoothing out of temperature differences between constant temperature surfaces, the frost line and a constant temperature surface lower down representing the average yearly surface temperature (about 10–20 m deep) below which there is no temperature variation at any level.

The temperature gradient immediately under the frost line is quite important since it controls the progress of ice formation. Some empirical data of the magnitudes of this gradient will be considered below.

The gradient is most important when freezing starts. Along with the “hoar frost” studies during the first winter frosts, temperature determinations were made to

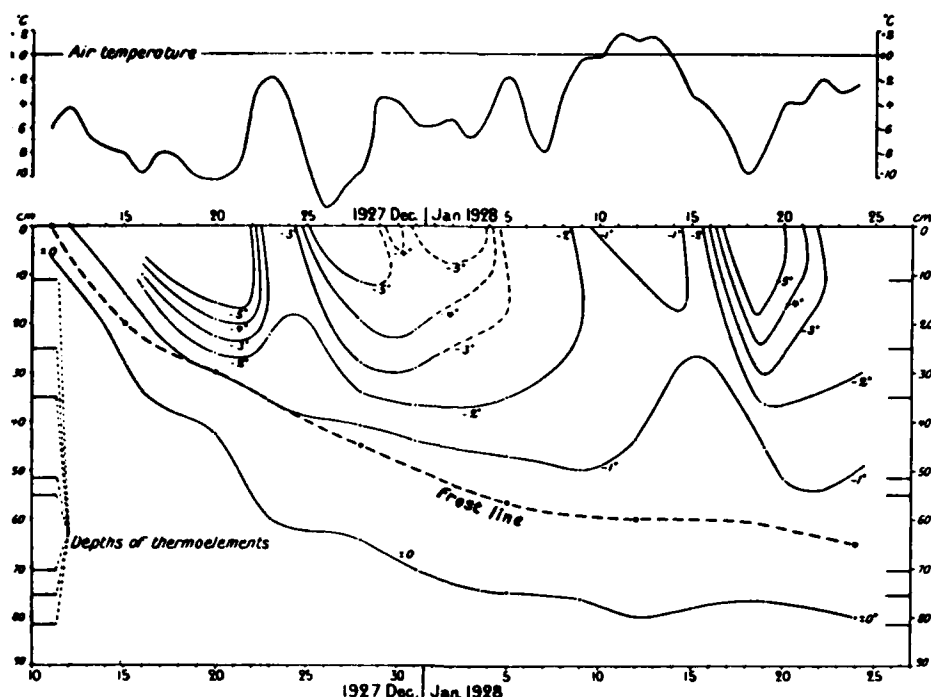


Figure 81. Diagram showing temperature variations and the downward movement of the frost line in a cylinder of saturated clay (lean clay from Ramvik), insulated on its sides and frozen outdoors (at the State testing grounds, Stockholm).

1 and 2 m depth. Small, accurate thermometers were inserted to different depths in the soil. Some of the temperature curves obtained are given in Figure 82. For such small depths the curves are practically straight lines; the gradient varies from $0.3^{\circ}\text{C}/\text{cm}$ early in November to almost $0.1^{\circ}\text{C}/\text{cm}$ in the middle of December. $0.1^{\circ}\text{C}/\text{cm}$ can be used as the average gradient at which freezing begins, for mineral soils in central Sweden.

Ångström obtained similar curves for different depths in sandy soil at the Experimental Station which agree quite well with these curves (Ångström 1928, Fig. 4). For the months of October and November temperature differences of about $4\text{--}3^{\circ}\text{C}$ were obtained for depths of 10 to 50 cm, i.e., $0.1\text{--}0.075^{\circ}\text{C}/\text{cm}$. That is, then, the average gradient between 10 and 50 cm. In the top decimeter the gradient is, of course, somewhat larger. Later on the gradient decreases and is in December about $0.065^{\circ}\text{C}/\text{cm}$, in January $0.04^{\circ}\text{C}/\text{cm}$, in February $0.03^{\circ}\text{C}/\text{cm}$, and in March $0.02^{\circ}\text{C}/\text{cm}$.

These values agree with other tests made at places having similar climate. In Table 23 some of these values are given.

We can conclude the following from the results: At the beginning of freezing, the gradient at the frost line is about $0.1^{\circ}\text{C}/\text{cm}$, but decreases during the winter rather rapidly, and at the turn of the year is about $0.05^{\circ}\text{C}/\text{cm}$ and at the end of winter is about $0.02\text{--}0.03^{\circ}\text{C}/\text{cm}$.

For colder regions, where the average surface tem-

perature is lower, the gradient at the end of winter and in the spring is considerably less. Likewise for warmer regions, the gradient is larger at the end of winter.

E. Greatest possible frost depth, melting from below

The value of the temperature gradient below the frost line is important in determining the greatest frost depth and melting from below. During the entire winter, a continuous upward flow of heat occurs, and the amount decreases as the winter progresses. The heat flow below the frost line is, of course, directly proportional to the temperature gradient. The amount of heat that flows is quite large (see Table 27, column d), and is about $4\text{--}10\text{ cal}/\text{cm}^2\text{ day}$. In order for freezing to take place, the heat conducted away must be greater than the heat absorbed. For this to happen, a certain minimum temperature gradient must exist. The temperature at the frost line is essentially constant and is somewhat under 0°C ; for a given thickness and a given temperature gradient under the frost line, a certain minus temperature is required in order that freezing can continue. Or conversely: if at the ground surface a certain temperature exists, and under the frost line there is a certain temperature gradient, then the frost can only grow to a certain thickness, this thickness depending on the difference in temperature between the surface and the frost line, and the temperature drop below the frost line.

If we consider only the temperature gradient caused

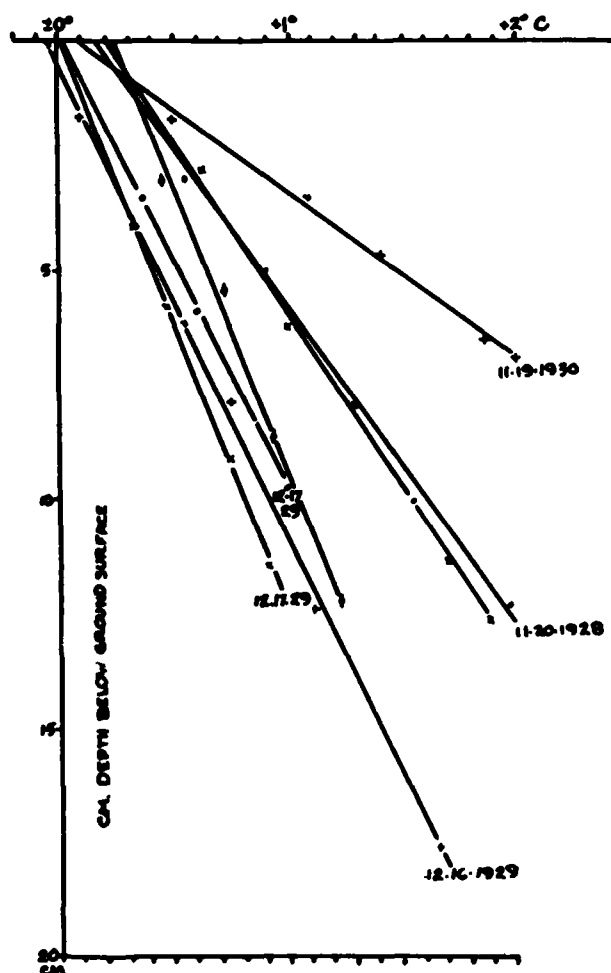


Figure 82. Temperature gradient in the upper part of mineral soils (clay, silt and silty moraine material), in ground near Stockholm during late fall, measured by means of thin thermometers under fresh hoar frost.

by the heat flow from within the soil, which for loose soils is about $0.06^\circ/\text{m}$, the maximum frost depth is about 15–35 m for each degree difference between the average surface temperature and the frost line temperature. The depths of frost of the perpetually frozen arctic regions are found in this manner.

The effect then of the ground surface temperature is that it determines the greatest possible frost depth.

If the temperature at the frost line is t_0 , and the surface temperature is t_y , the gradient under the frost line $dt/dx = a$ (degrees per unit length), and the heat conductivity coefficient in unfrozen soil is λ_1 , and in frozen λ_2 , the maximum frost depth L_M is

$$L_M = \frac{t_0 - t_y}{a} \cdot \frac{\lambda_2}{\lambda_1}$$

With very slight approximation the ratio of λ_2 to λ_1 can

be said to equal unity and the formula becomes

$$L_M = \frac{t_0 - t_y}{a} \text{ approx.}$$

In regions where perpetually frozen soil is not found, but where there is yearly winter freezing, which has to work against the stored up summer heat,* the conditions are entirely different. The gradient is here many times the geothermal value. For gradients of 2, 3 and $4^\circ\text{C}/\text{m}$, (i.e., 0.02, etc., $^\circ\text{C}/\text{cm}$) the frost depth L for each degree of $t_0 - T_y$, is $L = 0.5, 0.33$, and 0.25 m.

As long as the ground remains bare, the average surface temperature is about the same as the average air temperature, at least during overcast weather. During clear weather there is some difference due to the sun's radiation. During the winter the air can absorb considerable radiant heat from the direct sun rays, and the surface temperature can then be much less than the air temperature. In early winter in central and northern Sweden, the average air temperature is from a few up to 10°C below zero. In the cooler regions, such as inland Norrland (northern province of Sweden) there is very little cloudy weather, and due to the effect of radiation, the actual soil temperature is much less than the air temperature.

It is then obvious that in these regions the rate of penetration of frost during early winter can be quite large, especially if the ground is bare, or the snow mantle is very thin.

Later on in the winter the ground is usually covered with a rather thick, loose snow blanket. This blanket is a very poor heat conductor and acts as an insulator. Even for extremely cold weather the heat transmitted through the blanket is small.

The conductivity of the snow blanket depends principally on its looseness, and porosity. The conductivity can then be assumed to be approximately a function of the density of the snow. In Figure 83 a number of determinations are given graphically and an approximate curve drawn through them.

There are quite a lot of data on the density of snow under various conditions (see Keraänen 1929, p. 282–283). Newly fallen snow usually has a density less than 0.1. The average density of the snow covering in the cooler climates is usually less than 0.2; during the periods of melting or during the spring thaws, the density increases to between 0.2 and 0.3. The following gives the monthly data obtained by H. Abels (in Keraänen 1929, p. 283).

* It should be noted that besides the conditions for perpetual frost, the constant temperature surface 10–20 m deep (which is greater than the average yearly temperature of the surface) is often quite a bit greater than zero degrees.

Table 23. Temperature gradients (°C/cm) in the upper first meter of natural mineral soils.

	Nov	Dec	Jan	Feb	Mar
New Stockholm (Beskow)	(0.3)–0.1	—	—	—	—
New Stockholm (Ångström)	0.08	0.065	0.04	0.03	0.02
Pavloosk* (Leyst)	0.075	0.088	0.042	0.039	0.028
Leningrad† (Luboslawsky)	0.13	0.054	0.04	0.04	0.035
Koänigsberg* (Schmidt)	—	0.05	0.035	0.02	—

* Monthly mean, after Schubert 1930.

† Monthly mean, after Keränen.

Average density of the snow covering

Nov.	Dec.	Jan.	Feb.	Mar.	April
0.139	0.182	0.193	0.189	0.233	0.279

Comparing this with the curve in Figure 83, we find that the heat conductivity coefficient during the early part and the middle of winter, where there hasn't been much packing and alternate melting and freezing, is about 0.00025 to 0.0004, and is hardly one-tenth of that of the ground. When melting occurs it rises to 0.0004 to 0.0008, and at its greatest possible density may be as high as 0.001.

The greatest possible frost depth X can then be computed, when the following values are known:

The thickness of the snow covering = ℓ , the surface

temperature of the snow covering = t_y , the temperature at the frost line = t_o , and the temperature gradient at the frost line is dt/dx , and the ratio between the conductivity of the frozen soil and the snow is λ/λ_1 .

The equation is then

$$\ell \frac{\lambda}{\lambda_1} \cdot \frac{dt}{dx} + X \cdot \frac{dt}{dx} = t_o - t_y$$

$$X = \frac{t_o - t_y}{\frac{dt}{dx}} - \ell \cdot \frac{\lambda}{\lambda_1}$$

If melting starts from below, t_o can be taken as 0°C; when $-t_y$ is large, t_o can also in other cases be taken as

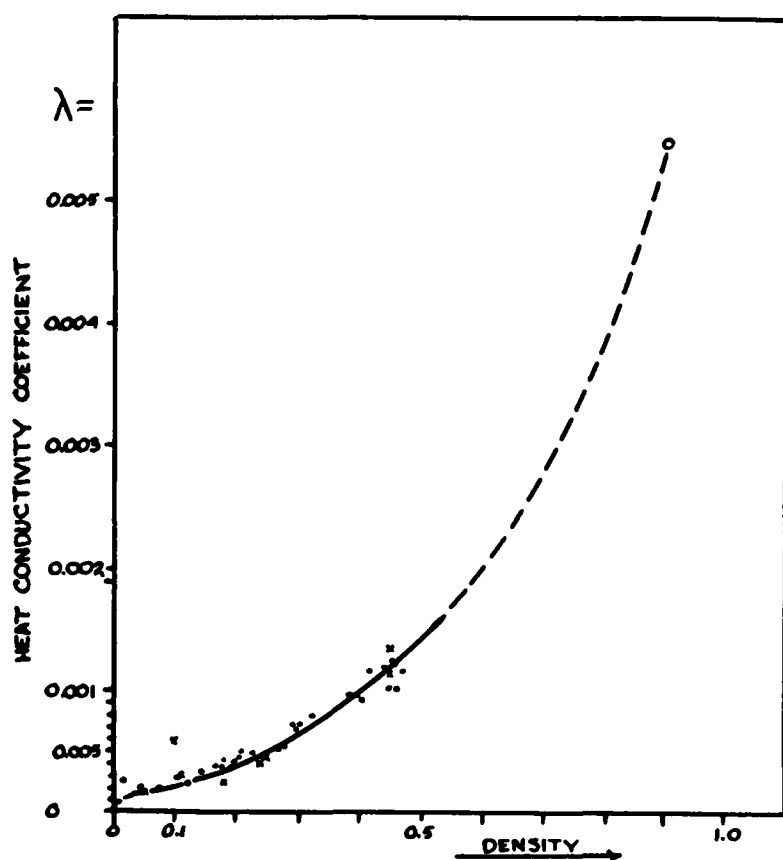


Figure 83. Diagrams showing the heat conductivity coefficient of snow as a function of its density. Graphical summary of data from Landolt-Börnstein, and especially Jansson (1901).

Table 24. Greatest frost depth possible with various snow depths and temperatures.

a λ/λ_1	b dt/dx (°C/m)	c ℓ (m)	d t_y for $X = 0$ (°C)	e Maximum frost depth X (m)					
				$t_y = -2^\circ\text{C}$	$t_y = -5^\circ\text{C}$	$t_y = -10^\circ\text{C}$	$t_y = -15^\circ\text{C}$	$t_y = -20^\circ\text{C}$	$t_y = -30^\circ\text{C}$
10	4	0.5	-20	—	—	—	—	0	2.5
		0.3	-12	—	—	—	0.75	2	4.5
		0.2	-8	—	—	0.5	0.75	3	5.5
		0.1	-4	—	0.25	1.5	2.75	4	6.5
		0	0.0	0.5	1.25	2.5	3.75	5	7.5
	3	0.5	-15	—	—	—	0	1.67	5
		0.3	-9	—	—	0.33	2	3.67	7
		0.2	-6	—	—	1.33	3	4.67	8
		0.1	-3	—	0.67	2.33	4	5.67	9
		0	0	0.67	1.67	3.33	5	6.67	10
7	2	0.5	-10	—	—	0	2.5	5	10
		0.3	-6	—	—	2	4.5	7	12
		0.2	-4	—	0.5	3	5.5	8	13
		0.1	-2	0	1.5	4	6.5	9	14
		0	0	1	2.5	5	7.5	10	15
	3	0.5	-10.5	—	—	—	1.5	3.17	6.5
		0.3	-6.3	—	—	1.23	2.9	4.57	7.9
		0.2	-4.2	—	0.27	1.93	3.6	5.27	8.6
		0.1	-2.1	—	0.97	2.63	4.3	5.97	9.3
		0	0	0.67	1.67	3.33	5	6.67	10
7	2	0.5	-7	—	—	1.5	4.0	6.5	11.5
		0.3	-4.2	—	0.4	2.9	5.4	7.9	12.9
		0.2	-2.8	—	1.1	3.6	6.1	8.6	13.6
		0.1	-1.4	0.3	1.8	4.3	6.8	9.3	14.3
		0	0	1	2.5	5	7.5	10	15
	1	0.5	-3.5	—	1.5	6.5	11.5	16.5	26.5
		0.3	-2.1	—	2.9	7.9	12.9	17.9	27.9
		0.2	-1.4	0.3	3.6	8.6	13.6	18.6	28.6
		0.1	-1.7	1.3	4.3	9.3	14.3	19.3	29.3
		0	0	2	5	10	15	20	30

a— λ/λ_1 where λ = heat-conductivity coefficient in the frozen zone, and λ_1 = for snow.

b—Temperature gradient dt/dx at the frost line.

c—Thickness of snow layer = ℓ .

d—Highest temperature t_y at which the soil can freeze beneath the snow layer.

e—Greatest possible frost-depth in meters, for various temperatures t_y at the surface of the snow.

0°C. The equation then simplifies to

$$X = \frac{t_y}{\frac{dt}{dx}} - \ell \cdot \frac{\lambda}{\lambda_1}$$

For an average value of λ/λ_1 , 10 can be taken for loose snow, and from 9 down to 5 for more compact snow. Table 24 gives the possible depth X for a number of different depths of snow covering, temperature gradients, and ratio λ/λ_1 .

The table illustrates what we can conclude from the formula: Especially in moderately cold climates, where the temperature at the surface of the snow cover is only a few degrees below zero, this cover is very important. For a snow surface temperature of -2°C , and 1 dm thick, all soil freezing is prevented, and the previously formed

frozen soil begins to melt from underneath. We assume here that the temperature gradient is 2°C/m or larger; also, in somewhat warmer regions, where the winter is more mild, such as in southern Sweden, the gradient never is as large as this. In middle and northern Sweden, where the snow covering during midwinter is usually about 1 or 2 dm the effect of the covering is enhanced.

If we compute with a gradient of only 3°C/m we find (column d) that the surface temperature for 2 dm of snow covering is -6° , for 3 dm, -9° and for 5 dm, -15°C . If, therefore, earlier during the winter, frozen soil of depth X meters is formed, and snowfall occurs, to a depth of the given values above, the average temperature of the snow surface must be the same as the corresponding temperatures above, plus an additional temperature of $-3 X^\circ\text{C}$.

Table 25 gives a number of average monthly temper-

atures for different Swedish localities. From this we see that for upper Norrland (northern province in Sweden) the average temperature during midwinter is about -10°C , for lower Norrland, -5 to -8°C , and for central Sweden about -2 , -3 to -4°C .

The temperature at the snow surface is somewhat lower than these air temperatures. The difference, of course, depends upon the amount of sunshine. In Norrland, the difference is the greatest. At a place in Finland (Sodankyla) with a climate similar to that in Norrland, this difference was measured (Keränen 1939, p. 289) and was found to be (air temperature minus snow surface temperature) during 1917–16 for Nov 1.2° , Dec 0.9° , Jan 1.7° , Feb 1.2° , Mar 1.2° , and April 2.2°C .

Of course these values are comparatively small, so that the air temperature can be used approximately for the snow surface temperature.

We find then that in upper Norrland a snow covering of $1\frac{1}{2}$ m is required to prevent any further soil freezing. In central Norrland, a thickness of 3 to 4 dm is required, and in Central Sweden a covering of 2 to 3 dm. When these depths are exceeded, melting from underneath begins. (This, of course, is only considering the average temperature, for large variations may cause a considerable departure.)

Since the snow depth during midwinter, as a rule, exceeds these values, it is not surprising that during the latter part of the winter melting begins from underneath under normal conditions. Usually, the first big cold spell occurs before snowfall, and the bare ground freezes. If snow should fall before the ground freezes, and is of considerable depth the whole winter and does not melt away or compress into ice, it can happen that the ground doesn't freeze at all. Such winters are, as we know, not rare at all. During the winter of 1930–31 the ground was unfrozen over most of the country. In 1928 the author found that all along the coast of Norrland there was no ground frost at all.

We have considered above the case of the ground being covered with an undisturbed snow blanket of normal density and thickness. When we are concerned with roads, the situation is, of course, quite different, for here we have a roadbed, with a thin compact snow cover and drainage with a large snow depth. The roadbed then becomes less insulated and the sides become more insulated than normally.

Before there was general snow clearing of roads, the snow could be quite deep and loose over the roadbed. A thickness of 0.4 to 0.5 m was not at all unusual even on the main roads. This snow covering was very often sufficient to prevent further freezing, and melting started from below. From Figure 84 we see that this was the case in late winter of 1926 in South Sunderbyn and in later winter 1927 in Brännland.

Table 25. Average monthly temperatures ($^{\circ}\text{C}$) for different Swedish localities.

	Nov	Dec	Jan	Feb	March	April
Haparanda	-4.8	-9.5	-10.9	-11.6	-8.0	-1.6
Stensele	-6.1	-11.5	-12.1	-11.5	-7.6	-0.7
Umeå	-2.9	-7.0	-8.0	-8.3	-5.5	+0.2
Osterund	-2.4	-7.4	-8.4	-7.6	-4.7	+0.8
Härnösand	-0.9	-5.2	-6.2	-6.1	-3.4	+1.3
Falun	-0.8	-5.0	-6.0	-5.5	-2.7	+2.7
Karlstad	+1.2	-2.4	-3.4	-3.4	-1.2	+3.8
Stockholm	+1.5	-1.7	-2.9	-3.1	-1.3	+3.4
Linköping	+1.7	-1.5	-2.4	-2.4	-0.5	+4.3
Göteborg	+3.6	+0.7	-0.5	-0.6	+1.1	+5.6
Växjö	+1.8	-1.2	-2.3	-2.3	-0.4	+4.4
Lund	+3.4	+0.5	-0.7	-0.7	+1.2	+5.3

In both cases, melting occurred in February and March, when the snow covering was greatest, and it was still not compacted by thawing weather. In 1926 there was, however, a fairly large increase in the frost depth during the month of April; in both of these places it almost doubled. This cause is, of course, as weather observations show, that thawing weather occurred, followed by a big cold spell. Thus the snow covering became compacted into ice, and its conductivity became much greater, and the subsequent cold spell caused the ground to freeze deeper. (On the other hand, data have shown that at other places where there was no snow covering, there was no further penetration of frost during this cold spell.) The curves for the following year, 1927, do not show any further penetration of the frost line towards the end of winter (only a period of delayed thawing).

Nowadays, with general snow clearing and heavy traffic, the snow covering is very small, at most only a few decimeters, and it is quickly turned into ice during short periods of mild weather. The effect is then that the covering has very little effect, and there is very little retardation and no retreat of the frost line, and that it progresses downward continuously with a decreasing rate. It is then, of course, obvious that the frost depth becomes much greater.

The drainage ditches, however, due to snow removal have a very thick snow covering which insulates the ground underneath so well, that the conduction upward in the snow is much less than the flow of heat to the surface from underneath. Only during winters of very little snow, or during extremely long periods of very low temperature, is the ground under the ditches frozen towards the end of winter. In most cases, and as a rule, the bottom of the ditches are unfrozen, which is practically quite useful, since the melted water during the spring thaw can seep into the ground through the ditch-

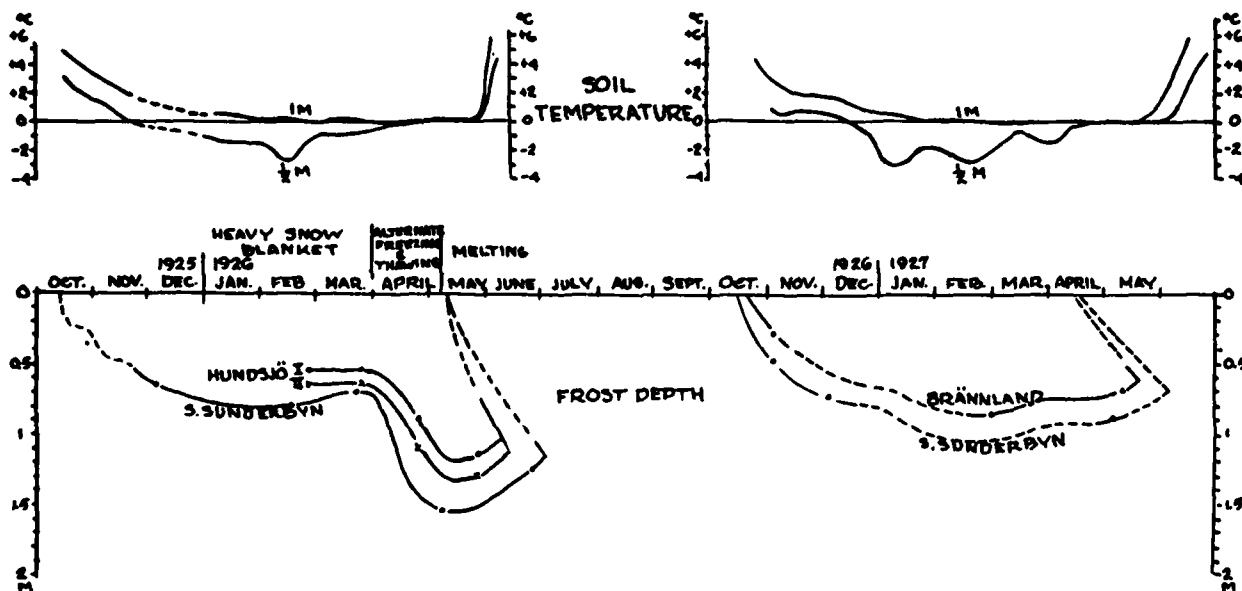


Figure 84. Below: Downward and upward movement of the frost line for different localities in Northern Sweden, determined by measurements in the road base. It should be noted that these are only horse plowed roads. The upper soil temperature curves are for $1\frac{1}{2}$ m depth below the fields in S. Sunderbyn. In 1926, a thaw occurred in April, followed by a severe cold spell. The snow melted on the roads during the thaw and then froze forming a much better heat conducting ice layer. Thus the frost line took a severe drop but not the soil temperature, because in the fields a snow cover still remained.

es, and decreases the difficulties experienced on roads during the thawing. Of course the opposite is true, when due to local conditions, or thin snow covering, the ditches remain frozen throughout the winter.

Of great importance is the melting of the ground from below during the period of general thawing, when the snow disappears and thawing begins to penetrate from above. Quantitative computations are quite simple, since the entire frozen mass now has about the same temperature, just under 0°C , and the heat from underneath is entirely used to melt the ice. Knowing the temperature gradient under the frost line, as well as the temperature conductivity coefficient, the quantity of ice that is melted in unit time from the heat from below can easily be computed, and also, if we know the amount of ice in the frozen soil, the thickness which is melted in unit time can be found.

The heat of melting of ice is 80 cal./gr; if the heat conductivity coefficient is λ and the temperature gradient is dt/dx , the quantity of ice melted Q is

$$Q = \frac{\lambda \cdot dt}{80 \cdot dx} \text{ [gr/cm}^2 \cdot \text{sec].}$$

If the ice content is $\alpha\%$ by weight, the thickness of the melted layer is

$$l = \frac{100 \cdot Q}{\alpha(1 - 0.001 \alpha)} = \frac{\lambda \cdot dt}{0.8 \alpha \cdot (1 - 0.001 \alpha)} \text{ [cm].}$$

In this formula it is assumed that there is a decrease in volume of $\frac{1}{10}$ of the ice when melted. If the frozen soil is very porous, such as a sandy soil, in which the ice does not fill all the pore space, the formula is simplified to

$$l = \frac{100 \cdot Q}{\alpha} = \frac{\lambda \cdot dt}{0.8 \cdot \alpha}.$$

Even in the above cases, when exact computations are not necessary, this simplified formula can be used, since the approximation is usually quite small.

As we have said, we must assume that we know the temperature gradient under the frost line. Direct determinations of this gradient indicate that for central Sweden the gradient towards the end of winter is of the order of $0.02\text{--}0.03^{\circ}\text{C/cm}$. But we cannot say very definitely what the values are for northern Sweden. However, the gradient for certain years can be computed in another manner, by knowing the average surface temperature.

Empirical observations and mathematical computations of the penetration of the temperature variations downward show that towards the end of winter the temperature-depth curve has the approximate shape of a hyperbola, becoming asymptotic at a depth of about 10–20 m. The temperature at the frost line (after melting has begun) is everywhere $= 0^{\circ}\text{C}$, and the remaining variable is then the temperature at the depth where the curve is asymptotic. The temperature at this depth is practically

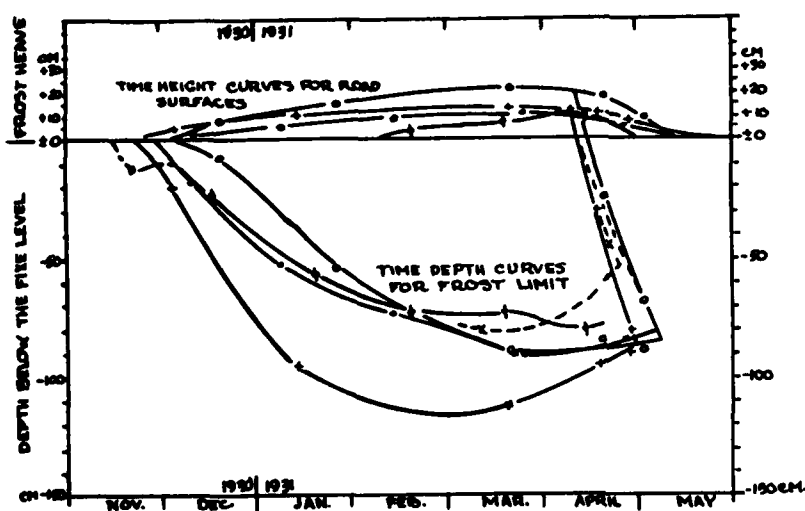


Figure 85. Curves of movement of frost line and amount of frost-heave for various roads in the Trondheim district of Norway. These roads were plowed and open to motor traffic all winter (data courtesy of A. Rode, Trondheim)

everywhere constant, and is about the average temperature of the ground surface (or better yet the average temperature at a depth of a meter) plus the geothermal increment.

The average surface temperature is largely controlled by the air temperature. But it is not identical with it but diverges from it due to different factors, the most important being the snow covering. In regions having a low winter temperature and a large snow covering, the difference can be quite large, due to the insulating effect of the snow. The ground surface temperature during the entire winter stays around 0°C , which is much higher than the air temperature, and the heat gain is greater than the loss that occurs when the snow melts, when during a shorter period the ground is at 0°C while the air is at a higher temperature. In regions where the winter is not so cold, the effect of the snow covering in preserving heat is less, and can in some cases even be reversed, if

the winter temperature is very close to zero so that the heat gain is very small, while the heat loss during thawing is large.

The effect of vegetation is, as a rule, to lower the temperature, since it is a better insulator during the warm period of the year than during the cold. For moss and turf the effect can be quite large, due to the fact that during the summer the surface layer is dry and a poor conductor, and in the fall and winter it is wet and a good conductor. On mossy ground in Norrland one can especially observe how on hot sunny days during the summer there is still some frost left at amazingly small depths.

Table 26 contains average values of air and soil temperatures and their differences for different localities in Sweden, taken from the annual data of the State Meteorological and Hydrographical Institute.

The table shows clearly how the excess temperature between the ground and the air decreases from north to

Table 26. Annual means of air-temperature (t_1) and soil-temperature at $1/2$ m depth (t_2) and their differences ($t_2 - t_1$), for a number of places in Sweden.

Place	Province	Elevation above sea level (m)	Latitude	Observation (years)	Air temperature t_1 ($^{\circ}\text{C}$)	Soil temperature (at $1/2$ m) t_2 ($^{\circ}\text{C}$)	Difference $t_2 - t_1$ ($^{\circ}\text{C}$)
Abisko	Norrbottnens	388	$68^{\circ}21'$	1926-28	-1.17	+0.92	+2.09
Kiruna	Norrbottnens	504	$67^{\circ}51'$	20-28	-1.62	+2.34	+3.96
Braännberg	Norrbottnens	88	$65^{\circ}48'$	25-28	+0.13	+2.84	+2.71
S. Sunderbyn	Norrbottnens	20	$65^{\circ}42'$	25-28	+0.73	+3.05	+2.32
Luleå	Norrbottnens	5	$65^{\circ}37'$	27-28	+1.15	+3.97	+2.82
Gissleås	Jämtlands	320	$63^{\circ}42'$	22-28	+0.40	+2.65	+2.21
Björkä bruk	Västernorrlands	30	$63^{\circ}11'$	24-28	+2.28	+4.75	+2.47
Uppsala	Uppsala	24	$56^{\circ}51'$	19-28	+5.16	+6.72	+1.56
Värpnäs	Värmlands	68	$59^{\circ}24'$	21-28	+5.14	6.17	+1.03
Suburb	Stockholm	14	$59^{\circ}22'$	20-28	+5.96	+7.23	+1.97
Skara	Skaraborgs	115	$58^{\circ}24'$	22-28	+5.30	+6.01	+0.71
Flahult	Jämsköpings	224	$57^{\circ}42'$	19-28	+4.66	+6.10	+1.44
Svalöv	Malmöhus	72	$55^{\circ}55'$	22-28	+6.54	+7.28	+0.74
Alnarp	Malmöhus	10	$55^{\circ}39'$	23-28	+7.55	+8.35	+0.80

south, corresponding to differences in winter climate. In upper Norrland it is about 2–3°C, and in southern Sweden it drops to less than 1°C. The average soil temperatures are therefore—even in the coldest regions of the country—quite high: along the coast of Norrland about plus 3°C, in central Norrland 4–5, in Svea and Gotland 6–8. Thus there is always a considerable positive temperature gradient during the latter part of the winter and in the spring, which causes an appreciable melting from below.

The above explanation for the melting from below during late winter and spring is contrary to the explanation given by Keränen (1929, p. 260–263). Keränen treats thawing as occurring by heat flow from underneath, but assumes that this only takes place during the early part of winter, since most of the stored up excess heat from the summer is still in the ground. When spring comes, all of this heat has been used up and there is therefore almost no conduction of heat from below and there is no melting from below. Though he observed melting from below during the spring as well as the winter, he attributed it to the heating due to the melted water that seeps down. To substantiate his thesis, he gives an example from Sodankylä, which shows a very small amount of thawing from underneath during the spring.

The error in Keränen's reasoning is, of course, that

he only considered the excess heat from the summer, and neglects to take into consideration that constant average yearly temperature which occurs at a certain level, against which, as shown above, the gradient and the rate of thaw is proportional. When the temperature at this level is quite low, the gradient and the consequent thawing is small. This is, of course, the case in Sodankylä. The average air temperature is quite small there (a few degrees below 0°C) and the snow covering is large, and is the reason why the average yearly temperature of the ground is about 1°C.

The temperature gradient can be found from the graphical construction of the temperature curves. With small approximation, we can assume that the slope of the curve at the frost line is proportional to the constant average temperature at a lower level—therefore proportional to the average ground surface temperature, computed in degrees over 0°C plus an increment due to geothermic heat increase, which is about 0.6°C.

In Figure 86, the construction is carried out for average ground temperature of plus 1, 3, 5, 7 and 10°C. Even if the figure is not quantitatively exact—it is only to illustrate qualitatively the relation between the gradient at the frost line and the average ground temperature. The possible error is not large, and the curve then gives an approximate measure of the magnitude of this gradient.

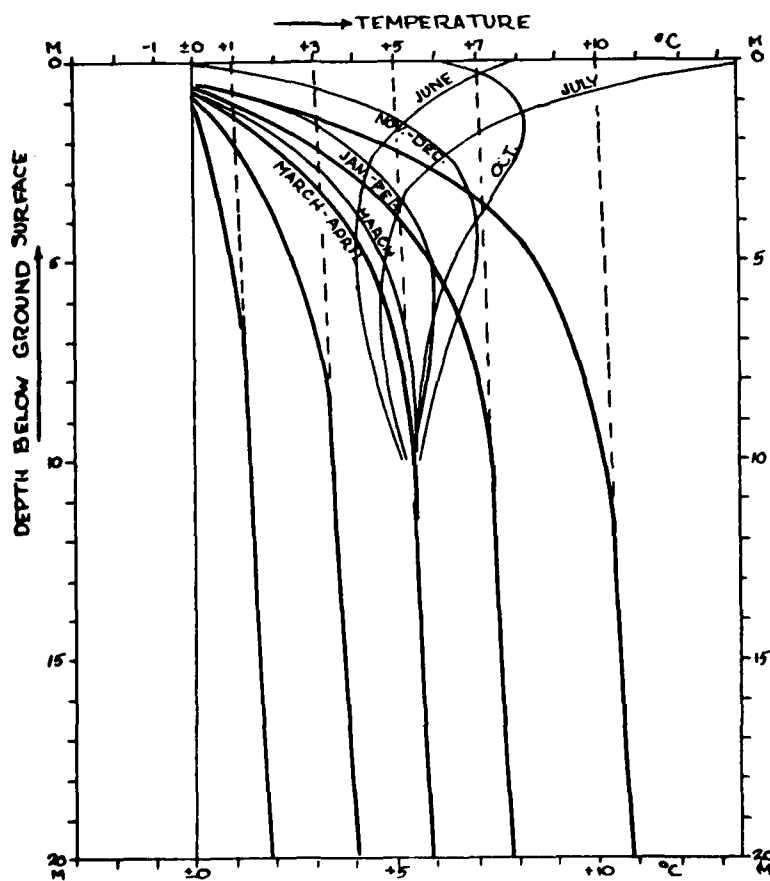


Figure 86. Schematic curves of ground temperatures assuming the following different values of mean yearly temperatures at the ground surfaces: +1, +3, +5, +7 and +10°C. The solid curves are for the periods when there is frozen ground and the dashed lines are for the periods when there is no frozen ground. The slope of the straight portions of the line is the geothermal gradient.

Table 27.

dt/dx a Temperature gradient (°C/cm)	λ		$q = \lambda \cdot (dt/dx)$	$Q = \lambda \cdot (dt/dx) \cdot 80$
	b	c	d	e
	Heat conductivity coefficient		Amount of	Ice melted Q
	cal	cal	heat Q	cm H ₂ O
	(cm × degree × sec)	(cm × degree × day)	(cal/day × cm ²)	(gram H ₂ O/cm ²) per day
0.01	0.002	173	1.73	0.0217
	0.003	259	2.59	0.0324
	0.004	346	3.46	0.0433
0.02	0.002	173	3.46	0.0433
	0.003	259	5.18	0.0648
	0.004	346	6.92	0.0865
0.03	0.002	173	5.19	0.0648
	0.003	259	7.77	0.0972
	0.004	346	10.38	0.130
0.04	0.002	173	6.93	0.0866
	0.003	259	10.36	0.1295
	0.004	346	13.84	0.173

For an average ground temperature of plus 3°C, it is about 0.7°C/m, and for plus 7°C (central Sweden) it is about 2.5°C/m. Notice that this latter value for average temperature of 6–7°C agrees with the empirically determined value for the Stockholm district. From this empirical value we can either—without graphical construction—compute the value of the gradient $dt/dx = G$ for different average ground temperatures (t), or from the general formula:

$$\frac{G}{G_a} = \frac{t + 0.6}{t_a + 0.6} \text{ (approx.)}$$

where G_a is the known gradient at a place of known ground temperature t_a . For the Stockholm district the values $t_a = 7 \pm 1^\circ\text{C}$ and $G_a = 2.5 \pm 1^\circ\text{C/m}$ can be used.

In Table 27, the quantity of ice melted Q is computed for a number of values of gradients (dt/dx) and heat conductivity coefficients λ , which include the range of values found in Scandinavia.

As the end column in the table shows, we can therefore compute, assuming that after thawing has begun and the temperature of the frozen soil is close to zero, the heat per day conducted upwards causing melting is enough to melt 0.03 to 0.12 gr of ice/cm², corresponding to a water layer 0.3–1.2 mm thick. Just how thick a layer of frozen soil this corresponds to, depends upon the quantity of water in the particular frozen soil. The total water content in capillary saturated freezing soil before freezing varies between 30–40% by volume. To this we must add the amount of water sucked up during heaving. This latter quantity, consisting of ice strata, is the part that is frozen longest. If we compute for the small amount of cold that occurs at the under surface of the

thawing—the frozen soil has never been cooled much below 0°C—on the average between $1/2$ and $1/4$ of the original water content is frozen, we get as the minimum ice content (in which the water sucked up is taken as zero):

$$1/4 \times 15 \text{ to } 1/2 \times 40 = 3.5\text{--}10\% \text{ by volume.}$$

As an average value we can take

$$1/3 \times 30 = 10\%.$$

To this we must add the water sucked up during freezing, which can vary from zero up to 100%.

We find then, that the thickness of the frozen soil layer that can be melted per day from underneath, for very sparsely stratified frozen soil is about $(0.03 \rightarrow 0.1 \rightarrow 0.12)/0.1 = 0.3 \rightarrow 1 \rightarrow 1.2$ cm per day. For every stratified frozen soil, the rate of thawing is only one-tenth as large.

For average stratified frozen soil, which shows an appreciable amount of heave, the rate of thawing from below can be taken at about $1/4$ and at the most 1 cm/day. For large frost depths, this highest value can be exceeded, since this part of the frozen soil has less ice and also because the gradient is somewhat larger. For moderate frost depths, and no deeper post-winter freezing, the rates of melting is about $1/2$ cm/day, in ice stratified frozen soil.

It should be pointed out that the above determinations refer to ordinary fairly flat ground. For roadways, other factors may enter, which change the magnitude of the gradient considerably, in either direction. For snow cleared roads on land with a thick snow cover, the frost

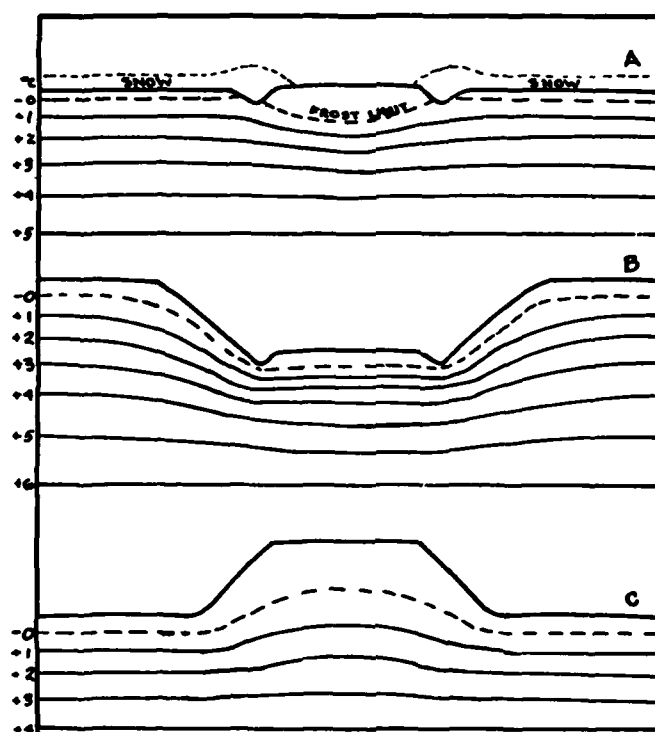


Figure 87. Schematic cross-sections showing the isotherms and the frost line (dashed line) alongside and below roads towards the end of winter. A shows the effect of deep snow cover on the side of a road. The frost line becomes curved downward, causing a closer spacing of isotherms, i.e., a steep temperature gradient and therefore rapid thawing from below. B shows the effect of road cuts—also a steep temperature gradient and rapid thawing from below. C shows the effect of fills—isothers become wider-spaced and a flat temperature gradient and hence slow thawing from below.

depth under the road becomes larger than usual, and therefore—when it is not a question of high banks—the frost line under the surface is warped downwards (Fig. 87a). Due to this the isotherms become pressed together, i.e., the temperature gradient is larger than under the surrounding ground. Cuts have the same effect and if the cut is deep, the effect can be quite large (Fig. 87b). The effect in fills is opposite that of a cut. For normal embankments and normal snow covering this effect is small, but for high banks there is a considerable decrease in the gradient (Fig. 87c). The effects are similar if the road cuts through a steep side slope.

Now how do these computed values of rate of thawing compare with the values obtained by direct observation? From the data given before, the temperatures at different depths, this comparison can be made. The data are given graphically in Figures 84 and 85 and in Table 28, which summarizes the values computed from the curves for the rate of melting below the frost line.

As the table shows, the empirically determined values agree with the computed. The individual variations could certainly be explained if more details were known.

Table 28.

Locality	Maximum rate of melting from below (cm/day)
S. Sunderbyn	0.1
Sör Tröndelag No. 1	0.4
Sör Tröndelag No. 2	1.2
Sör Tröndelag No. 3	0.7
Sör Tröndelag No. 4	0.14

In the case of S. Sunderbyn this is easy to do: the very rapid freezing during the latter part of the winter in 1926 occurred after thawing weather and ice formed on the roads, and was followed by cold, which affected only the roads, and not the surrounding ground—as the temperature observations from cultivated land in the same locality show. Thus a downward warped curvature of the frost line occurred, similar to that in Figure 87a, and in addition a very small increase in the ice content, both factors of which cause a rapid thawing from below.

Table 29. Summary of mechanical analysis of the remolded soils used in this study.

Soil type	See page(s):	Percent by weight									
		>20 (mm)	20-6 (mm)	6-2 (mm)	2-0.6 (mm)	0.6-0.2 (mm)	0.2-0.06 (mm)	0.06-0.02 (mm)	0.02-0.006 (mm)	0.006-0.002 (mm)	<0.002 (mm)
Atterberg's sedimentation method											
Sediments											
1	117	—	0.7	1.2	1.5	2.6	13.0	31.8	27.3	8.5	13.4
5	115	—	0.2	1.4	3.5	20.2	54.3	11.2	3.3	1.0	4.7
7	115,117	—	—	—	—	1.9	15.4	35.1	24.0	10.1	13.5
8	115,117	—	—	—	0.2	15.2	54.2	16.4	6.9	2.3	4.8
10	117	—	—	—	0.5	0.5	4.1	11.4	14.7	21.0	48.0
12	117	—	—	—	0.9	4.0	21.8	52.3	11.9	2.3	6.8
45990	115	—	—	—	0.1	0.2	14.0	19.0	13.4	15.6	37.7
VI: 7	115	—	—	—	0.2	2.3	66.3	18.8	4.3	1.3	7.0
A 84	115,117	—	—	—	0.7	1.0	5.9	38.6	31.1	7.4	15.3
Bw 1	115,117	—	—	—	0.1	0.2	53.9	28.0	8.0	2.4	7.4
Bw 2	115	—	2.3	5.3	10.6	21.1	30.1	16.4	5.0	1.8	7.3
Bw 7	115	1.6	1.0	1.4	4.3	6.5	6.1	11.0	35.6	16.1	16.2
Bw 10	115	—	0.5	10.7	25.0	27.8	13.4	6.2	3.1	1.2	12.1
Bw 13	115	1.1	2.5	5.4	7.4	13.6	27.5	16.9	9.7	4.5	11.2
Bw 14	115	—	—	—	0.3	0.7	6.9	63.0	18.6	3.0	7.5
Bw 19	115	—	—	—	0.5	2.4	14.6	20.7	37.2	15.0	9.6
Bw 21	115	—	—	—	—	0.5	2.2	19.4	54.2	16.4	7.3
Bw 22	115	—	—	—	0.3	0.4	8.4	25.4	40.5	17.1	7.9
Bw 23	115	—	—	—	1.2	13.8	53.2	14.8	9.3	2.9	4.8
Bw 24	115	—	—	—	0.6	0.3	1.4	9.2	34.7	23.0	30.8
Bw 25	115	—	—	—	0.1	0.2	53.8	38.5	3.7	0.8	2.9
Bw 26	115	—	—	—	0.2	0.3	5.0	31.5	32.8	14.0	16.2
GB 2	115	—	—	—	0.3	1.0	6.7	9.4	13.5	29.9	39.2
GB 5	115	—	—	—	2.3	11.9	20.7	19.2	12.4	9.6	23.9
GB 9	115	—	—	—	0.6	5.1	30.7	27.8	16.5	6.5	12.8
GB 15	115	—	—	—	0.2	0.2	6.1	46.8	29.5	5.4	11.8
GB 16	115	—	—	—	0.2	0.4	1.7	16.5	43.3	15.2	22.3
GB 17	115	—	—	—	0.1	0.2	3.0	9.9	18.4	24.2	44.0
GB 18	115	—	—	—	0.7	1.4	5.4	26.8	37.6	11.4	16.7
GB 20	115,117	—	—	—	0.1	2.4	25.9	40.9	16.2	2.7	11.8
GB 21	115	—	—	—	2.9	2.2	2.5	21.4	27.5	13.6	29.9
M 102	115,117	—	—	—	1.6	2.3	8.8	57.9	21.1	2.5	5.8
Moraine soils											
2	—	—	8.8	7.7	7.5	13.4	32.6	13.8	6.4	2.4	7.4
V: 2	—	2.9	9.8	18.4	23.9	18.7	12.1	5.9	3.2	1.0	4.0
V: 4	—	—	9.3	9.0	18.5	24.1	20.4	9.1	4.3	1.1	4.2
B 9, 640	—	9.2	10.7	9.7	19.8	27.0	13.8	5.6	2.01	0.5	1.6
Bw 4	—	6.3	15.6	16.3	13.9	18.2	15.2	7.2	3.2	1.2	2.0

Soil type	See page:	Percent by weight								
		16 (mm)	16-8 (mm)	8-4 (mm)	4-2 (mm)	2-1 (mm)	1-0.5 (mm)	0.5-0.25 (mm)	0.25-0.125 (mm)	0.125 (mm)
Road pavements, Gravel and moraine soilsSieve Analyses (Tyler sieves)										
102	104, Table 9	19.3	11.6	9.2	7.7	7.7	11.1	14.9	7.7	10.8
BI 4	104, Table 9	—	—	—	14.7	21.2	28.6	14.7	20.7	12.6
		—	0.5	2.2	25.7	21.0	16.4	14.6	6.9	
So 4	104, Table 9	—	—	—	29.3	22.8	20.4	9.6	17.6	11.6
		0.5	19.9	34.9	11.9	6.4	5.3	5.2	4.3	
44	104, Table 9	—	—	—	19.5	16.2	15.9	13.1	35.3	17.4
		—	7.3	6.0	8.7	17.0	20.7	14.9	8.0	
Ga 6	104, Table 9	—	—	—	21.8	26.6	19.1	10.2	22.3	32.9
		4.1	5.2	5.8	7.0	7.7	9.6	14.9	12.8	
209	104, Table 9	—	—	—	10.0	12.4	19.1	16.4	42.2	9.4
		38.7	9.8	8.9	8.0	8.3	7.5	6.2	3.2	
1934, I:10	104, Table 9	—	—	—	24.0	21.7	17.9	9.4	27.1	11.5
		36.1	9.9	7.7	8.3	8.2	7.6	6.5	4.1	
29	104, Table 9	—	—	—	21.6	20.0	17.1	10.8	30.4	12.4
		37.5	12.0	6.9	5.2	5.6	8.0	7.8	4.5	
4/11 33, 8 b	104, Table 9	—	—	—	14.6	20.8	20.3	11.7	32.4	12.6
		2.1	17.7	18.0	16.3	10.4	8.7	8.8	5.3	
		—	—	—	22.7	19.0	19.2	11.6	27.5	

Table 29(cont'd). Summary of mechanical analysis of the remolded soils used in this study.

Soil type	See page:	Percent by weight								
		16 (mm)	16-8 (mm)	8-4 (mm)	4-2 (mm)	2-1 (mm)	1-0.5 (mm)	0.5-0.25 (mm)	0.25-0.125 (mm)	0.125 (mm)
Road pavements, Gravel and moraine soilsSieve Analyses (Tyler sieves)										
B1 14	104, Table 9	—	1.7	7.2	19.6	15.5	14.2	13.9	8.9	19.0
		—	—	—	21.7	19.9	19.5	12.4	26.6	
4/11 33, 4	104, Table 9	3.4	8.1	10.2	13.5	16.0	14.0	11.5	6.6	16.3
		—	—	—	24.8	21.7	17.7	10.2	25.3	
47	104, Table 9	19.1	9.7	8.0	9.1	12.5	14.9	9.5	4.1	13.1
		—	—	—	23.1	27.6	17.5	7.6	24.2	
1934, II:1	104, Table 9	—	12.0	11.1	18.0	19.4	14.7	9.3	4.3	11.2
		—	—	—	33.0	25.0	15.8	7.3	19.0	
B1 5	104, Table 9	6.6	11.2	8.2	8.0	10.3	13.3	14.8	7.6	20.0
		—	—	—	15.6	20.2	22.4	11.5	30.3	
201	104, Table 9	43.1	11.0	6.5	5.7	6.6	5.9	5.5	3.5	12.2
		—	—	—	19.6	17.5	16.3	10.4	36.2	

NOTE: Second line for each soil gives the analysis of the material passing the 2-mm sieve.

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Progress of Scandinavian Soil Frost Research from 1935 to 1946

Fil. dr. Gunnar Beskow (Ph.D.)
Swedish Road Institute, Stockholm

Supplement: Some Results of Scandinavian Soil Frost Research 1935-1946

GUNNAR BESKOW

I. INTRODUCTION

During the 12-year period since the original Swedish publication, considerable research work on soil freezing and its mechanical effects has been performed by Norwegian investigators. Experiments concerning the frost depth as a function of total winter coolness, and the speed of frost penetration as a function of heat conductivity and water content have been made.

Publication of these works began in 1937 with a paper by Arne Eriksen of the Norwegian Northern Railroads, and was continued with reports from A. Watzinger, E. Kindem and B. Michelsen, at the Norwegian Technical University in Trondheim; technical-economic conclusions were given by K. Heje of the same University. H. Brudal, T.B. Riise and O.A. Gjörv of the Norwegian Government Highway Board have made important contributions. Sverre Skaven Haug and H. Fleischer of the Norwegian Government Railroads have been especially concerned with the use of moor insulation for railroads.

The extremely cold winters of 1939-40, 1941-42 and 1946-47 caused many unusual frost action occurrences. From 1940 to 1944, the author performed a series of investigations regarding frost action problems of the Swedish railroads, and also problems encountered in heavy foundations. These investigations were of a practical-technical nature, but they also dealt with the same principal questions as the Norwegian researchers referred to above. Another investigation was made to study the effects on frost action of excess water from other than ordinary groundwater sources. In addition, the effect of soil stabilization chemicals on frost action is now the subject of laboratory and field test studies.

II. CALCULATION OF FROST DEPTH IN DIFFERENT MATERIAL

The simplest method of calculating frost depth as a function of time is as follows: For freezing of pure water, i.e., the growing of an ice-layer on the surface

from still water and assuming constant air temperature below 0°C, Stefan gives the following formula:

$$S = \sqrt{\frac{2 \cdot \lambda \cdot T}{e \cdot q}}$$

where S = thickness of ice after time T

t = temperature (in °C)

λ = heat conductivity coefficient of ice

e = density

q = specific heat of ice.

This formula is, of course, an approximation. It doesn't take into account the energy liberated by cooling ice below 0°C nor the heat conductivity between ice surface and air (in reality temperature t should be defined as the surface temperature of the ice). The formula can be adapted for the freezing rate of wet soil, using, of course, the correct constants. The product $e \cdot q$, or the heat of melting per unit of volume depends on the (freezing) water content of soil, and is proportional to that water content in per cent by volume. But the lower the water content, the more important becomes the effect of heat capacity of the soil substance itself.

In case of pure water freezing, the melting energy of ice (80 cal/gr = 72 cal/cm³) is quite large compared to the heat capacity of ice (0.45 cal/cm³); thus at a surface temperature of -10°C (mean ice temperature = -5°C) the heat capacity of ice is responsible only for $5 \times 0.45 = 2.25$ calories against the 72 of melting energy. But in case of a sandy soil with 5% water per unit volume, and a heat capacity of about 0.35 cal/cm³ for the mineral grains, plus 0.025 cal for ice = 0.375 cal/cm³, the proportion would be $5 \times 0.375 = 1.87$ cal. against $0.05 \times 80 = 4$ cal. Thus the heat capacity plays an important role. (When water content = 0, heat capacity is, of course, the only factor.)

Thus it is necessary to place in the formula the sum of the heat of melting and the heat capacity of both soil matter and soil water. That sum the Norwegians call "frost-storing capacity," Q .

Thus,

$$Q = f \cdot 80 + \frac{f}{2} \cdot (f \cdot 0.45 + s \cdot 0.55) \text{ cal/cm}^3$$

where t = surface temperature

f = water content per unit volume

s = mineral matter per unit volume.

The sum within the parentheses represents the heat capacity of (frozen) soil, composed of the heat capacity of ice and of dry soil matter. Pure empirical data (e.g., in case of organic soils) could be substituted for $s \cdot 0.55$. Also one could use an approximate figure for the heat capacity of different soil types (frozen), as the variation is rather small (0.5–0.4 for wet clay, 0.40 for saturated sand, 0.38 for moist sand, and 0.35 cal/cm³ for dry sand).

For most cases (mineral soils, except fat clays) a good approximation which can be used is:

$$Q = f \cdot 80 + \frac{1}{2} \cdot 0.4 \text{ cal/cm}^3.$$

Thus the formula becomes

$$s = \sqrt{\frac{2 \cdot t \cdot \lambda \cdot T}{Q}}$$

where Q = "frost storing capacity."

A. The concept of "cold quantity"

It was assumed that the air temperature t remains constant, a condition which will never be realized in practice. In place of a varying temperature, a *mean temperature* for a given time period should be substituted. This figure is obtained planimetrically from the measured time-temperature curve. Since the formula deals with the product of temperature and time, it is convenient to take this product directly by measuring the *area under the time-temperature curve below the zero (°C) line*. In case of a frost period broken by intervals with temperatures *above* freezing, these periods (i.e., the areas above the zero line) have to be *subtracted* from the area below the zero line. The resulting sum expressed in degree hours is called "*frost quantity*." Because of its fundamental importance, it is valuable to know the "frost quantity" for different localities: to know both the "normal" value and the value for extremely cold winters ("maximum frost quantity"). These data can usually be obtained from the meteorological statistics. The Norwegian Government Railroad Board has edited maps of Norway with equilogical curves for different frost quantities (5,000, 10,000, 15,000, etc., up to 90,000 degree hours) (see Skaven Haug 1944).

B. Complicating factors

The above formulas neglect the following factors: heat exchange conditions between surface and air; radiation (essentially the heat radiation from the surface to

empty space); heat conduction from below; a covering layer of snow; the water freezing conditions in fine-grained soils (only a small part of the total water content is frozen in clay near the frost line). Of these factors, the first two cause some difference between the surface temperature (the uppermost surface, either soil or pavement surface, or surface of covering snow) and the air temperature. The effect may be rather variable, depending upon the frequency of clear or cloudy sky, the height of the sun in winter, the darkness of the surface, etc. In subarctic-continental climate the surface temperature is lower than the air temperature; at Sodankyla in northern Finland the difference is about 1–2°C (referred to in Section V). In less extreme climate the difference is rather small, and may be neglected.

1. Heat conduction from below

A more important factor is the effect of heat conductivity from below. This heat conduction toward the frost line, which reduces the "cold conduction from above," is proportional to the temperature gradient below the frost line. This reduction can be calculated as follows:

Assume a frost depth of 0.5 m, a surface temperature of –6°C, i.e., the mean gradient in the frozen layer is 12°C/m. The gradient below the frost line is 4°C/m (see Fig. 88). Thus the heat conduction from below is $\frac{1}{3}$ of the total heat conduction from the frost line to the surface; that is, freezing is reduced by $\frac{1}{3}$. (This could be visualized graphically as follows: the gradient line from below extended to the soil surface shows the surface

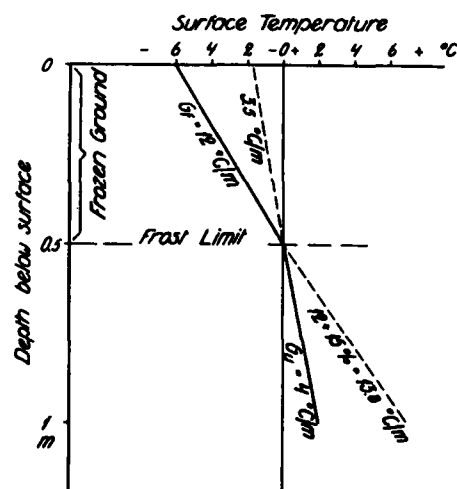


Figure 88. Influence of heat conductivity from below. Ratio of heat conductivity of frozen to unfrozen = $\lambda_f/\lambda_u = 1.15$; thus a corresponding adjustment is made to the actual temperature gradient curves when extended across the frost line.

temperature for equilibrium, i.e., no freezing nor melting, that is at -2°C . Thus the *active frost excess* is only $6 - 2 = 4^{\circ}\text{C}$, or $2/3$ of the surface cold. (See Fig. 88).

Generally the conditions are somewhat more complicated, since for the same soil, at the same moisture content, heat conduction may be somewhat higher in the frozen state. For clay it is about the same, but for coarser soils the heat conductivity in frozen soil is about 10–20% higher (see Section V, subsection A). Thus the equivalent temperature gradient curves in reality are broken at the frost line, as shown in Figure 88. In the given example, a gradient of $12^{\circ}\text{C}/\text{m}$ in the frozen soil corresponds to $115/110 \times 12 = 13.8^{\circ}\text{C}/\text{m}$ in the unfrozen, assuming that the heat conductivity in the frozen soil is 15% higher than in the unfrozen.

Thus in the formula the correction for heat conductivity from below has to be made by substituting for the temperature t , a quantity $t - \delta G$, where δ = depth of frozen layer, G = temperature gradient below frozen layer. (Theoretically, G should really be multiplied by λ_u/λ_f , where λ_u = heat conductivity coefficient for unfrozen soil and λ_f = for frozen soil:

$$t - \frac{\lambda_u}{\lambda_f} \cdot \delta G$$

but as λ_u/λ_f for clay soils is about 1.0, and for sandy soils is about 0.9 or 0.8, this correction is not important when considering other approximations.)

Practically, then, frost depth does not depend only on $t \cdot T$ or "frost quantity." Moderately low temperatures over a long time period do not cause the same frost depth as a severe cold spell over half the time, even though the products $t \cdot T$ are the same. When t/δ reaches the same value as the temperature gradient below the frost line, then $t - \delta \cdot G = \delta \cdot (t/\delta - G)$ becomes zero and no freezing occurs. Such extreme consequences are not unusual, especially when there is heavy snow (see Fig. 84, the curve for "S. Sunderbyn," February–March 1926).

How then can one make a correction for heat conductivity from below in the formula for frost depth as a function of time, when frost depth is increasing and gradient G is decreasing? Any exact mathematical treatment is not possible. An approximation is possible because of the fact that G decreases as winter proceeds in somewhat the same proportion as t/δ . Thus, the correction calculated for the middle or end period of freezing could be extended to the whole period. (For example, at a mid-winter temperature of $t = -8^{\circ}\text{C}$ and a frost depth $\delta = 1 \text{ m}$, $t = -8^{\circ}\text{C}/\text{m}$, and $G = 3^{\circ}\text{C}/\text{m}$, the correction would be $3/8$. For finding the gradient G , for the end of winter. (See Section V, subsection E, and Fig. 86.)

2. Effect of snow

The effect of a layer of snow (or any other material)

on the surface can most simply be taken into account as an equivalent increase in thickness of frozen soil. The rate of heat transfer from the freezing zone (or freezing speed) is proportional to the temperature gradient in the frozen layer, which is $t \cdot \delta_f$, where δ_f = thickness of the frozen layer. When heat has to pass through a top snow layer, temperature gradients in both layers (of course after equilibrium is reached) are inversely proportional to their heat conductivity λ . The snow layer has the same effect as a frozen soil layer of thickness

$$\delta_s \cdot \frac{\lambda_f}{\lambda}$$

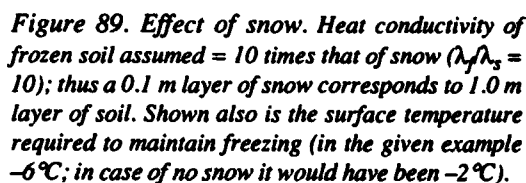
where δ_s = thickness of snow, λ_f and λ_s = heat conductivity of frozen soil and snow respectively. Thus, in the calculation add to the existing soil frost depth the thickness of snow cover, δ_s , multiplied by λ_f/λ_s . Since heat conductivity λ_s of the snow is rather low (about $1/10$ to $1/7$ of that of ordinary mineral soil (Fig. 83 and Table 24), the effect of snow is very large and thus corresponds to a soil layer seven or ten times as thick.

The above approximation, where heat capacity is excluded from the calculations, i.e., heat energy liberation restricted to the frost line, always gives too large an effect of snow cover and the value of equivalent soil thickness should be somewhat reduced. In case of wet soils (except clays), the error is moderate, and thus very small where the calculations deal with moderately thick snow fallen on ground already frozen to some depth (Fig. 89). Assuming that the surface air temperature does not increase sharply, the heat capacity of a frozen soil layer would play a very unimportant role in the freezing formula, and the approximation is nearly correct.

3. Freezing conditions at the frost line

The calculations above are nearly correct for all but the fine-grained soils. In fine silts and especially clays the decrease of freezing temperature (Section II, subsection B) becomes important. When calculating frost depth from temperature readings (isotherms) the 0°C -isotherm may *not* be regarded as the frost line (Fig. 81), as the temperature of the frost line may be some tenths or near -1°C . And in clays quantity of water frozen in the lower frost layers is only a small part (some 10%) of the total water content, the quantity of ice to water successively increasing with decreasing temperature (Section II, subsection B, Fig. 19 and 23). Thus, in the case of clays especially, rather large corrections are necessary.

Concerning the practical use of calculating the frost depth (or freezing speed), its value is essentially for *comparison* (i.e., frost depth in material b when frost is at a given depth in material a, and the necessary properties of both materials are known). Because of the many and serious uncertainties (or approximations) the calcu-



III. "FREEZING RESISTANCE" OF DIFFERENT MATERIALS

In Norway, this matter has been studied extensively. The "freezing resistance" of any layer is expressed in "cold quantity," or $-^{\circ}\text{C}/\text{hours}$. For the top layer, this "freezing resistance" Ω is given from the formula of frost depth, thus

$$\Omega_1 = t \cdot T = \frac{Q_1}{\lambda_1} \cdot \frac{\delta_1^2}{2} \text{ } ^\circ\text{C hr.}$$

$$\Omega_2 = \frac{Q_2}{\lambda_2} \cdot \frac{\delta_2^2}{2} + Q_2 \cdot \delta_2 \cdot \frac{\delta_1}{\lambda_1}$$

λ = heat conductivity coefficient.

$$\Omega_n = \frac{Q_n}{\lambda_n} \cdot \frac{\delta_n^2}{2} + q_n \cdot \delta_n \cdot \sum \frac{\delta_o}{\lambda_o}$$
$$\sum \frac{\delta_0}{\lambda_0}$$

As seen from the formula, the "freezing resistance" Ω for any layer depends upon the thickness and material constants of the layer, as well as its depth below surface, or thickness of covering layers, and their heat conductivity.

In order to obtain small frost depths, it is best to combine a lower layer with high "frost storing capacity" q , i.e., wet moor, with a covering upper layer of material with low heat conductivity (e.g., porous slag, charcoal), the upper layer acting as a heat insulating barrier, saving as long as possible the "heat storage" of underlying water saturated (but of course not in itself frost heaving) material.

A disadvantage is the considerable settlements in the moor material, compression from overlaying masses, elastic compression from passing traffic load, as well as long-time settlement caused from biochemical decomposition within the organic substance. These conditions reduce the applicability of moor layers especially for

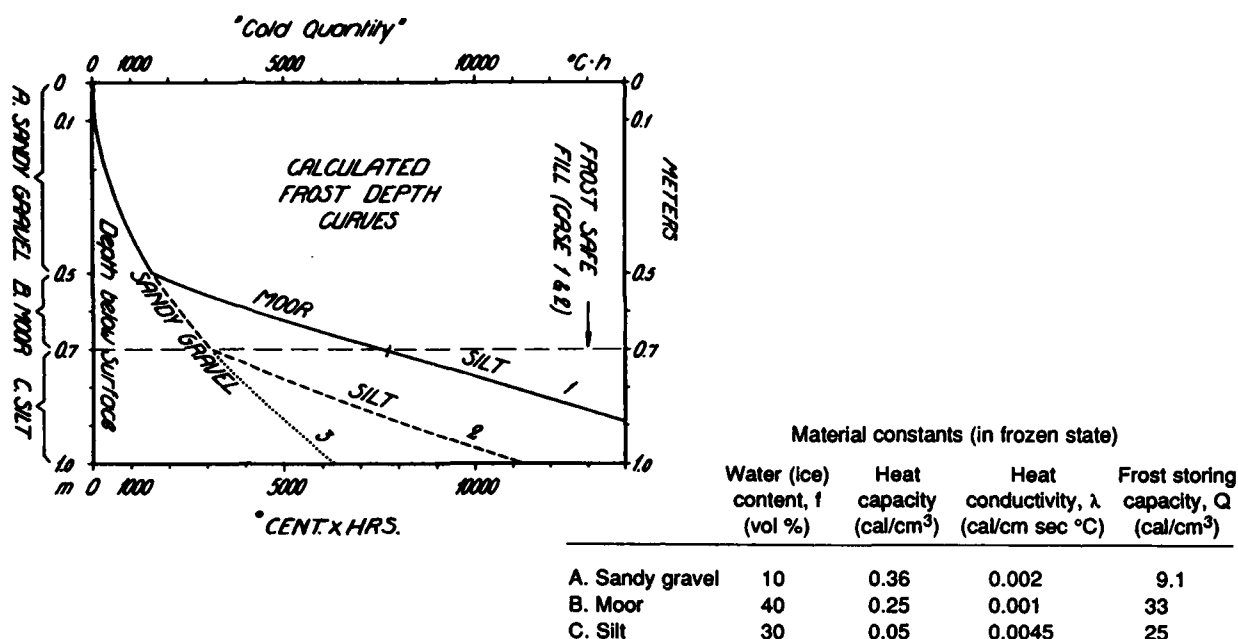


Figure 90. The effect on depth of frost penetration of a 20-cm layer of moist moor (graphical example of Norwegian practice). Curve 1 shows the frost depth for sandy gravel-moor-silt; no. 2 for 70 cm on sandy gravel on silt (without moor layer); no. 3 for sandy gravel alone. For this excavation of 70 cm, the bottom layer of moist moor has extended the period of no frost heaving from 3100 °C × hr (for 70 cm sand only) to 7700 °C × hr. Notice: No reduction made for heat conduction from below. For temperate climate the empirical "cold quantity" ought to be reduced by about 30% which is, however, a very rough approximation.

roads with rigid pavements, which can stand but small deflections. Nevertheless, for railways and gravel roads (the latter rather common in Scandinavia, with its widespread population and easy sources of good material for gravel roads) moor is rather useful. Surely the compression during the first years may be about $\frac{1}{4}$ to $\frac{1}{2}$ of the original thickness, but the use of strongly pre-compressed moor bundles does reduce fill settlement of 10% or less. Both railroads and gravel roads could rather easily be readjusted during the settling period. With a moor layer of about 20–40 cm thickness, its upper surface on about 0.5 m depth, the elastic deformation by traffic is not a disadvantage, either for rails or for gravel road surfaces, even if the latter are perfectly hard and well-bound.

IV. PRACTICAL MEASURES AGAINST FROST DAMAGE

Although this book deals with the basic physics of frost action in soil and not with the techniques of frost damage prevention (these matters being handled in a series of other publications), it may be convenient to conclude the American edition with a brief summary of the practical methods used against frost damage, and the principles for their application.

A. Objectives: Security from frost heaving and frost boils

The troubles caused by frost action may be quite different for different structures. For buildings, as well as heavy road or runway pavements, the heaving in itself causes the destruction. The damage depends mainly on unevenness of heaving, different degrees of heaving at different parts of the structure causing breakage and deflections.

In most cases, a practical objective is to prevent frost heaving from reaching disastrous degrees of inequality. sometimes, as in the case of most buildings, it is necessary to prevent all or nearly all movement, usually by excavation to "frost-safe depth" and refilling with non-frost-heaving material. But in the case of roads, runways and railways, a rather large amount of heaving may be tolerated, the crux being to avoid abruptness (both for sake of pavement and ease of traffic movement). Here one should aim at moderating and smoothing of heave action, e.g., through cutting off the peaks of the frost heave curve. It is not difficult to minimize frost action by excavation and filling when expense doesn't matter, but it takes expert judgment to select the simplest and cheapest way to reach the right reduction in frost heaving.

On the other hand, the main frost damage of roads is due to the softening of soil from melting ice lenses,

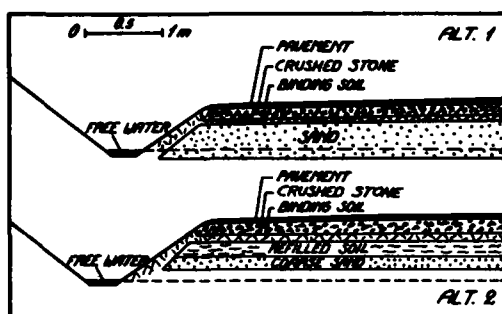


Figure 91. Use of sand layer to prevent frost damage. Alternative 1. Thick sand fill, acting as a non-frost-sensitive back fill and sub-base. Alternative 2. Thin insulating bottom layer of rather coarse sand, acting as an insulation layer, breaking capillary connection with overlying refilled frost sensitive soil material.

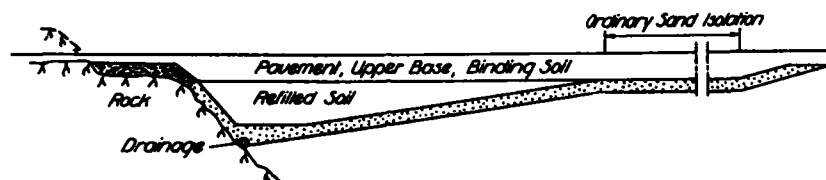


Figure 92. Use of "sand-wedge" construction, for smooth transition between different frost heaving road sections (avoiding abrupt ledges on surface of frozen road). Figure represents a soil cut (strongly frost-heaving) with rock base. Deep transition wedge between rock and frost-heaving soil; also the ordinary sand insulation layer in the soil cut ending with a short wedge.

causing a decrease of bearing strength in the soil. For silts, especially, the mechanical effect of this water excess may be tremendous ("frost boils"). This melting phenomenon is not dealt with in this book; for roads without concrete pavements, it is the really disastrous consequence of frost action. Frost preventive measures practiced in Scandinavia are aimed principally to guard against this softening due to melting of the subgrade soil, thus, e.g., the depth of excavation and insulating back fill are dimensioned purely in order to insure that the bearing capacity is sufficient. In these cases, the back fill thus does function as a sub-base layer.

1. Excavation-Insulation

As already said, this measure has the dual function of both diminishing or preventing frost heaving action, and of forming a sub-base necessary during the subgrade softening in spring. In Sweden, as a rule, the overwhelming most important function is that of being a sub-base, dimensioned for that purpose. The ordinary material is sand or sandy gravel, which is rather common all over the country.

From practice, confirmed by test sections, the min-

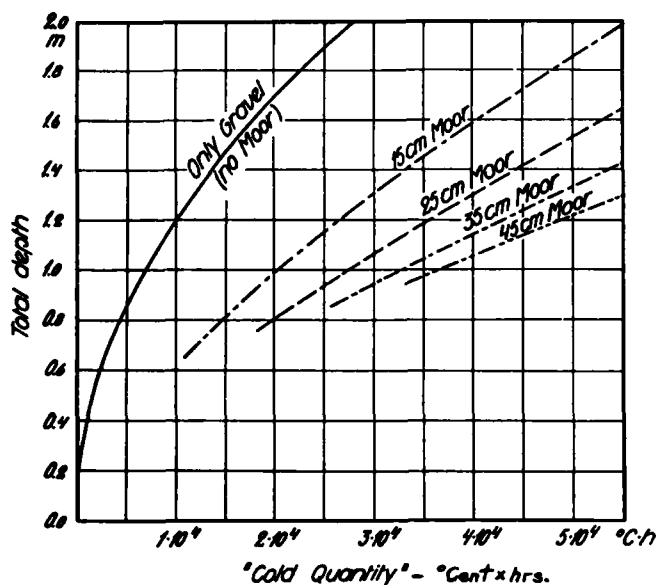
imum depth from road surface to bottom of sandfill necessary for the wheel-load allowed in Swedish highway traffic ($2\frac{1}{2}$ tons—of course for non-rigid pavements) is 60 cm for very sensitive soils, namely, silts and light silty clays, 50 cm for moderately sensitive soils, clays and moraine soils. In the U.S. Army Engineers' Design curves, these figures of base sub-base thickness for $2\frac{1}{2}$ tons wheel-load corresponds to a CBR value of about 1.0 to 1.5% respectively.

The function of the sand fill as a non-frost-sensitive sub-base material is quite simple. But sand is also effective as a rather thin bottom layer, covered with refilled frost-sensitive soil. In this case, the sand layer has a real insulation function, namely, cutting the capillary connection between the overlying soil and the subgrade, thus preventing the capillary water suction to the frost line. It is essential that the sand layer itself does not be-

come water saturated, i.e., it must be situated above the free water table (Fig. 91).

Thus, in practice the sand sub-base may be reduced to a rather thin insulating layer. In order to secure a non-interrupted, open porosity all over the area, a minimum thickness of 12 cm is stipulated. The refill of very frost sensitive material (silt) should not exceed 20 cm thickness.

This principle of a thin sand insulation layer, with frost heaving soil refilled above, presumes that there is such good drainage that the distance from the sand insulation layer down to free water level (during the period of freezing) exceeds the capillarity of the sand, with a margin of safety of at least 10 cm. With the top of the sand at a depth of, e.g., 45 cm (from road surface), and a sand of capillarity 15 cm, the free water must be kept at a depth of $45 + 15 + 10 = 70$ cm; that is, the bottom of ditches or other drainage must be still some centimeters deeper. As the standard depth of ordinary open roadside ditches does average about 60–70–80 cm, it is important to get coarse, clean sand material with low capillarity, of some 7–10–15 cm. For ordinary, thicker sand sub-bases, any sandy material could be



Material constants		
	Water (ice) content, f (vol %)	Heat conductivity, λ (cal/cm sec °C)
Gravel	7	0.0014
Moor	70	0.0025
	Frost storing capacity, Q (cal/cm ³)	
Gravel	7	
Moor	58	

Figure 93. (Redrawn from Brudal, 1945.) Frost safe excavation depth for gravel back fill on a layer of wet moor. Minimum depth to top of moor layer = 0.5 m, calculated for four different dimensions of moor layer. Note: No reduction for heat conduction from below. In reality the necessary depths are somewhat smaller. Compare text (Fig. 90).

used, even a rather fine or "dirty" sand, provided it is non-frost-heaving (see Section III, subsection B). But for a thin sand insulation layer below refilled soil, the sandy material must be a rather coarse, well-sorted, clean sand with low capillarity.

For special purposes, the depth down to sand bottom may be increased. In case of an abrupt step from non-frost-heaving to frost-heaving, e.g., at a rock border in combined rock-soil cuts, a transition may be required, consisting of a "sand-wedge," with its base at about a meter's depth or more, the base depth slowly rising toward the frost heaving side (see Fig. 92).

In the case of railroads and highways with concrete pavements on strongly and unevenly frost-heaving ground, the principle of heat insulating materials, or more generally a fill with highest possible frost resistance, has a special economic value, i.e., to keep a limited depth of excavation. The formulas for calculation of frost depth in such materials and combinations of materials have already been given. A graphical example is shown in Figure 93.

The following practical approximation can be stated: Frost depth in wet moor is about 50-60% of that in moist sand. When used as fill in roads with non-rigid pavements, the minimum thickness of base and pavement must be 0.5 m. A 30-cm layer of moor, down to a depth of 0.8 m, would hold out against freezing about as long as a sand filled down to 1.1 m. Assuming the moor is covered by a layer of very porous slag of say 20 cm, in itself a good sub-base material, the frost resistance of this combination, with a depth of only 0.8 m, would be equivalent to a sand fill of about 1.3- to 1.5-m depth.

In the case of a *deep* sand fill, it is most expedient to keep the sand rather moist, especially near the bottom,

thus increasing the frost resistance. Thus such deeper excavations with sand fill *should not be drained* at the bottom. Also, rather fine sand, with high capillarity, may be chosen, in order to keep the lower part of the sand fill saturated (of course the sand should not be frost heaving). For the top layer of such a deep sand fill, a coarse, well-sorted sand should be chosen, forming a layer with a high percentage of air voids and a low heat conductivity.

The Norwegian Government Railroads use hard molded blocks of moor, 1.0 times 0.5 m, and 0.3, 0.4 or 0.5 m thick (depending on the climatic conditions). Such blocks are easily laid to a uniform moor layer, and the compression settlement in the following years (which is one of the main disadvantages for moor insulation) is both uniform and greatly reduced, from about 50% for ordinary Sphagnum moor blocks. The water content in these Sphagnum moor layers is found to remain rather high and constant, about 80-85% by volume; the organic decomposition is thus extremely slow (no serious effect in half a century).

2. Deep drainage

Usually this method is used for a simple practical purpose: to reduce the height of groundwater where it stands especially high, i.e., increase the depth to groundwater where it is especially small. Thus both frost heaving is moderated and smoothed and frosts boils prevented.

The terrain conditions favorable for deep drainage are roads with side slopes. Here a deep drainage placed along the up slope side will cut the groundwater flow, i.e., force the groundwater level down to the drainage level, this influence extending rather far to the other

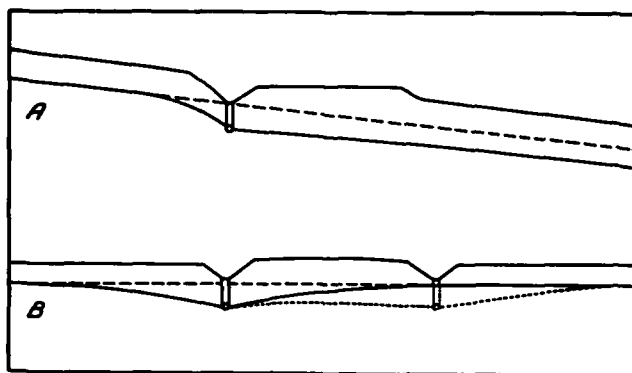


Figure 94. Effect of deep drainage on groundwater level. Broken line: groundwater level before drainage. A—Side slope. Only one drain, along the upper road side. B—No side slope. Drain causes a depression in the groundwater surface, the narrower the depression, the denser the soil. Thus as a rule two drains are necessary.

side. Thus only one deep drain is required along the roadside. But in case of rather flat ground, or the road going in the direction of slope, the drain would cause only a trough in the groundwater level, and a rather narrow one, if the soil has low permeability. Such terrain conditions require deeper double drains, one on each side of the road (see Fig. 94).

Very extensive experience has shown that for *preventing frost boils*, the effective depth of drainage below road surface has to be at least 1.7 m in silts, and 1.5 m in clay and moraine soil. As an example, a road suffering from frost boils at special locations where the soil type and groundwater are unfavorable, such deep drainage along the affected parts will reduce the heaving and eliminate the frost boils. Of course, the effect of deep drainage in each actual case could be predicted more exactly by studying changes in groundwater water depth during the freezing period. Some soil conditions may be especially favorable for deep drainage, such as the existence of a sand layer, which could be cut and emptied of water, after that acting as a natural sand insulation layer.

3. Chemical treatment

Chemical treatment is the newest and most promising method of preventing damage from frost action. For the last few decades it has been known that any substance reducing the thickness of soil water films would reduce frost heaving (see Section III, subsection B). Tests have been made on road fills with such substances as sulphite leach, CaCl_2 , sodium humate, etc. Most substances used for soil stabilization show a frost heave reducing effect, but common chemical soil stabilization is a matter of surface treatment. For frost action, the treatment must affect a rather thick subgrade soil layer, and that raises practical problems. The substance must

invade at least a half meter of soil. A thorough mechanical mixing would be much too expensive, and a suction-diffusion method must bring the chemical into the soil, i.e., the substance must possess some solubility in water. It must be movable in the soil-water system, and yet gradually become absorbed in order to avoid washing out with time. Systematic research on this object is now in progress.

4. Frost-safe foundations for buildings

The first rule is to extend the foundation down to "frost-safe depth" (or to non-frost-heaving subgrade bottom, where such a level is above the frost line). But even if the foundation is laid deep below frost line, heaving may occur by "side grasp" action. If the ground alongside the foundation heaves, and the space next to the foundation wall is packed with hard-freezing fine soil, the connection between frozen soil and foundation wall may be strong enough to cause the foundation and building to follow the frozen soil crust as the soil heaves. Of course, this action depends largely on the shape of foundation wall surface. Where it is rough and uneven this "side grasp" effect is highly facilitated. Also, where the foundation wall shows some inward slope, heaving occurs. We can thus make a general rule: Between the foundation wall and the soil, a fill of non-hard-freezing material should be applied preferably coarse sand or fine gravel. This vertical insulation wall permits the heaving movement of the side ground to proceed, leaving the foundation unaffected. The thickness needed is about 25–30 cm. This sideward insulating fill would be needless where the foundation wall surface is smooth and slopes slightly outwards with depth. For example, concrete foundation blocks for columns, etc., in frost heaving ground should be given a pyramidal shape with a side-slope of some 10° , in which cases no sand insulation is needed.

In reference to the frost-safe depth, one has to consider not only the climate and the soil, but also the heat conducted by the foundation wall itself. For example, a footing block for a column construction may cause a serious depression of the frost line, especially in cases where the concrete block extends above the grass and snow cover.

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